

THE NATIONAL EXAMINATIONS COUNCIL OF TANZANIA



**CANDIDATES' ITEMS RESPONSE ANALYSIS FOR
ACSEE 2015**

141 BASIC APPLIED MATHEMATICS

THE NATIONAL EXAMINATIONS COUNCIL OF TANZANIA



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ACSEE 2015**

**141 BASIC APPLIED MATHEMATICS
(School Candidates)**

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FOREWORD

The National Examinations Council of Tanzania is delighted to issue this booklet on the analysis of the candidates' responses to the Basic Applied Mathematics items that were examined in the Advanced Certificate of Secondary Education Examination (ACSEE) 2015. This booklet has been produced in order to give feedback to students, teachers and other education stakeholders on how the candidates responded to the examination questions.

The analysis of the candidates' responses indicates that, the candidates had good performance on the questions that were set from the topics of Statistics, Linear Programming, Computing Devices and Algebra; average performance on the questions that were set from the topics of Probability, Differentiation and Functions and weak performance on the questions that were set from the topics of Matrices, Trigonometry and Integration.

The reasons which have contributed to the candidates' weak performance on the topics of Matrices, Trigonometry and Integration include; inability of candidates to apply the technique of substitution and the standard formula to evaluate indefinite integrals of polynomials, inadequate knowledge and skills to perform matrix operations, failure of candidates to use trigonometrical identities in answering questions, lack of skills to sketch graphs, poor algebraic/arithmetic skills and occurrence of sign errors that affected the quality of their answers.

The feedback given in this report will enable the education stakeholders to identify proper measures to be taken in order to improve candidates' performance on Basic Applied Mathematics in forthcoming examinations administered by the Examinations Council.

The Council will highly appreciate comments and suggestions from students, teachers, education stakeholders and the public in general, that can be used to improve future writing of this booklet.

Lastly, the Council would like to thank all the Examination Officers, Examiners and others who contributed in the preparation of this report.



Dr Charles Msonde
EXECUTIVE SECRETARY

1.0 INTRODUCTION

This booklet contains the analysis of the candidates' responses for the Basic Applied Mathematics items that were examined in the ACSEE 2015. The analysis highlights the strengths and weakness that were noted while marking the candidates' scripts. The National Examination Council of Tanzania produces these booklets in order to provide feedback to students, teachers and other education stakeholders on the performance of the candidates on this examination.

The Basic Applied Mathematics paper consisted of ten (10) compulsory questions that were set in accordance with the 2011 Examination Format, which was derived from the 2010 Revised Syllabus. Each question carried ten (10) marks.

A total of 17,462 candidates sat for the Basic Applied Mathematics examination in 2015, out of which 12,934 (74.10%) candidates passed. In 2014, a total of 14,742 candidates sat for the examination, out of which 11,370 (77.39%) candidates passed. This represents 3.29 percent drop in the number of candidates who passed.

The analysis for each item is presented in the next section. It comprises a brief description of the requirement of the item and the performance of the candidates. The possible factors that contributed to the good and poor performance of the candidates are pointed out and illustrated by using samples of candidates' responses. The candidates' performance in each question was categorized as good, average, or weak if the percentage of candidates who scored 30 percent or more of the marks allocated for the question lies in the intervals 50 – 100, 30 – 49, 0 – 29 respectively.

The analysis has also shown the topics with weak, average and good performance. Moreover, it has shown the factors which have contributed to the lower performance in the 2015 Basic Applied Mathematics examination as compared the 2014 examination results and concluded by putting forward recommendations to raise the standard of performance in this subject.

2.0 ANALYSIS OF QUESTIONS

2.1 Question 1: Calculating Devices

In this question, the candidates were required to evaluate the following items with the help of a calculator and write the answers correct to 2 decimal places:

(a) $\cos^{-1}\left(\frac{2}{3}\right) + \sin^{-1}\left(\frac{3}{4}\right)$.

(b) $\sqrt[3]{8 \sin 25^\circ \cos 55^\circ}$.

(c) $\log_8 17 - \ln\left(\frac{5}{12}\right)$.

(d) $T(t) = 280 + 920e^{-0.9108t}$ at $t = 10$ given that $e \cong 2.72$.

(e) The number of ways for 20 people to be seated on a bench if only 5 seats are available.

(f) The value of the function $f(x) = \left(1 + \frac{1}{x}\right)^x$ when $x = 10, 100, 1000, 10000$ and hence comment on the value of $f(x)$ when x gets very large.

This question was attempted by 96.2 percent of the candidates. Many candidates (66.9%) scored from 3 to 10 marks with 0.7 percent of them scoring all the 10 marks, indicating that the question had good performance.

The candidates who scored full marks in this question had the necessary mathematical knowledge and skills to use scientific calculators in performing computations and rounded the answers to 2 decimal places as required, see Extract 1.1.

Extract 1.1

$$1. \quad a) \cos^{-1}\left(\frac{2}{3}\right) + \sin^{-1}\left(\frac{3}{4}\right) = 96.78^\circ$$

$$b) \sqrt[3]{8 \sin 25^\circ \cos 55^\circ} = 1.247 \approx 1.25$$

$$c) \log_8 17 - \ln\left(\frac{5}{12}\right)$$

$$= \frac{\log 17}{\log 8} - \ln 5 + \ln 12$$

$$= 2.238$$

$$\log_8 17 - \ln\left(\frac{5}{12}\right) = 2.24$$

$$1d. \quad T(t) = 280 + 920e^{-0.1108t}$$

$t = 10$
 $e \approx 2.72$

$$T(10) = 280.01$$

$$e) \quad {}^{20}P_5 = \frac{20!}{(20-5)!} = \frac{20 \times 19 \times 18 \times 17 \times 16 \times 15!}{15!}$$

$$= 1860480 \text{ ways}$$

$$f) \quad f(x) = \left(1 + \frac{1}{x}\right)^x$$

$$* f(10) = 2.59$$

$$f(100) = 2.70$$

$$f(1000) = 2.72$$

$$f(10000) = 2.72$$

when x gets very large, $f(x)$ become ≈ 2.72

Extract 1.1 is a sample answer from one of the candidates who was able to use a scientific calculator correctly in working out the required answers in all the items of this question.

However, some candidates could not use calculators to compute the values of items (a) to (f) correctly. The analysis shows that 5.3 percent of the candidates who attempted this question scored a 0 mark. They provided incorrect answers in all the items of this question, indicating that they lacked knowledge and skills on how to use scientific calculators appropriately.

The analysis also shows that some candidates scored low marks in this question because they did not adhere to the instruction of writing the answers in each item correctly to 2 decimal places. For instance the candidate in Extract 1.2 computed the values of items (a) to (f) correctly but failed to write the final answers in the required number of decimal places.

It was noted that items (c) and (e) were poorly performed as compared to others. For example, in item (c), a number of candidates wrote the answer as “1.10” instead of 2.24. These candidates lacked knowledge on logarithm and hence failed to change $\log_8 17$ to either log base 10 or “e”. They did not realize that in scientific calculators they could not work with log base 8. It was also observed that some candidates were mixing the concepts of permutation and combination in part (e). They were computing ${}^{20}C_5$ instead of ${}^{20}P_5$ as the number of ways for the 20 people to be seated on a bench with 5 seats. This shows that, they did not know how the concepts of combinations and permutations are applied in solving daily life problems.

Extract 1.2

$$1. a) \cos^{-1} \frac{x}{y} + \sin^{-1} \left(\frac{y}{x} \right) = \underline{\underline{96780.1}}$$

$$b) \sqrt[3]{8 \sin 25^\circ \cos 55^\circ} = \underline{\underline{1247.0}}$$

$$c) \log_k 17 - \ln \left(\frac{5}{12} \right) =$$

$$d) \int t = 280 \text{ at } t = e^{-0.9101t} = \underline{\underline{280101.3}}$$

$$e). 5 \times 20 = 5 \times 4 \times 3 \times 2 \times 1 = 120 \text{ ways}$$

$$f) f(x) = \binom{x+1}{x}$$

$$\text{for } x=10, \text{ Value of } f(x) = 2594.$$

$$x=100, \text{ Value of } f(x) = 2705.$$

$$x=1000 \text{ Value of } f(x) = \del{2716} 2717$$

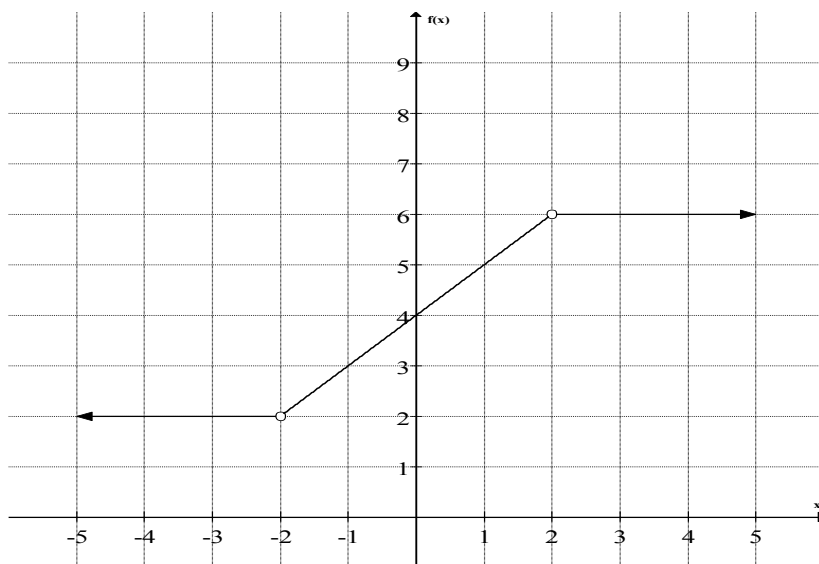
$$x=10,000 \text{ Value of } f(x) = 2718.$$

In Extract 1.2, the candidate computed the answers for items (a), (b), (d) and (f) correctly but could not approximate them to 2 decimal places as required. For example, in item (a), the candidate wrote the answer as 96780.1 instead of 96.78 and in item (d) wrote 280101.3 instead of 280.10.

2.2 Question 2: Functions

This question had parts (a), (b) and (c). In part (a), the candidates were required to find the coordinates of the points where the line $y - 2x + 5 = 0$ meets the curve $3x^2 - 4y^2 = 10 + xy$. In part (b), they were given the

following graph of a function $f(x)$ and then required to use it to determine:
 (i) the function $f(x)$ and (ii) the domain and range of $f(x)$.



In part (c), the candidates were required to find the asymptotes and the intercepts of the function $f(x) = \frac{3x - 7}{x + 2}$ and then to sketch its graph.

This question was attempted by 86.3 percent of the candidates of which 29.5 percent scored from 3 to 10 marks and among them 0.1 percent scored all the 10 marks. This question was therefore averagely performed.

The candidates who performed well in part (a), were able to substitute the equation of the line $y - 2x + 5 = 0$ into the curve $3x^2 - 4y^2 = 10 + xy$ to obtain the quadratic equation $3x^2 - 17x + 22 = 0$ which was essential in determining the coordinates of intersection $\left(\frac{11}{3}, \frac{7}{3}\right)$ and $(2, -1)$. As illustrated in Extract 2.1(a), these candidates had adequate knowledge and skills of solving simultaneous equations that involves linear and quadratic equations.

Extract 2.1(a)

Qn2(a) $\left\{ \begin{array}{l} 3x^2 - 4y^2 = 10 + xy \quad \text{--- (i)} \\ y - 2x + 5 = 0 \quad \text{--- (ii)} \end{array} \right.$

from equation (ii)

$$y = 2x - 5 \quad \text{--- (iii)}$$

Substituting (iii) into (i).

$$3x^2 - 4(2x - 5)^2 = 10 + x(2x - 5)$$
$$3x^2 - 4(4x^2 - 20x + 25) = 10 + 2x^2 - 5x$$
$$3x^2 - 16x^2 + 80x - 100 = 10 + 2x^2 - 5x$$
$$3x^2 - 16x^2 - 2x^2 + 80x + 5x - 100 - 10 = 0$$
$$-15x^2 + 85x - 110 = 0$$
$$x = 2 \quad \text{or} \quad x = \frac{11}{3}$$

\therefore But for $y = 2x - 5$

$$y = 2(2) - 5 \quad \text{or} \quad y = 2\left(\frac{11}{3}\right) - 5$$
$$y = -1 \quad \quad \quad y = \frac{7}{3}$$
$$(x, y) = (2, -1) \quad \text{or} \quad \left(\frac{11}{3}, \frac{7}{3}\right)$$

\therefore Coordinates of point where lines meet are at $(2, -1)$ and $(\frac{11}{3}, \frac{7}{3})$.

In Extract 2.1 (a), the candidate had good algebraic skills that enabled him/her to obtain the correct points of intersection of the line and the curve.

It was noted that, some candidates managed to substitute the linear equation $y - 2x + 5 = 0$ into the curve $3x^2 - 4y^2 = 10 + xy$ to obtain $3x^2 - 4(2x - 5)^2 = 10 + x(2x - 5)$ but they were unable to successfully complete answering this item because they could not expand the brackets to obtain a simplified quadratic equation which was important for subsequent steps. It was also noted that, while substituting the linear equation into the given curve, some candidates forgot to square the term $(2x - 5)$ and hence obtained the incorrect equation $3x^2 - 4(2x - 5) = 10 + x(2x - 5)$ and eventually ended up with wrong coordinates. It was further noted that several candidates completely lacked knowledge and skills of solving simultaneous equations. Extract 2.1(b) illustrates this case.

Extract 2.1(b)

2a	<p>Solution From $y - 2x + 5 = 0$ $y = 2x + 5$ When $x = 0$ $y = 5$ When $y = 0$ $x = -2.5$</p> <p>Then $3x^2 - 4y^2 = 10 + yy$ $3(-2.5)^2 - 4(5)^2 = 10 + 5x - 2.5$ $18.75 - 100 = 10 + 5x - 2.5$ $18.75 - 100 \geq 10 + -12.5$ $-81.25 = 22.5$</p>
----	---

In Extract 2.1(b), the candidate failed to solve the given simultaneous equations. The candidate instead computed the x and y intercepts using the equation of the line and then substituted the values obtained in the equation of the curve and hence ended up with meaningless calculations.

Majority of the candidates attempted parts (b) and (c) of this question, but only a few managed to get correct answers. The candidates who scored full marks in both part (b) and (c) were able to identify the required function from the given graph, state its domain and range correctly and sketched the correct graph of the given rational function. Extract 2.2(a), is a sample answer illustrating how these candidates provided correct responses in both parts (b) and (c).

Extract 2.2(a)

2(b)(i)	$g(x) = \begin{cases} 2 & \text{if } x < -2 \\ x+4 & \text{if } -2 < x < 2 \\ 6 & \text{if } x > 2 \end{cases}$
(ii)	Domain is a set of all real values except -2 and 2 . Range is a set of all real values provided $2 < y < 6$

(c) Vertical asymptote.

$$x+2=0.$$

$$x=-2$$

∴ Vertical asymptote = -2

Horizontal asymptote

$$y = \frac{\frac{3x}{x} - \frac{7}{x}}{\frac{x}{x} + \frac{2}{x}} \quad \lim_{x \rightarrow \infty}$$

2c. $y = \frac{3 - 7/x}{1 + 2/x} \quad \lim_{x \rightarrow \infty}$

Substituting $x = 0$.

$$y = \frac{3-0}{1+0}$$

$$y = 3$$

∴ Horizontal asymptote is 3.

∴ y-intercept ; $x = 0$.

$$y = \frac{3(0) - 7}{0 + 2}$$
$$= \frac{0 - 7}{2}$$

$$y = -\frac{7}{2}$$

x-intercept ; $y = 0$.

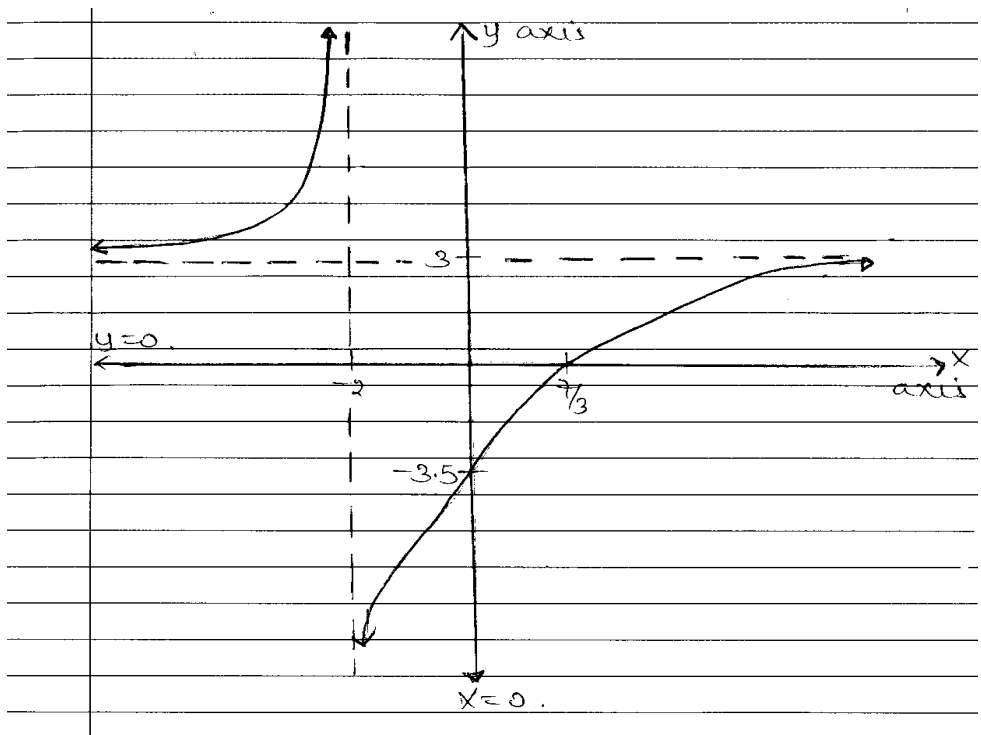
$$y = \frac{3x - 7}{x + 2}$$

$$0 = \frac{3x - 7}{x + 2}$$

$$0 = 3x - 7$$

$$7 = 3x$$

$$x = \frac{7}{3}$$



Extract 2.2(a) shows that the candidate had sufficient knowledge that enabled him/her to identify the function and to state the domain and the range of the function correctly. The candidate also applied knowledge of intercepts and asymptotes correctly in sketching the graph of the rational function.

The candidates who scored a 0 mark in part (b) were not able to identify the function $f(x)$. They did not recognize that the graph of $f(x)$ had some gaps and therefore the function should be defined differently in the domain

$$x < -2, -2 < x < 2 \text{ and } x > 2 \text{ as } f(x) = \begin{cases} 2 & \text{for } x < -2 \\ x + 4 & \text{for } -2 < x < 2. \\ 6 & \text{for } x > 2 \end{cases}$$

It was noted that some candidates managed to identify the constant functions for the domain $x < -2$, and $x > 2$ but failed to find the function over the domain $-2 < x < 2$. They did not realize that for the interval $-2 < x < 2$ they were supposed to find the equation of the straight line joining points $(-2, 0)$ and $(2, 0)$. Furthermore, several candidates were stating the domain as the set of all real numbers i.e. $x = \{x \in \mathcal{R}\}$ instead of

$x = \{x \in \mathbb{R} : x \neq 2, 2\}$ and the range as $y = \{y \in \mathbb{R}\}$ instead of $2 \leq y \leq 6$, indicating that these concepts were not fully understood. The candidates who performed poorly in part (c), lacked the skills to find asymptotes, intercepts and draw graphs. A sample answer, illustrating how the candidates failed to answer parts (b) and (c) is shown in Extract 2.2(b).

Extract 2.2(b)

2b i, $f(x)$
 Let $f(x) = y$; $y = \{2, 6\}$
 $y = \{\text{All real number}\}$
 $\therefore f(x) = \{\text{All real numbers}\}$
 $\therefore f(x) = \{2, 6\}$

ii, Domain = value of x-axis
 = $\{\text{All real number}\}$

Range = values of y-axis
 = $\{2, 6\}$
 \therefore Range = $\{2, 6\}$

\therefore The Domain and range of $f(x)$ are $\{\text{All real number}\}$ and $\{2, 6\}$ respectively

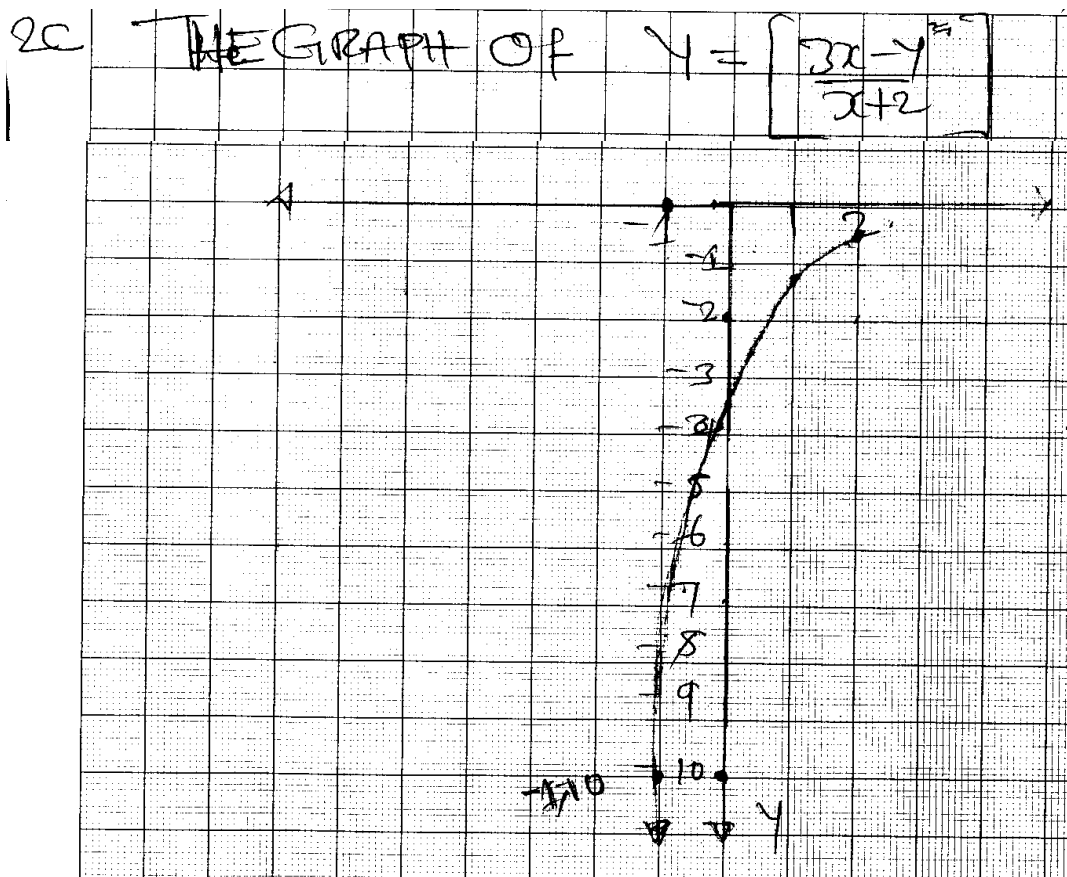
2c

Asymptotes
 Let $f(x) = y$
 $y = \frac{3x-7}{x+2}$

$y = \frac{3x-7}{x+2}$

Table Value

$\frac{3x-7}{x+2}$	-1	0	1	2
y	-10	-3.5	-1.3	0.25



In Extract 2.2(b), the candidate generalizes that the domain and the range of the function are real numbers instead of looking at the nature of the function. Also instead of finding the horizontal and the vertical asymptotes, the candidate created a table of values contrary to the requirement of the question to sketch the incorrect graph.

2.3 Question 3: Algebra

This question had parts (a) and (b). In part (a), the candidates were given the series $-1 + 1 + 3 + \dots$ and were required to;

- (i) Express it in the form $S_n = \sum_{r=1}^n f(r)$.
- (ii) Give one reason as to whether the series is an arithmetic or a geometric progression.
- (iii) Determine the value of n for which $S_n = 575$.

In part (b), the candidates were required to find the possible values of the first term in a geometric progression whose second term exceeds the first term by 20 and the fourth term exceeds the second term by 15.

This question was attempted by 78.3 percent of the candidates, out of which 50.1 percent scored from 3 to 10 marks with 0.4 percent of them scoring full marks, showing that the performance for this question was good.

In part (a)(i), several candidates were able to express the given series in sigma notation as $\sum_{r=1}^n (2r - 3)$. In part (a)(ii), a number of candidates were able to recognize that the series given in part (a)(i), is an arithmetic progression with the common difference $d = 2$ and the n^{th} term $A_n = 2n - 3$. In part (a) (iii), a number of candidates were also able to determine the value of n for which $S_n = 575$. The candidates applied the formula for the sum of the first n terms of an arithmetic progression to formulate the equation $575 = \frac{n}{2}[2 \times -1 + (n - 1) \times 2]$ and then solved it correctly to obtain the required value of $n = 25$.

In part (b), only a few candidates were able to formulate the equations $ar - a = 20$ and $ar^3 - ar = 15$ from the given information and then solved the equations successfully to obtain the values of the common ratio $r = \frac{1}{2}, -\frac{3}{2}$ and the values of the first terms $a = -40, -8$ as required.

Extract 3.1 is a sample answer from one of the candidates who answered this question correctly.

Extract 3.1

3(a) Given $-1 + 1 + 3$

$$(i) S_n = \sum_{r=1}^n f(r)$$

the series is Arithmetic progression
since the common difference $= d = 2$.

$$\text{from } A_n = A_1 + (n-1)d$$

$$= -1 + (n-1)2$$

$$= -1 + (n-1)2$$

$$= -1 + 2n - 2$$

$$= -1 - 2 + 2n$$

$$A_n = 2n - 3$$

$$A_n = f(r) = 2n - 3 = 2r - 3$$

$$\therefore S_n = \sum_{r=1}^n 2r - 3$$

2(a) (ii) The Series is Arithmetic Since there is the common difference (d)

$$d = A_2 - A_1 = A_3 - A_2$$

$$= 1 - (-1) = 3 - 1$$

$$d = 2 = 2$$

$$(iii) S_n = 575 = \frac{n}{2} (2A_1 + (n-1)d)$$

$$575 = \frac{n}{2} (2(-1) + (n-1)2)$$

$$575 \times 2 = n(-2 + 2n - 2)$$

$$= n(-2 + 2n - 2)$$

$$575 \times 2 = n(2n - 4)$$

$$575 \times 2 = 2n^2 - 4n$$

$$0 = 2n^2 - 4n - 1150$$

$$\text{on solving } n = 25 \text{ or } -23$$

Since number (n) can't be negative
hence $n = 25$.

$$3b) \quad a_2 - a_1 = 20.$$

$$a_4 - a_2 = 15$$

for Geometric progression

$$a_n = a_1 r^{n-1}$$

$$a_2 = a_1 r^{2-1} = a_1 r$$

$$a_4 = a_1 r^{4-1} = a_1 r^3$$

$$a_1 r - a_1 = 20 \quad \text{--- (i)}$$

$$a_1 (r-1) = 20 \quad \text{--- (i)}$$

$$\text{from } a_4 - a_2 = 15$$

$$a_1 r^3 - a_1 r = 15$$

$$a_1 (r^3 - r) = 15 \quad \text{--- (ii)}$$

$$\text{take (ii)} \div \text{(i)}$$

$$\frac{a_1 (r^3 - r)}{a_1 (r-1)} = \frac{15}{20}$$

$$\frac{r^3 - r}{r-1} = \frac{15}{20}$$

$$\frac{r-1}{r(r^2-1)} = \frac{20}{15}$$

$$\frac{r-1}{r(r-1)(r+1)} = \frac{20}{15}$$

$$\frac{\cancel{r-1}}{r(\cancel{r-1})(r+1)} = \frac{20}{15}$$

$$2(b) \quad \frac{1}{r(r+1)} = \frac{20}{15}$$

$$15 = 20r(r+1)$$

$$15 = 20r^2 + 20r$$

$$0 = 20r^2 + 20r - 15$$

on solving $r = 0.5$ or -1.5

from $C_2 = C_1 r = C_1 r^{0.5}$ or $C_1 r^{-1.5}$

$$C_2 = 0.5C_1 \quad \text{or} \quad -1.5C_1 = C_2$$

from $C_2 - C_1 = 20$

$$0.5C_1 - C_1 = 20$$

$$-0.5C_1 = 20$$

$$C_1 = -40$$

or when

$$C_2 = -1.5C_1$$

from $C_2 - C_1 = 20$

$$-1.5C_1 - C_1 = 20$$

$$-2.5C_1 = 20$$

$$C_1 = -8$$

\therefore Possible values of C_1 are -8 or -40

In Extract 3.1, the candidate was able to recall and apply correctly the formula for the n^{th} term of arithmetic and geometrical progressions and the formula for the sum of the first n terms of arithmetic progression in answering question 3.

About 50 percent of the candidates who attempted this question scored low marks from 0 to 2.5 with 13.1 percent of them scoring a 0 mark. The candidates who scored a 0 mark in part (a)(i), could not find the general term of the series and as a result failed to express the series in sigma notation. In part (a)(ii), these candidates did not recognize that the series in part (a)(i) is an arithmetic progression, indicating a lack of understanding of the basic concepts of series. In part (a)(iii), the candidates were unable to

apply the formula $S_n = \frac{n}{2}[2A + (n-1)d]$ to obtain the quadratic equation $2n^2 - 4n - 1150 = 0$ which was a key requirement in finding the required value of n . The candidates failed to obtain this quadratic equation because some expanded the brackets wrongly and others made arithmetic and sign errors. The candidates failed to answer part (b) correctly because they could not formulate the equations involving the first, second and fourth terms of the geometric progression from the given information, see Extract 3.2.

Extract 3.2

3	(a) <u>Soln</u>
	$-1 + 1 + 3 \dots$
	(i)
	From $S_n = \sum_{r=1}^n f(r)$
	$S_n = \sum_{r=1}^3 f(r)$
	$\therefore = S_n \sum_{r=1}^3 1$
	(ii) The series in is an geometric progression due to its formula $\sum_{r=1}^n f(r)$.
	(b) <u>Soln</u>
	From G_1
	$G_1(n-r')r$
	$S_n = G_1 = G$
	$G_2 = G_1r$
	$G_3 = G_2r$
	$S_n = G_1 = G_2r + G_1r$
	$= 20 + 15 = 35$
	\therefore First term = 35.

Extract 3.2 is a sample answer from one of the candidates who failed to apply the concepts of arithmetic and geometric progressions correctly in answering question 3.

2.4 Question 4: Differentiation

The question had parts, (a), (b) and (c). In part (a), the candidates were required to find $\frac{dy}{dx}$ from first principle given $y = 2x^2$. In part (b), they

were required to find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ in terms of x given that, $x = 2t + 9$ and

$y = (t + 1)^4$. In part (c), they were given $f(x) = x^3 - 2x^2 + x - 7$ and required to;

- (i) Find the stationary values of the function.
- (ii) Find the equation of the tangent line to the curve at the point (0, -7).
- (iii) Draw the graph of this function for $-2 \leq x \leq 3$ and indicate on the graph the stationary points and the equation of the tangent line obtained in part (c) (ii).

This question was averagely performed. It was attempted by 85.1 percent of the candidates, of which 45.4 percent scored from 3 to 10 marks. The analysis has also shown that 2.1 percent of the candidates who attempted this question scored high marks from 8 to 10. The candidates who scored full marks in part (a), managed to apply the definition $\frac{dy}{dx} = \lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x}$ correctly in finding the first derivative.

In part (b), they either applied the chain rule or expressed y as a function of x and determined the first and the second derivative as required. It was noted that the common error in this part was that some candidates were expressing the first and the second derivative in terms of t instead of x and hence lost some few marks.

The candidates who answered correctly part (c), they managed to apply the concepts of differentiation in finding the stationary values of the given function as well as the equation of the tangent line and indicated them correctly on the graph of the function. A sample answer from one of the candidates who applied correctly the concepts of differentiation in answering this question is shown in Extract 4.1.

Extract 4.1

4. @

Soln

$$y = 2x^2.$$

$$f(x) = 2x^2.$$

$$f(x+\Delta x) = 2(x+\Delta x)^2.$$

from first principle.

$$\frac{dy}{dx} = \lim_{\Delta x \rightarrow 0} \left(\frac{f(x+\Delta x) - f(x)}{\Delta x} \right)$$

$$= \lim_{\Delta x \rightarrow 0} \left(\frac{2(x+\Delta x)^2 - 2x^2}{\Delta x} \right)$$

$$= \lim_{\Delta x \rightarrow 0} \left(\frac{2x^2 + 4x\Delta x + 2(\Delta x)^2 - 2x^2}{\Delta x} \right)$$

$$= \lim_{\Delta x \rightarrow 0} \left(\frac{4x\Delta x + 2(\Delta x)^2}{\Delta x} \right)$$

$$= \lim_{\Delta x \rightarrow 0} (4x + 2\Delta x)$$

$$\Delta x \rightarrow 0 \quad 2\Delta x \rightarrow 0$$

$$= 4x.$$

$$\therefore \frac{dy}{dx} = 4x.$$

4. (5)

Solve

$$x = 2t + 9, \quad y = (t+1)^4.$$

$$\text{from } x = 2t + 9.$$

$$t = \frac{x-9}{2}.$$

$$\frac{dt}{dx} = \frac{1}{2}.$$

$$y = (t+1)^4$$

Let

$$u = t+1$$

$$\frac{dy}{du} = 4u^3.$$

$$y = u^4$$

$$\frac{dy}{du} = 4u^3.$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= 4(t+1)^3.$$

$$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}.$$

$$= 4(t+1)^3 \cdot \frac{1}{2}.$$

$$= 2 \left(\frac{x-9}{2} + 1 \right)^3.$$

$$= 2 \left(\frac{x-7}{2} \right)^3.$$

$$\therefore \frac{dy}{dx} = 2 \left(\frac{x-7}{2} \right)^3.$$

$$4. \textcircled{a} \quad \frac{dy}{dx} = 2 \left(\frac{x-7}{2} \right)^3.$$

$$\frac{d^2y}{dx^2} = ?$$

$$f(u) = 2 \left(\frac{x-7}{2} \right)^3.$$

$$\text{Let } u = \frac{x-7}{2}.$$

$$\frac{du}{dx} = \frac{1}{2}.$$

$$f(u) = 2 u^3.$$

$$\frac{df(u)}{du} = 6 u^2.$$

$$\frac{d^2y}{dx^2} = \frac{df(u)}{du} \cdot \frac{du}{dx}.$$

$$= 6 \left(\frac{x-7}{2} \right)^2 \cdot \frac{1}{2}.$$

$$= 3 \left(\frac{x-7}{2} \right)^2.$$

$$\therefore \frac{d^2y}{dx^2} = 3 \left(\frac{x-7}{2} \right)^2.$$

$$4. \textcircled{b} \quad f(x) = x^3 - 2x^2 + x + 7.$$

① The stationary point values.

$$\frac{dy}{dx} = 3x^2 - 4x + 1.$$

At stationary value $\frac{dy}{dx} = 0.$

$$3x^2 - 4x + 1 = 0.$$

$$3x^2 - 3x - x + 1 = 0$$

$$3x(x-1) - 1(x-1) = 0$$

$$(3x-1)(x-1) = 0$$

$$x = \frac{1}{3} \quad \text{or} \quad x = 1.$$

Then y when $x = 0$

$$y = x^3 - 2x^2 + x - 7$$

$$= -7$$

$y =$ when $x = \frac{1}{3}$.

$$y = \left(\frac{1}{3}\right)^3 - 2\left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right) - 7$$

$$= -6\frac{23}{27}$$

\therefore The stationary points are $P(1, -7)$ and $Q\left(\frac{1}{3}, -6\frac{23}{27}\right)$

ii) The equation of the tangent to the curve at $(0, -7)$

Slope, $\frac{dy}{dx} = 3x^2 - 4x + 1$

$$= 3(0)^2 - 4(0) + 1$$

$$M = 1$$

4. (i) $m = \frac{y - y_1}{x - x_1}$

$$1 = \frac{y - (-7)}{x - 0}$$

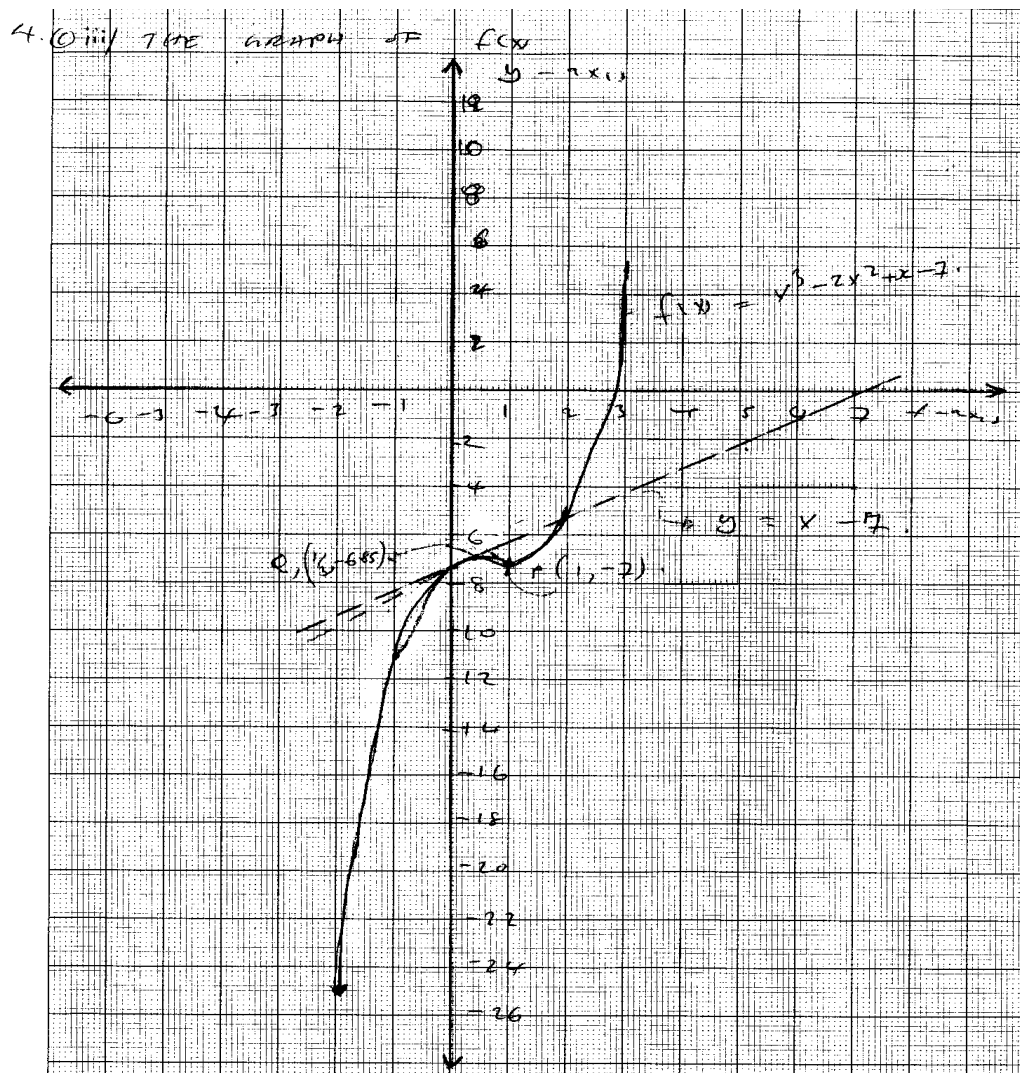
$$x = y + 7$$

$$y = x - 7$$

\therefore The equation is $y = x - 7$.

iii) The graph of the function.

x	-2	-1	0	1	2	3
y	-25	-11	-7	-17	-5	5



In Extract 4.1, the candidate demonstrated good understanding on the examined concepts of differentiation and also had good drawing skills.

On the other hand, 54.6 percent of the candidates who attempted the question scored low marks from 0 to 2.5 out of 10 marks. The candidates who performed poorly in part (a) could not apply correctly the definition for differentiating from first principles. They were unable to substitute $f(x) = 2x^2$ and $f(x + \Delta x) = 2(x + \Delta x)^2$ in the definition, and then simplify it while taking into account that Δx is a very small number that approaches zero. Extract 4.2, illustrates this case. It was also noted that some

candidates were finding the derivative using the standard formula

$$\frac{d(ax^n)}{dx} = nx^{n-1} \text{ contrary to the instructions that were given.}$$

In part (b), some candidates could not apply the chain rule $(\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx})$

to obtain the first derivative $\frac{dy}{dx}$ and as a result failed also to determine the

second derivative. In part (c)(i), some candidates were unable to find the stationary values of the given function because they could not either differentiate the given function correctly or find the correct values of x for which the first derivative is zero.

In part (c)(ii), some candidates were unable to find the equation of the tangent line to the curve at the given point $(0,-7)$ because they could not relate the concept of a slope and the definition of a derivative. In part c (iii), some candidates could not draw the graph of the function and indicate on it the stationery points and the equation of the tangent line. These candidates lacked the drawing skills as well as general understanding on the concepts of differentiation.

Extract 4.2

4.	$\frac{dy}{dx}$ - from
	$\frac{d}{dx}$
	from $y = 2x^2$.
	first principle.
	$\lim_{h \rightarrow 0} = \frac{(x+h)^2 - x^2}{h}$
	$\lim_{h \rightarrow 0} = \frac{(x+h)^2 + 2x}{h}$
	$\lim_{h \rightarrow 0} = \frac{(x+h)(x+h) + hx^2}{h}$
	$\lim_{h \rightarrow 0} = x^2 + 2xh + hx^2 + x^2$
	- 4x

$$4(a) \quad \lim_{h \rightarrow 0} = 4x$$

$$\therefore \frac{dy}{dx} = 4x.$$

$$e) \frac{dy}{dx} = \frac{V \frac{dy}{dx} - U \frac{dy}{dx}}{V^2}$$

$$= \frac{(2t+9) \cdot 2t - (t+1)^2 \cdot 4(t+1)^3}{(2t+9)^2}$$

$$\frac{dy}{dx} = \frac{4t^2 + 9t - (t+1)^2 \cdot 4(t+1)^3}{(2t+9)^2}$$

$$e) \frac{dy}{dx} = \frac{V \frac{dy}{dx} - U \frac{dy}{dx}}{V^2}$$

$$= \frac{(2t+9) \cdot 2t - (t+1)^2 \cdot 4(t+1)^3}{(2t+9)^2}$$

$$\frac{dy}{dx} = \frac{4t^2 + 9t - (t+1)^2 \cdot 4(t+1)^3}{(2t+9)^2}$$

4(c) Given:

$$f(x) = x^3 - 2x^2 + x - 7.$$

$$y = 3x^2 - 4x + x - 7.$$

$$y = 3x^2 - 5x - 7.$$

for $(0, -7)$.

To solve for y .

$$y = 3(0)^2 - 5(0) - 7.$$

$$y = -7.$$

To solve for x .

$$y = 3x^2 - 5x - 7.$$

$$0 = 3x^2 - 5x - 7$$

$$x = 2.5 \quad (x, y) = (2.5, -7)$$

4(d) To find equation of the curve $(0, -7)$.

$(x_2, y_2) = (2.5, -7)$.

from $\frac{y_2 - y_1}{x_2 - x_1}$

slope = $\frac{y_2 - y_1}{x_2 - x_1}$.

slope = $\frac{2.5 - (-7)}{2.5 - 0}$

= $\frac{-7 - (-7)}{0}$

Extract 4.2 shows that the candidate did not understand how to apply the concepts of differentiation. For instance, in part (a), the candidate applied the first principle wrongly and in part (b) applied the quotient rule instead of the chain rule.

2.5 Question 5: Integration

This question had three parts (a), (b) and (c). In part (a), the candidates were required to evaluate the integrals: (i) $\int x(x+9)^{\frac{1}{2}} dx$ and (ii) $\int x \cos(5x+9) dx$.

In part (b), the candidates were required to find the value of the constant a from the integral $\int_2^4 \left(3x^2 - ax - \frac{16}{x^2} \right) dx = 40$ and in part (c), they were required to sketch the graph of the curve $y = x^3 - 3x^2 - 2x$ and then find the area bounded by the curve and the x -axis.

This question was attempted by 64.1 of the candidates, of which many candidates (85.1%) scored below 3 out of 10 marks with 26.9 percent of them scoring a 0 mark, indicating that this question was poorly performed. This was the most poorly performed question in this examination.

In part (a)(i), only a few candidates were able to use the technique of integration by substitution in evaluating the given integral. The majority of the candidates were unable to use a correct substitution such as letting $u = x + 9$; $du = dx$ in order to reduce the given indefinite integral into the standard integral $\int ax^n dx = \frac{a}{n+1} x^{n+1} + c$, see Extract 5.1. Part (a)(ii) was not marked because it required application of integration by parts, a technique which is not indicated in the syllabus. The marks for this part were distributed to other parts of this question.

In part (b), only few candidates were able to evaluate the given definite integral and substituted the limits of integration correctly to obtain the equation $(68 - 8a) - (16 - 2a) = 40$ which they solved for the required value of $a = 2$. The majority of the candidates lacked the basic knowledge and skills of evaluating definite integrals of polynomials. It was noted that a number of candidates scored low marks in this part because they managed to carry on the integration but they could not perform correctly the basic algebraic operations after substituting the lower and upper limits of integration. Generally, many candidates lacked knowledge and skills of evaluating integrals.

In part (c), many candidates could not sketch the graph of the given cubic equation because they were using either incorrect table of values or incorrect turning points and intercepts. Also, the candidates could not apply the formula $\text{Area} = \left| \int_a^b f(x) dx \right|$ to find the required area. Some candidates

were using incorrect limits of integration while others were unable to carry on the integration correctly. It was noted that a number of candidates were finding the area using the formula $\text{Area} = \int_a^b f(x)dx$ instead $\text{Area} = \left| \int_a^b f(x)dx \right|$ and hence ended up with the answer 0 instead of $\frac{1}{2}$ square units.

Extract 5.1

$$\begin{aligned}
 & 5. \quad a) \quad \int dx \\
 & \quad i) \quad \int x(x+9)^{1/2} dx \\
 & = \int x\sqrt{x+9} dx \\
 & = \int \sqrt{x^2+9x} dx \\
 & = \int \sqrt{x^2+9x^2} dx \\
 & = \int (x^2+9x^2)^{1/2} dx \\
 & = \frac{(x^3+9x^2)^{1/2+1}}{\frac{1}{2}+1} + c \\
 & = \frac{(x^3+9x^2)^{3/2}}{3/2} \\
 & = \frac{2}{3} \sqrt{(x^3+9x^2)^3} \\
 & \quad \therefore = \frac{2}{3} \sqrt{(x^3+9x^2)^3}
 \end{aligned}$$

$$5) \quad b) \quad \int_2^4 \left(3x^2 - ax - \frac{16}{x^2} \right) dx = 40$$

Soln

$$\text{let } u = 3x^2 - ax - \frac{16}{x^2}$$

$$u = 3x^2 - ax - 16x^{-2}$$

$$\frac{du}{dx} = 6x - a + 32x$$

$$\frac{du}{dx} = 38x - a$$

$$\int_2^4 \frac{u \cdot du}{38x - a} = 40$$

$$\frac{1}{38x - a} \int_2^4 u^2 \cdot du = 40$$

$$\frac{1}{38x - a} \left[\frac{u^3}{3} \right]_2^4 = 40$$

$$\frac{1}{38x - a} \left[\frac{3x^2 - ax - 16x^{-2}}{2} \right]_2^4 = 40$$

$$\frac{1}{38x - a} \left[\frac{3(4)^2 - a(4) - 16(4)^{-2}}{2} - \frac{3(2)^2 - a(2) - 16(2)^{-2}}{2} \right] = 40$$

$$\frac{1}{38x - a} \left[\frac{48 - 4a - 1}{2} - \frac{27 - 2a - 4}{2} \right] = 40$$

$$5) \quad c). \quad \text{Given}$$

$$y = x^3 - 3x^2 + 2x$$

$$A = \int_0^1 y^2 \cdot dx$$

$$y = x^3 - 3x^2 + 2x$$

$$\frac{dy}{dx} = 3x^2 - 6x + 2$$

$$3x^2 - 6x + 2 = 0$$

$$x = 1.5 \quad x = 0.4$$

$$A = \int_{0.4}^{1.5} (x^3 - 3x^2 + 2x)$$

In Extract 5.1, the candidate could not apply the techniques of integration in answering parts (a) and (b). In part (c), the candidate applied the formula $\text{Area} = \int_a^b y^2 dx$ instead of $\text{Area} = \int_a^b |y| dx$ in finding the area, indicating a poor mastery of the concepts of integration.

Despite the general poor performance in this question, there were some good responses which deserved full marks. The candidates were able to apply correctly the knowledge and skills on the topic of integration in answering this question. A sample answer from one of these candidates is shown Extract 5.2.

Extract 5.2

5.	
①	
(c)	$\int x(x+9)^{1/2} dx.$
	let
	$(x+9)^{1/2} = u$
	$x+9 = u^2.$
	$x = u^2 - 9$
	$dx = 2u du$
	$\int (u^2 - 9) u \cdot 2u du$
	$2 \int (u^2 - 9) u^2 du$
	$2 \int u^4 - 9u^2 du$
	$2 \int u^4 du - 18 \int u^2 du$
	$\frac{2u^5}{5} - \frac{18u^3}{3} + C.$
	but $u = (x+9)^{1/2}.$
	$\therefore \int x(x+9)^{1/2} dx = \frac{2}{5} (x+9)^{5/2} - 6(x+9)^{3/2} + C$

5(b)

$$\int_2^4 (3x^2 - ax - \frac{16}{x^2}) dx = 40$$

$$\int_2^4 3x^2 dx - \int_2^4 ax dx - \int_2^4 \frac{16}{x^2} dx = 40$$

$$\left[x^3 \right]_2^4 - a \left[\frac{x^2}{2} \right]_2^4 + 16 \left[\frac{1}{x} \right]_2^4 = 40$$

$$(4^3 - 2^3) - \frac{a}{2} (4^2 - 2^2) + 16 \left(\frac{1}{4} - \frac{1}{2} \right) = 40$$

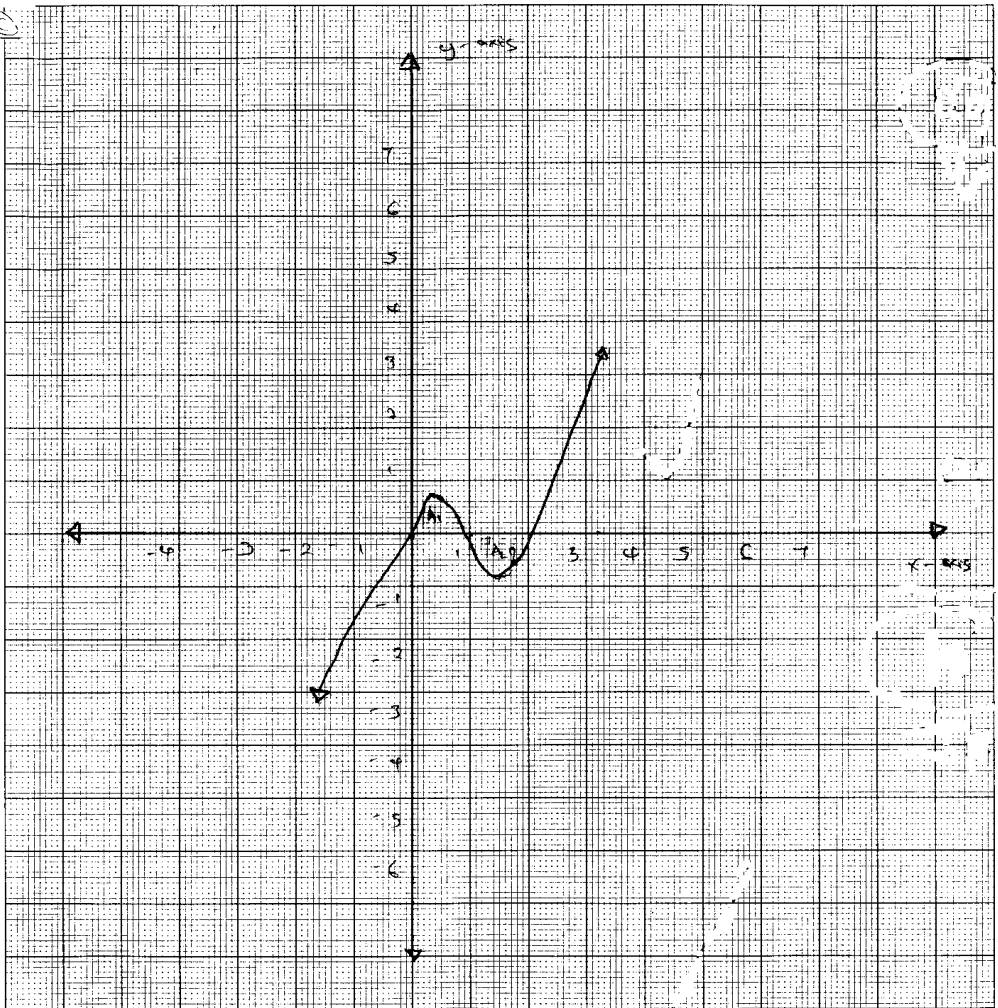
$$56 - 6a - 4 = 40$$

$$52 - 6a = 40$$

$$6a = 12$$

$$\therefore a = 2$$

5(c)



5.6	
	Area bounded by the Curve
	$A = \int_0^2 y \, dx$
	$A = \int_0^1 x^3 - 3x^2 + 2x \, dx + \int_1^2 x^3 - 3x^2 + 2x \, dx$
	$A = \left[\frac{x^4}{4} - x^3 + x^2 \right]_0^1 + \left[\frac{x^4}{4} - x^3 + x^2 \right]_1^2$
	$A = 0.25 + 0.25 \text{ unit square}$
	$\therefore A = 0.5 \text{ unit square}$

In Extract 5.2, the candidate demonstrated good understanding on how to evaluate definite and indefinite integrals and how to find area under a curve.

2.6 Question 6: Statistics

In this question, the candidates were given the scores obtained by 22 students from Sarawak Secondary School in a mathematics classroom test as; 49, 64, 38, 60, 46, 64, 68, 42, 38, 68, 57, 63, 76, 51, 54, 66, 62, 63, 58, 59, 47, 55. The candidates were required to;

- Summarize the scores in a frequency table with equal class intervals of size 5 by taking the lowest limit to be 35.
- Find the mean score by using the data in part (a).
- Find the interquartile range.
- Find the number of students who scored above the mean score.

This question was attempted by 95.6 percent of the candidates. The majority of the candidates (84.8%) scored from 03 to 10 marks and among them 4.6 percent scored all the 10 marks. The question had a good performance and it was the best performed question in this examination.

In part (a), the candidates were able to prepare the required frequency table as they managed to prepare the class intervals correctly and counted the number of scores for each class accurately. In part (b), they managed to calculate the mean score correctly using either the formula $\bar{X} = A + \frac{\sum fd}{N}$ or

$\bar{X} = \frac{\sum fx}{N}$. The candidates also managed to use the formulas:

$Q_1 = L + \frac{\left(\frac{N}{4} - n_b\right)c}{n_w}$ and $Q_3 = L + \frac{\left(\frac{3N}{4} - n_b\right)c}{n_w}$ in finding the interquartile range $Q_3 - Q_1$. Extract 6.1 is a sample answer illustrating how these candidates answered this question correctly.

Extract 6.1

6 a) i) frequency table.

Class Interval	Frequency	Cumfreq.
35 - 39	2	2
40 - 44	1	3
45 - 49	3	6
50 - 54	2	8
55 - 59	4	12
60 - 64	6	18
65 - 69	3	21
70 - 74	0	21
75 - 79	1	22
Total	22	

6. b)

Class Interval	freq.	Cumfreq.	class mark	fx
35 - 39	2	2	37	74
40 - 44	1	3	42	42
45 - 49	3	6	47	141
50 - 54	2	8	52	104
55 - 59	4	12	57	228
60 - 64	6	18	62	372
65 - 69	3	21	67	201
70 - 74	0	21	72	0
75 - 79	1	22	77	77
Total	22		$\Sigma fx =$	1239

$$\text{Mean} = \frac{\Sigma fx}{N} = \frac{1239}{22}$$

$$\text{Mean}(x) = 56.32$$

6C) Interquartile range (IQR).

$$IQR = \text{Upper quartile} - \text{Lower quartile}$$

$$\text{Upper quartile} = L + \frac{\left(\frac{3N}{4} - Na\right) i}{nw}$$

$$\text{Upper quartile class } \frac{3}{4} \times 22 = 16.5$$

is 60-64

$$L = 59.5, \quad Na = 12$$

$$i = 5, \quad nw = 6$$

$$\text{Upper quartile} = 59.5 + \frac{(16.5 - 12) \times 5}{6}$$

6C. Upper quartile = 63.25

$$\text{Lower quartile} = L + \frac{\left(\frac{N}{4} - Na\right) i}{nw}$$

$$i = 5$$

$$\text{Lower quartile class } \left(\frac{22}{4} = 5.5\right)$$

$$\text{is } 45-49$$

$$L = 44.5$$

$$Na = 3$$

$$nw = 3$$

$$\text{Lower quartile} = 44.5 + \frac{\left(\frac{22}{4} - 3\right) \times 5}{3}$$

$$\text{Lower quartile} = 48.67$$

$$\text{Interquartile range (IQR)} = \text{Upper quartile} - \text{Lower quartile}$$

$$IQR = 63.25 - 48.67$$

$$IQR = 14.58$$

$$\text{Interquartile range is } 14.58$$

6d. Students who scored above the mean score 56.32 are 13 students

In Extract 6.1, the candidate was able to prepare the frequency distribution table correctly and used appropriately in calculating the mean, interquartile range and the number of students who scored above the mean score.

Only few candidates (15.2%) scored low marks (0 to 2.5 out of 10) in this question. Most of those who scored a 0 mark used class intervals that were contrary to the requirement of the question. Other candidates were not careful while counting the number of scores in some of the class intervals and as a result obtained incorrect frequencies leading to incorrect mean score and interquartile range. In addition, the candidates were unable to apply the formula for mean, lower quartile and upper quartile correctly. Extract 6.2 is a sample answer from one of the candidates showing some of the difficulties the candidates faced while answering this question.

Extract 6.2

6a	Class Interval	X	f	f x
	25 - 40	37.5	2	75
	41 - 46	43.5	2	87
	47 - 52	49.5	2	99
	53 - 58	55.5	4	222
	59 - 64	61.5	3	184.5
	65 - 70	67.5	1	67.5
	71 - 76	73.5	1	73.5
			$\Sigma f = 15$	$\Sigma fx = 740.5$
6b	Mean	=	$\frac{\Sigma fx}{N}$	= $\frac{740.5}{15}$
		=	32.66	49.37
6c		=	73.5 - 37.5	
		=	36	
6d	11 student above the mean scores			

In Extract 6.2, the candidate used the class size of 6 instead of the recommended class size of 5 to prepare the frequency table. Moreover, the candidate calculated the interquartile range as the difference between the highest class mark and the lowest class mark instead of the difference between the upper quartile and the lower quartile.

2.7 Question 7: Probability

This question had four parts. In part (a), the candidates were required to find $P(A \cup B)$ and $P(A' \cap B')$ given that A and B are two events such that $P(A) = \frac{1}{4}$, $P(B) = \frac{1}{2}$ and $P(A \cap B) = \frac{1}{8}$. In part (b), they were required to find $P(A/B)$ given that a fair die was rolled and the events A and B were recorded as $A = \{1, 3, 5\}$ and $B = \{2, 3, 4, 5\}$. In part (c), the candidates were given that in section B of CSEE Basic Mathematics Examination each candidate has to choose and answer four out of six questions. The candidates were then required to find how many choices are there for each candidate. In part (d), the candidates were given that, a box contains 4 ripe mangoes and 9 none ripe mangoes and they were required to find the probability that both will be ripe mangoes if two mangoes were to be chosen randomly from the box.

This question was attempted by 79.9 percent of the candidates. It was averagely performed as 46.2 percent of the candidates who attempted it scored from 3 to 10 marks. Only 0.3 percent of the candidates who attempted this question managed to score full marks.

The candidates who answered correctly part (a) had sufficient knowledge and skills to determine the probability of the combined events using the formulas $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ and $P(A' \cap B') = 1 - P(A \cup B)$. In part (b), a number of candidates managed to apply the definition of the conditional probability $P(A/B) = \frac{P(A \cap B)}{P(B)}$ in answering this part

correctly. In part (c), only some candidates managed to apply the concepts of combination correctly in calculating the required number of choices as ${}^6C_4 = \frac{6!}{4! \times (6-4)!} = 15$. In part (d), only a few candidates were able to apply

the concepts of combination or use tree diagrams to help them in calculating the required probability. A sample answer from one of the candidates who demonstrated good understanding of the tested concepts of probability and combinations is shown in Extract 7.1.

Extract 7.1

7. (a) Solution

$$\text{Given; } P(A) = \frac{1}{4}$$

$$P(B) = \frac{1}{2}$$

$$P(A \cap B) = \frac{1}{8}$$

$$(i) P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cup B) = \frac{1}{4} + \frac{1}{2} - \frac{1}{8}$$

$$= \frac{5}{8}$$

$$\therefore P(A \cup B) = \frac{5}{8}$$

$$(ii) P(A' \cap B')$$

From De Morgan's rule

$$P(A' \cap B') = P(A \cup B)'$$

$$\text{For, } P(E) + P(E') = 1$$

$$P(A \cup B) + P(A \cup B)' = 1$$

$$\frac{5}{8} + P(A \cup B)' = 1$$

$$P(A \cup B)' = \frac{3}{8}$$

$$\therefore P(A' \cap B') = \frac{3}{8}$$

(b)

Solusi

Luasan, A fair die with events A and B.

$$A = \{1, 3, 5\}$$

$$S = \{1, 2, 3, 4, 5, 6\}$$

$$B = \{2, 3, 4, 5\}$$

$$S = \{1, 2, 3, 4, 5, 6\}$$

7 (b)

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A) = \frac{3}{6} = \frac{1}{2}$$

$$P(B) = \frac{n(B)}{n(S)}$$

$$P(B) = \frac{4}{6} = \frac{2}{3}$$

$$P(A \cap B) = P(A) \times P(B)$$

$$= \frac{1}{2} \times \frac{2}{3}$$
$$= \frac{1}{3}$$

$$P(A/B) = \frac{\frac{1}{3}}{\frac{2}{3}}$$

$$P(A/B) = \frac{1}{3} \times \frac{3}{2}$$
$$= \frac{1}{2}$$

$$\therefore P(A/B) = \frac{1}{2}$$

7 (c) Solution

Given; Total number of questions = 6.
Questions required = 4.

Number of choices are found by combination

$${}^n C_r = \frac{n!}{(n-r)!r!}$$

Where n - total number of items
 r - taken items at a time.

$${}^6 C_4 = \frac{6!}{(6-4)!4!}$$
$$= \frac{6!}{2! \times 4!}$$

$$= \frac{6 \times 5 \times 4!}{2 \times 1 \times 4!}$$

$$= \frac{6 \times 5}{2}$$

$$= 15 \text{ choices}$$

\therefore Number of choices for each candidate = 15.

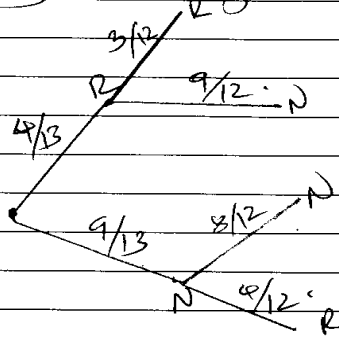
(d) Solution

Given; Number of ripe mangoes = 4.

Number of non-ripe mangoes = 9.

Total number = 13.

7 (d) Using tree diagram.



Probability of both being ripe = $P(R) \times P(R)$

$$= \frac{4}{13} \times \frac{3}{12}$$

$$= \frac{4}{13} \times \frac{1}{4} = \frac{1}{13}$$

\therefore Probability = $\frac{1}{13}$

Extract 7.1 shows that the candidate understood the question and applied correctly the acquired knowledge and skills on the topic of probability to obtain the required solution.

Nevertheless, there were some candidates (53.8%) who scored low marks (from 0 to 2.5) in this question and among them 15.8 percent scored a 0 mark. Parts (a) and (b) were generally not answered well mainly due to candidates unable to apply the knowledge of probability with its appropriate formulae in answering the questions. Part (c) was poorly answered by some candidates because they could not apply the concepts of combination in working out the required answer. It was observed that other candidates were wrongly applying the concepts of permutations as they could not distinguish permutation and combination. Likewise the candidates failed to answer part (d) correctly because they could not represent the given information in a tree diagram or apply the concepts of combination in working out the required probability. Extract 7.2 is a sample answer showing some of the difficulties the candidates encountered while answering this question.

Extract 7.2

7a.	Solution
	given, $P(A) = \frac{1}{4}$, $P(B) = \frac{1}{2}$ $P(A \cap B) = \frac{1}{8}$
	required, $P(A \cup B)$
	where,
	$P(A \cup B) = P(A) + P(B)$
	$= \frac{1}{4} + \frac{1}{2}$
	$P(A \cup B) = \frac{3}{4}$
	$\therefore P(A \cup B) = \frac{3}{4}$
	for, $P(A' \cap B') = (P(A \cap B) - P(A)) + (P(A \cap B) - P(B))$
	$= \left(\frac{1}{8} - \frac{1}{4}\right) + \left(\frac{1}{8} - \frac{1}{2}\right)$
	$= -\frac{1}{8} + -\frac{3}{8}$
	$= -\frac{1}{2}$
	$\therefore P(A' \cap B') = \frac{1}{2}$
7b)	given, $A = \{1, 3, 5\}$ $B = \{2, 3, 4, 5\}$
	required, $P(A/B)$
	Now,
	$\frac{1! \times 2! \times 3!}{2! \times 2! \times 4! \times 1!}$
	$\frac{1!}{2! \times 4!}$
	$\therefore P(A/B) = 0.021$

(e)	Total number of questions = 6 Number of questions to be chosen = 4
	Solution,
	Member of choices = $\frac{6!}{4!}$
	$= \frac{6! \times 4!}{4!} = \frac{6 \times 5 \times 4!}{4}$
	$= 30$
	\therefore There are 30 number of choices
76.)	Solution
	ripe mangoes = 4
	Non ripe mangoes = 9
	Total number of mangoes = 13
	Let ripe mangoes be P(R)
	$P(R) = \frac{\text{Total number of mangoes}}{\text{Ripe mangoes}}$
	$P(R) = \frac{13}{4} \times \frac{13}{4}$ (Since two mangoes are picked)
	$P(R) = 10 \frac{9}{16} = \frac{169}{16}$
	\therefore The Probability that both will be ripe mangoes is $\frac{169}{16}$

In Extract 7.2, the candidate did not master well the concepts and formulas for determining probability of combined events. For instance the candidate applied the formula $P(A \cup B) = P(A) + P(B)$ instead of $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ in answering part (a).

2.8 Question 8: Trigonometry

In part 8 (a), the candidates were required to evaluate without using a mathematical table or a calculator (i) $\cos(165^\circ)$ and (ii) $\tan(A+B)$ given that A and B are acute angles having $\sin(A) = \frac{7}{25}$ and $\cos(B) = \frac{5}{13}$.

In part 8 (b), they were required to:

- (i) Find the values of x that satisfy the equation $\sin 2x + \cos x = 0$ for $0^\circ \leq x \leq 360^\circ$.
- (ii) Verify that the solution of the equation in part (b)(i) can be obtained graphically by plotting the graph of $y = \sin 2x + \cos x$ for $0^\circ \leq x \leq 360^\circ$.

This question was attempted by 56.1 percent of the candidates, out of which only 21.3 percent managed to score from 3 to 10 marks. This was the least attempted and the second poorly performed question in this examination.

In part 8(a)(i), the candidates were unable to express the angle 165° as a sum or a difference of two special angles, for instance, $165^\circ = 135^\circ + 30^\circ$ and then apply the identity $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$ to obtain the required answer. In part 8(a) (ii), the candidates were unable to apply the identity $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$ or $\tan(A+B) = \frac{\sin(A+B)}{\cos(A+B)}$ to find

the required answer. Part 8(b) (i) was also poorly performed as the majority of the candidates could not reduce the given equation into the equation $\cos x[2 \sin x + 1] = 0$ and then to subsequently solve $\cos x = 0$ or $2 \sin x + 1 = 0$ to obtain the required values of x . Likewise part 8(b) (ii) was poorly done as many candidates could not sketch the required graph. Some of the candidates were using incorrect table of values while others lacked the skills to sketch the graph. Generally, the candidates lacked knowledge and skills on the topic of trigonometry, see Extract 8.1.

Extract 8.1

$$8. (i) \cos(165^\circ)$$

$$\cos \theta = \frac{\text{Adj}}{\text{Hyp}}$$

$$\cos(165^\circ)$$

$$\therefore \cos(165^\circ) \text{ is } -0.9659$$

$$(ii) \sin(A) = \frac{7}{25}$$

$$\cos(B) = \frac{5}{13}$$

$$\tan(A+B) = ?$$

$$8. (a) (i) \sin = \frac{\text{Opp}}{\text{Hyp}}$$

$$\text{Opp} = 7$$

$$\text{Hyp} = 25$$

$$\cos = \frac{\text{Adj}}{\text{Hyp}}$$

$$\text{Adj} = 5$$

$$\text{Hence } \tan = \frac{\text{Opp}}{\text{Adj}}$$

$$\tan(A) = \frac{7}{5}$$

$$\tan(A) = \frac{7}{5}, \tan(B) = \frac{13}{7}$$

$$\text{So } \tan(A+B)$$

$$\frac{7}{5} + \frac{13}{7}$$

$$\frac{49 + 65}{35} = 3.257142$$

$$\therefore \tan(A+B) = 3.2571$$

In Extract 8.2, the candidate computed the value of $\cos(165^\circ)$ using either a mathematical table or a calculator contrary to the requirement of the question. The candidate also used incorrect trigonometrical ratios for $\tan A$, $\tan B$ and incorrect identity for $\tan(A+B)$, indicating lack of knowledge on the tested concepts of trigonometry.

However, there were only a few candidates (0.4%) who managed to score above 7 out of 10 marks. The candidates who scored high marks had sufficient knowledge and skills on the concepts of trigonometry. Extract 8.2

is a sample answer from one of the candidates who performed well in this question.

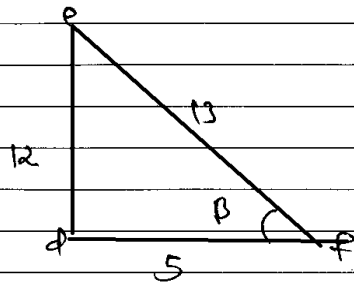
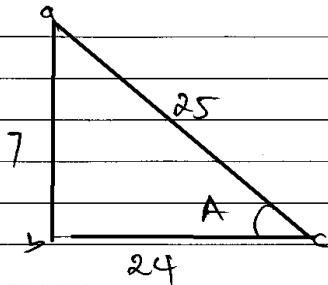
Extract 8.2

8	(a) soln
	(i) $\cos 165^\circ$
	$\cos 165^\circ = \cos (45 + 120)$
	from $\cos (A + B) = \cos A \cos B - \sin A \sin B$.
8	(a)
	(i) $\cos 165 = \cos (45 + 120)$
	$= \cos 45 \cos 120 - \sin 45 \sin 120$
	but
	$\cos 120 = -\cos 60^\circ$
	$\sin 120 = \sin 60^\circ$
	$\cos 165^\circ = -\cos 45 \cos 60^\circ - \sin 45 \sin 60^\circ$
	$\cos 45^\circ = \frac{\sqrt{2}}{2}$
	$\cos 60^\circ = \frac{1}{2}$
	$\sin 45^\circ = \frac{\sqrt{2}}{2}$
	$\sin 60^\circ = \frac{\sqrt{3}}{2}$
	$= -\frac{\sqrt{2} \times 1}{2} - \frac{\sqrt{2} \times \sqrt{3}}{2}$
	$\cos 165^\circ = -\frac{\sqrt{2}}{2} - \frac{\sqrt{6}}{2}$
	$\cos 165^\circ = \frac{-\sqrt{2} - \sqrt{6}}{2}$
	or
	$\cos 165^\circ = \frac{-\sqrt{2}(1 + \sqrt{3})}{2}$
	Therefore the value of
	$\cos 165^\circ = \frac{-\sqrt{2} - \sqrt{6}}{2}$ or $\frac{-\sqrt{2}(1 + \sqrt{3})}{2}$

8 (a) (ii) Soln
Dada.

$$\sin A = 7/25$$

$$\cos B = 5/13$$



$$\cos A = 24/25$$

$$\sin A = 7/25$$

$$\tan A = \frac{\sin A}{\cos A}$$
$$= 7/25 \div 24/25$$

$$\tan A = 7/24$$

$$\sin B = 12/13$$

$$\cos B = 5/13$$

$$\tan B = \frac{\sin B}{\cos B}$$
$$= \frac{12}{13} \div \frac{5}{13}$$

$$\tan B = 12/5$$

from

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$= \frac{7/24 + 12/5}{1 - 7/24 \times 12/5}$$

$$= \frac{7/24 + 12/5}{1 - 7/10}$$

$$\tan(A+B) = \frac{323}{36}$$

$$\therefore \tan(A+B) = \frac{323}{36}$$

8 (b) (i) solve

from

$$\sin 2x + \cos x = 0 \quad 0^\circ \leq x \leq 360^\circ$$

from

$$\sin 2x = 2 \sin x \cos x$$

$$2 \sin x \cos x + \cos x = 0$$

$$\cos x (2 \sin x + 1) = 0$$

$$\cos x = 0$$

$$2 \sin x + 1 = 0$$

for $\cos x = 0$

$$x = \cos^{-1}(0)$$

$$x = 90^\circ, 270^\circ$$

for

$$2 \sin x + 1 = 0$$

$$2 \sin x = -1$$

$$\sin x = -\frac{1}{2}$$

$$x = \sin^{-1}\left(-\frac{1}{2}\right)$$

$$x = 210^\circ, 330^\circ$$

Thus, the value of x that satisfy the equation are

$$x = 90^\circ, 210^\circ, 270^\circ \text{ and } 330^\circ$$

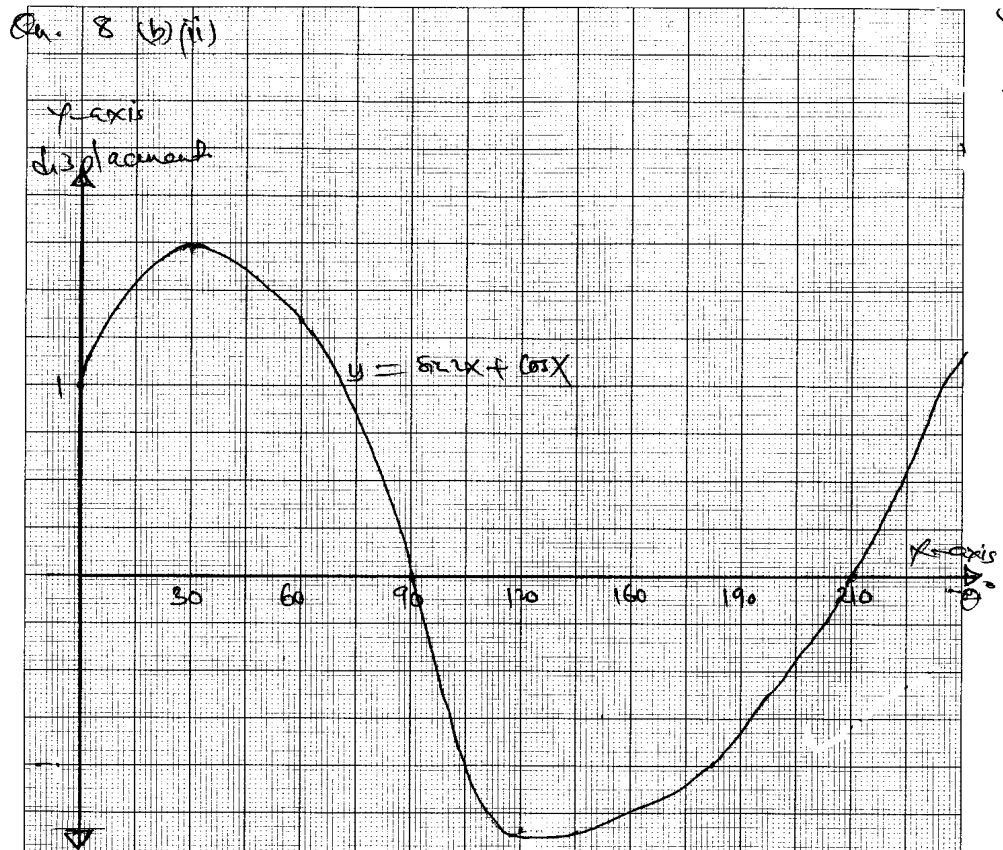
8 (b) (ii) \sin
 Equation $y = \sin 2x + \cos x$
 Table of values

X	90	210	270	330
$\sin 2x$	0	0.866	0	0.866
$\cos x$	0	-0.866	0	-0.866
y	0	0	0	0

For other angles

X	0°	30	45	60	120	180	270
$\sin 2x$	0	0.866	1	0.866	-0.866	0	0
$\cos x$	1	0.866	0.707	0.5	-0.5	-1	0
y	1	1.732	1.707	1.366	-1.366	-1	0

The graph is shown on the graph paper



Extract 8.1 shows that the candidate had an adequate knowledge and skills on the tested concepts of trigonometry.

2.9 Question 9: Matrices

This question had three parts (a), (b) and (c). In part (a), the candidates were given the matrices $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ -1 & 5 \end{pmatrix}$, $B = \begin{pmatrix} -2 & 3 & 4 \\ 3 & 2 & 1 \end{pmatrix}$, $C = \begin{pmatrix} 3 & 5 \\ 1 & 2 \end{pmatrix}$ and they were required to; (i), state with one reason as to whether the matrix operations AB , BA and BC are defined or not and (ii), find $2A + 3B^T$.

In part (b), they were required to verify that $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$.

In part (c), the candidates were given that $D = \begin{vmatrix} a & -4 & -6 \\ -8 & 5 & 7 \\ -5 & 3 & 4 \end{vmatrix}$ is the

inverse of matrix $E = \begin{vmatrix} 1 & 2 & -2 \\ 3 & b & 1 \\ -1 & 1 & -3 \end{vmatrix}$ and they were required to find the values of a and b .

The question was attempted by 82.9 percent of the candidates, of which only 21.6 percent managed to score from 3 to 10 marks and only 1 out of the 14,551 candidates who attempted this question managed to score full marks, indicating that this question had a poor performance.

The candidates who performed poorly in this question had inadequate knowledge and skills to transpose a matrix, multiply a matrix by another matrix and to determine the determinant of a matrix. It was noted that in part (a) (i), many candidates were unable to explain whether the matrix operations AB , BA and BC are defined or not because they did not know the requirements for matrix multiplications. The candidates were not aware that in order to multiply 2 matrices, say A and B , the number of columns in A must be equal to the number of rows in B . Thus if A is an $m \times n$ and B is an $r \times s$ then $n = r$. It was observed that some of the candidates multiplied the matrices instead of stating the condition required for matrices to be conformable for matrix multiplications. Apart from the few who did not seem to know what a transpose matrix was, the majority of the candidates

who answered part (a)(ii) gave correct answers. In part (b), the majority of the candidates were able to expand the determinant of the given matrix to obtain $(bc^2 - cb^2) - (ac^2 - ca^2) + (ab^2 - ba^2)$ but failed to re-arrange the terms of this expression in order to factorize it as required. Part (c) was performed well by only a few candidates. The majority of the candidates were unable to use the fact that $DE = I$, where I is an identity matrix to

obtain
$$\begin{pmatrix} a-6 & 2a-4b-6 & -2a+14 \\ 0 & -9+5b & 0 \\ 0 & -6+3b & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$
 and then to compare the

LHS and the RHS of this equation in order to determine the required values of a and b . Some candidates answered this part by using the alternative method of finding the inverse matrix of E and later on compare the elements of matrix D and those of the inverse matrix. Hardly any got the correct inverse matrix and as a result they obtained incorrect values of a and b . Extract 9.1 is a sample answer showing how the candidates failed to provide correct answers for question 9.

Extract 9.2

Q. ✓ A $\begin{pmatrix} 1 & 2 \\ 3 & 4 \\ -1 & 5 \end{pmatrix}$, B $\begin{pmatrix} -2 & 3 & 4 \\ 3 & 2 & 1 \end{pmatrix}$ and C $\begin{pmatrix} 1 & 3 & 5 \\ 1 & 2 \end{pmatrix}$

Ans. The matrices are defined since they are well arranged and can easily be calculated.

$$\begin{aligned}
 & \text{ii/ } 2A + 3B^T \\
 & \text{Sol.} \\
 & 2A = 2 \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 1 & 5 \end{pmatrix} \\
 & 2A = \begin{pmatrix} 2 & 4 \\ 6 & 8 \\ 2 & 10 \end{pmatrix} \\
 & 3B = 3 \begin{pmatrix} -2 & 3 & 4 \\ 3 & 2 & 1 \end{pmatrix} \\
 & 3B = \begin{pmatrix} -6 & 9 & 12 \\ 9 & 6 & 3 \end{pmatrix} \\
 & \therefore 2A + 3B^T = \begin{pmatrix} 2 & 4 \\ 6 & 8 \\ 2 & 10 \end{pmatrix} + \begin{pmatrix} -2 & 3 & 4 \\ 3 & 2 & 1 \end{pmatrix} \\
 & = \begin{pmatrix} -4 & 7 & 4 \\ 9 & 10 & 1 \\ 2 & 10 & 0 \end{pmatrix}
 \end{aligned}$$

In Extract 9.2, the candidate could not explain why the matrix operations AB and BA in part (a)(i) were defined. In part (a)(ii), the candidate was unable to find the transpose of matrix B and also could not recognize that matrices of different orders cannot be added. The candidate did not attempt parts (b) and (c).

Only 2 percent of the candidates who attempted this question managed to score from 7 to 10 marks. The candidates who scored high marks in this question managed to answer parts (a) and (c) correctly but could not factorize the determinant of the matrix in part (b) as required. Extract 9.2 is a sample answer illustrating this case.

Extract 9.2

9 (a) (i) - AB it is defined since number of columns of A equals to the number of rows of B

- BA it is defined since number of columns of B equals to the number of rows of A

- BC it is not defined since number of columns of B is not equal to the number of rows of C

$$(ii) \quad 2A + 3B^T$$

$$2A = 2 \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ -1 & 5 \end{bmatrix}$$

$$2A = \begin{bmatrix} 2 & 4 \\ 6 & 12 \\ -2 & 10 \end{bmatrix}$$

$$9 \quad (ii) \quad B^T =$$
$$B = \begin{bmatrix} -2 & 3 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$B^T = \begin{bmatrix} -2 & 3 \\ 3 & 2 \\ 4 & 1 \end{bmatrix}$$

$$3B^T = 3 \begin{bmatrix} -2 & 3 \\ 3 & 2 \\ 4 & 1 \end{bmatrix}$$

$$3B^T = \begin{bmatrix} -6 & 9 \\ 9 & 6 \\ 12 & 3 \end{bmatrix}$$

$$2A + 3B^T = \begin{bmatrix} 2 & 4 \\ 6 & 12 \\ -2 & 10 \end{bmatrix} + \begin{bmatrix} -6 & 9 \\ 9 & 6 \\ 12 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} (2-6) & (4+9) \\ (6+9) & (12+6) \\ (-2+12) & (10+3) \end{bmatrix}$$

$$2A + 3B^T = \begin{bmatrix} -4 & 13 \\ 15 & 18 \\ 10 & 13 \end{bmatrix}$$

$$(b) \quad (a-b)(b-c)(c-a) = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$$

taking the first row

$$1 \begin{vmatrix} b & c \\ b^2 & c^2 \end{vmatrix} - 1 \begin{vmatrix} a & c \\ a^2 & c^2 \end{vmatrix} + 1 \begin{vmatrix} a & b \\ a^2 & b^2 \end{vmatrix}$$

$$\begin{aligned} 9 \quad (b) &= (bc^2 - b^2c) - (ac^2 - a^2c) + (ab^2 - a^2b) \\ &= bc^2 - b^2c - ac^2 + a^2c + ab^2 - a^2b \\ &= a^2c - a^2b + ab^2 - b^2c + bc^2 - ac^2 \\ &= a^2(c-b) + b^2(a-c) + c^2(b-a) \end{aligned}$$

$$c \quad \begin{bmatrix} 9 & -4 & -6 \\ -8 & 5 & 7 \\ -5 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 & -2 \\ 3 & b & 1 \\ -1 & 1 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 9-12+6 & 2a-4b-6 & -18-4+18 \\ -8+15-7 & 9a+5b+7 & 16-5-21 \\ -5+9-4 & -10+3b+4 & 10+3-12 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$2a - 4b - 6 = 0$$

$$9a + 5b + 7 = 0$$

$$a - 12 + 6 = 1$$

$$a - 6 = 1$$

$$a = 7$$

$$2a - 4b - 6 = 0$$

$$14 - 4b - 6 = 0$$

$$b = 2$$

Extract 9.2 shows that the candidate had sufficient knowledge to carry out matrix operations. The only difficult the candidate faced was to factorize the expression for the determinant in part (b) and hence lost some few marks in this part.

2.10 Question 10: Linear Programming

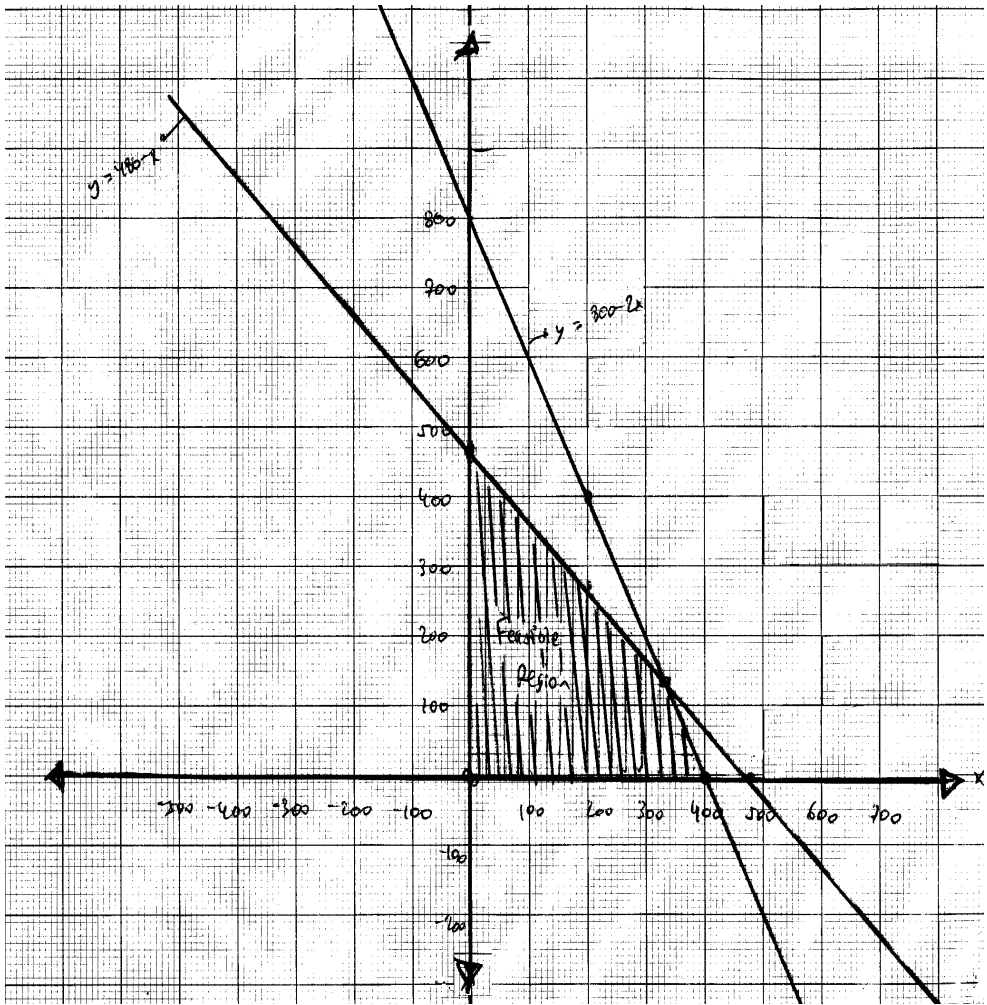
The question was; Mr. Taramise owns 480 acres of land on which he grows either maize or beans during the farming period. He normally gets a profit of Tshs 40,000/= per acre on maize and Tshs 30,000/= per acre on beans and he has 800 hours of labour available. If maize requires 2 hours per acre to raise and beans requires 1 hour per acre to raise, find how many acres of each, he should plant in order to get maximum profit.

This question was attempted by the majority of the candidates (87.1%), of which 84.2 percent scored from 3 to 10 marks and among them 9.9 percent scored all the 10 marks. The analysis has shown that this question had a good performance and it was the second best performed question in this examination.

The candidates who performed well in this question managed to formulate correctly the objective function and the inequalities for the constraints from the given information. They also managed to draw the graphs of these inequalities, identified the corner points and finally calculated the number of acres of maize and beans that Mr. Taramise should plant to obtain maximum profit. Extract 10.1 is a sample answer from one of the candidates who performed well in this question.

Extract 10.1

10	Let x be acres of maize			
	Let y be acres of beans			
	Objective function, $f(x,y) = 40,000x + 30,000y$			
	$x + y \leq 480$			
	$2x + y \leq 800$			
	$x \geq 0 \quad y \geq 0$			
	For function $x + y \leq 480$		For function $2x + y \leq 800$	
	$x + y = 480$		$2x + y = 800$	
	$y = 480 - x$		$y = 800 - 2x$	
	x	200	480	0
	y	280	0	480
	x	400	200	0
	y	0	400	800



From graph (in previous answer sheet)

Corner points are

$(0, 480)$ $(0, 0)$ $(400, 0)$ $(320, 160)$

↑

Intersection point of the two graphs

Corner Points	Objective function	Profit
0, 480	$40000(0) + 480(30,000)$	14,400,000
(0, 0)	$40,000(0) + 30,000(0)$	0
(400, 0)	$40,000(400) + 30,000(0)$	16,000,000
(320, 160)	$40,000(320) + 30,000(160)$	17,600,000
∴ He should plant 320 acres of maize and 160 acres of beans		

In Extract 10.1, the candidate formulated the constraints and the objective function correctly and drew correctly the graph which was used to obtain number of acres of maize and beans that was required for maximum profit.

On the other hand, there were a few candidates (15.8%) who scored low marks from 0 to 2.5 out of 10 marks in this question. The candidates who scored zero formulated incorrect inequalities for the constraints and incorrect objective function. Some candidates were able to obtain the required inequalities and the objective function but could not obtain the correct feasible region because they lacked skills to draw graphs. It was noted that other candidates were providing solutions that were not related to the requirement of the question, indicating that they were not familiar with how to solve linear programming problems. Extract 10.2 is a sample solution that illustrates how some of the candidates failed to answer this question.

Extract 10.2

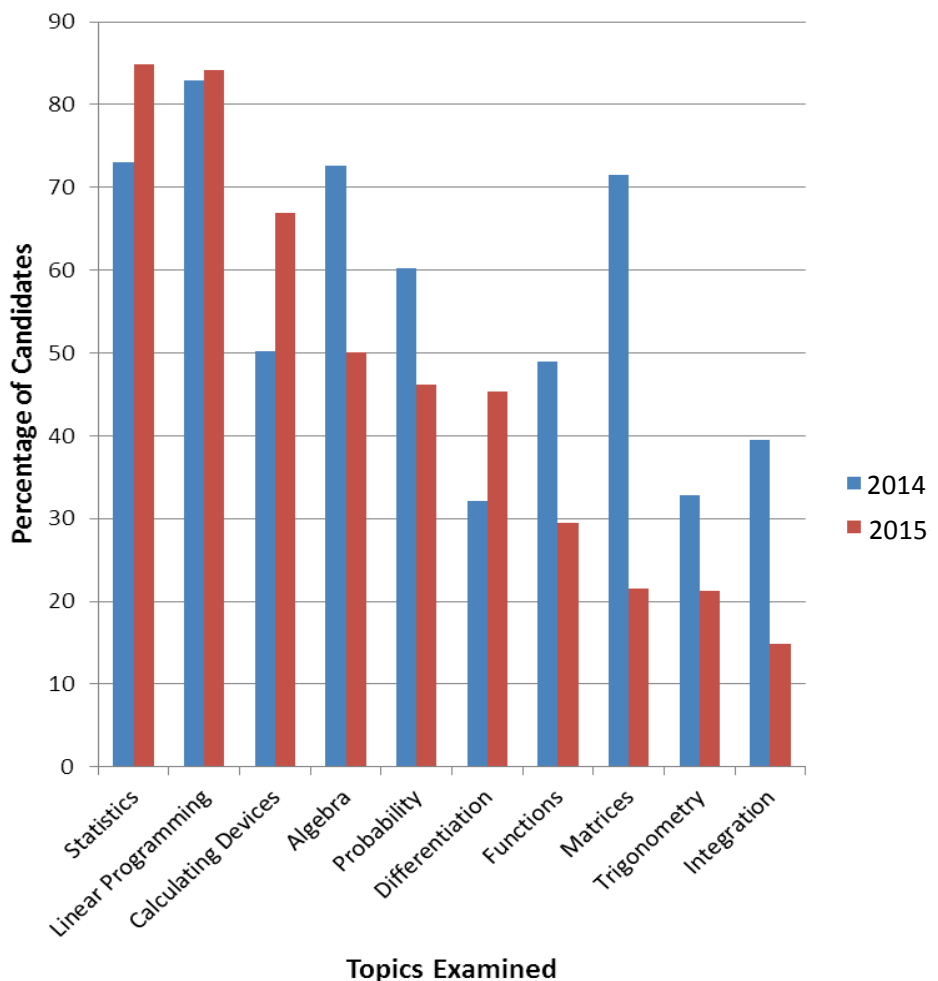
10	<p>data given:</p> <ul style="list-style-type: none"> - Present area 480 acres. - Profit on maize £40000 - " " on beans 30,000/2 - Labour hrs 800 - Maize require 2hrs. - Beans require 1hr. <p>How many acres for maximum profit?</p>
	$\frac{30,000 \times 800 \times 1}{40,000 \times 800 \times 2}$
	$= \frac{24,000,000 \times 1}{32,000,000 \times 2}$
	$= \frac{24,000,000}{64,000,000}$
	$= \frac{3}{8}$
	$\therefore \frac{3}{8} \times 480$
	$= 180$
	<p>He should cultivate 180 acres to get maximum profit.</p>

In Extract 10.2, the candidate did not formulate the constraints and the objective function and instead performed meaningless calculations on the numbers that were given in the question, indicating a general lack of knowledge and skills on solving linear programming questions.

3.0 ANALYSIS OF CANDIDATES' PERFORMANCE PER TOPIC

This section identifies the topics which had weak, average and good performance in the ACSEE 2015 Basic Applied Mathematics examination. The criteria for weak, average and good performance was that the average percentage of the candidates who scored 30 percent or more of the marks in all the questions tested from the same topic should lie in the intervals 0 – 29, 30 – 49 or 50 – 100 respectively. The section also compares the candidates' performance in ACSEE 2014 and 2015 in Basic Applied Mathematics examination topic wise. The analysis is presented in the Appendix and in the Bar Chart below.

The comparison of the Candidates' Performance per Topic in ACSEE 2014 and 2015



It is evident from the Appendix and the Bar Chart that, out of the 10 topics that were examined in 2015 Basic Applied Mathematics Examination, four (04) topics of *Statistics*, *Linear Programming*, *Calculating Devices* and *Algebra* have good performance; three (03) topics of *Probability*, *Differentiation* and *Functions* have average performance while three (03) topics of *Matrices*, *Trigonometry* and *Integration* have weak performance.

The comparison of the 2014 and 2015 candidates' performance per topic shows that:

- Four (04) topics of *Statistics*, *Linear Programming*, *Calculating Devices* and *Algebra* have consistently remained with good performance in 2014 and 2015.
- Two (02) topics of *Differentiation* and *Functions* have consistently remained with average performance in 2014 and 2015.
- In 2014 the topic of *Matrices* had a good performance while the topics of *Trigonometry* and *Integration* had an average performance whereas in 2015 all these topics have a weak performance. Therefore the performance on these topics has significantly dropped.

4.0 CONCLUSION AND RECOMMENDATIONS

4.1 Conclusion

The analysis of the candidates' performance per topic shows that the average percentage of candidates who scored 30% or more on the questions examined from same topics in 2014 and 2015 are 56.4 and 46.5 respectively, showing that the candidates' performance has dropped in 2015. The topics on which the candidates had weak performance are *Matrices*, *Trigonometry* and *Integration*.

The reasons which have contributed to the candidates' weak performance on the topics of *Matrices*, *Trigonometry* and *Integration* include: inability of candidates to apply the technique of substitution and the standard formula to evaluate indefinite integrals of polynomials, inadequate

knowledge and skills to perform matrix operations; failure by candidates to use trigonometrical identities in answering questions; lack of skills to sketch graphs, poor algebraic/arithmetic skills and also occurrence of sign errors that affects the quality of their answers.

4.2 Recommendations

In order to improve future candidates' performance in this subject it is recommended that the students need to have an understanding of all the topics in the syllabus while putting more emphasis on the topics of Matrices, Trigonometry and Integration on which the candidates had significant lower scores when comparing the ACSEE 2014 and 2015 Basic Applied Mathematics examinations.

On the other hand, teachers should ensure that the specific objectives for all the subtopics that are stated in the syllabus must be achieved during the teaching learning process, to enable the students acquire adequate knowledge and skills. In addition the teachers should make sure that the topics which had weak performance should be given more attention during teaching. Furthermore, the factors which have contributed to candidates scoring low marks should be notified to the students and rectified in order to improve the performance in this subject.

Finally, the Ministry of Education and Vocational Training is advised to make use of this report to influence their policies and operations and to make a follow up on the teaching and learning in order to raise the standard of performance in this subject.

Appendix

Analysis of candidates' performance per topic in Basic Applied Mathematics

S/N	Topic	Number of Questions	2014		2015	
			The % of Candidates who Scored an Average of 30% and Above	Remarks	The % of Candidates who Scored an Average of 30% and Above	Remarks
1.	Statistics	1	73.00	Good	84.8	Good
2.	Linear Programming	1	82.90	Good	84.2	Good
3.	Calculating Devices	1	50.2	Good	66.9	Good
4.	Algebra	1	72.60	Good	50.1	Good
5.	Probability	1	60.20	Good	46.2	Average
6.	Differentiation	1	32.10	Average	45.4	Average
7.	Functions	1	48.90	Average	29.5	Average
8.	Matrices	1	71.50	Good	21.6	Weak
9.	Trigonometry	1	32.80	Average	21.3	Weak
10.	Integrations	1	39.50	Average	14.9	Weak
	Average Performance per Topic		56.40		46.50	

In this Appendix, green, yellow and red colours represent good, average and weak performance respectively.

