THE NATIONAL EXAMINATIONS COUNCIL OF TANZANIA

CANDIDATES' ITEMS RESPONSE ANALYSIS FOR ACSEE 2015

142 ADVANCED MATHEMATICS

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142 ADVANCED MATHEMATICS

(School Candidates)

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FOREWORD

The Items Response Analysis Report for Advanced Certificate of Secondary Education Examination (ACSEE) 2015 was prepared to provide feedback to candidates, teachers, parents, policy makers and public in general on how the candidates answered the Advanced Mathematics examination questions. It is a booklet with analytical information that shows candidates general performance in each question and a comparison of the performance with the previous year.

The Advanced Certificate for Secondary Education Examination is a summative evaluation, which among other things shows the effectiveness of the education system in general and the education delivery system in particular. The responses of the candidates to the examination questions reflect what the education system was able or unable to offer to the candidates in their two years of Advanced Secondary Education.

The analysis presented in this report aims to inform the stakeholders about the candidates' general performance in the Advanced Mathematics examination. The report highlights the factors that made the candidates perform well in 13 out of the 18 questions that were examined. These factors include the ability to identify the task and the requirement of the question, sufficient knowledge and skills on mathematics concepts, use of appropriate drawings and sufficient skills in drawing the graphs and the proper following of the questions' instructions.

However, the report pinpoints the reasons for the failure of some of the candidates to score high marks. Such reasons include insufficient skills to apply appropriate mathematics techniques and formulas, incompetence to manipulate equations, poor computation skills and inability to recall theorems, laws and principles.

The feedback provided in this booklet will assist the educational administrators, school managers, teachers, students and other stakeholders to identify proper actions to be taken in order to improve the candidates' performance in the future examinations conducted by the Council.

The National Examinations Council of Tanzania will highly appreciate remarks and suggestions from all stakeholders that can be used to improve future Items Reponse Analysis Report in Advanced Mathematics.

Dr. Charles E. Msonde

EXECUTIVE SECRETARY

1.0 INTRODUCTION

This report analyses the performance of candidates in Advanced Mathematics for the Advanced Certificate of Secondary Education Examination (ACSEE) that was done in May 2015. The examination assessed candidates' competences in accordance to 2010 Advanced Level Mathematics syllabus and adhered to the 2011 examination format.

The examination had two papers; 142/1 Advanced Mathematics 1 (paper 1) and 142/2 Advanced Mathematics 2 (paper 2). Paper 1 had ten (10) questions which carried 10 marks each and the candidates were supposed to answer all questions. Paper 2 had two sections, A and B, with a total of 8 questions. Section A had four (4) questions of 15 marks each and the candidates were required to answer all questions. Section B had four (4) questions of 20 marks each and the candidates were required to answer only two (2) questions.

In 2015, a total of 9,144 candidates sat for the examination out of which 85.02 percent passed the examination. In 2014 however, a total of 9,549 candidates sat for the examination, out of which 89.40 percent passed the examination. Therefore in 2015 there is a 4.38 percentage drop in the number of candidates who passed.

The analysis of the individual questions is presented in the next section. The presentation highlights the requirements of each question, the way the candidates answered and the analysis of their responses. The Extracts of the candidates' responses are used to illustrate the cases presented.

Finally, the report provides the analysis of questions topic-wise considering the performance in the statistical intervals 0 – 29 (weak), 30 – 49 (average) and 50 – 100 (good). The report also provides the conclusion and recommendations which will be useful to different stakeholders such as the candidates, teachers, parents, educationists and the Government at large. It is expected that, the report will enhance teaching and learning of Advanced Mathematics and improve the performance of candidates on the topics that were performed poorly.

2.0 ANALYSIS OF PERFORMANCE IN EACH QUESTION

2.1 142/1 – ADVANCED MATHEMATICS 1

2.1.1 Question 1: Calculating Devices

The question comprised of parts (a) and (b) and it required candidates to:

(a) Use non–programmable calculators to:

(i) Calculate $\log_e\left(e^4 + 2\ln 5\right) + \log 5$ correct to six decimal places.

(ii) Obtain the value of $\sqrt{\dfrac{(4.03)^3 \times (814765)^{0.5}}{\sqrt{5}}}$ correct to three significant figures.

(iii) Find the value $\left(\dfrac{{}^6C_2 \times \ln 2}{\sqrt[3]{43}}\right) \times \begin{vmatrix} 2e & \ln 2 \\ e & \ln 2 \end{vmatrix}$ correctly to four decimal places.

(b) Evaluate $\sum_{y=2}^{3} e^{\ln y}\left(1 + (y+1)\ln y\right)$ to four significant figures.

A total of 9,147 (99.3%) candidates responded to the question whereas 73.6 percent scored from 3 to 10 marks with 15.8 percent scoring 10 marks. However, few candidates (26.4%) scored from 0 to 2.5 marks out of which 9.9 percent scored a 0 mark. Thus the candidates' general performance on this question was good.

The candidates who got full marks performed the given mathematical computations accordingly an indication that they had sufficient skills of using non-programmable calculators. Extract 1.1 shows a sample of responses from the script of one of the candidates who applied the skills of using calculators and mathematical manipulations accurately.

Extract 1.1

1	a) i) 4.756253
	ii) 163

	iii) 5.5917
	b) 22.34

Extract 1.1 shows the work of one of the candidates who was able to use a non-programmable calculator correctly in solving the given items.

However, 905 (9.9%) candidates who failed to use the non-programmable calculator lacked the skills of using it. Some of them did not have knowledge of the rounding off techniques such as making approximations into decimal places or significant figures. Extract 1.2 shows one of these mistakes.

Extract 1.2

ii) $\sqrt{\dfrac{(4.03)^3 \times (814765)^{0.5}}{\sqrt{5}}}$

$\Rightarrow \sqrt{\dfrac{65.450827 \times 902.6433404}{2.236067978}}$

$\Rightarrow 162.545.$

$\sqrt{\dfrac{(4.03)^3 \times (814765)^{0.5}}{\sqrt{5}}} = 162.545$

In Extract 1.2, the candidate expressed the answer in three decimal places while he/she was supposed to express it in three significant figures an indication of having inadequate skills in writing significant figures.

2.1.2 Question 2: Hyperbolic Functions

This question was one of the three questions that were well performed. The question consisted of parts (a), (b) and (c). The candidates were demanded to:

(a) (i) Express $4\cosh\theta + 5\sinh\theta$ in the form $r\sinh(\theta+\alpha)$ giving the values of r and $\tanh\alpha$.

(ii) Prove that $\cosh^{-1} x = \ln\left(x + \sqrt{x^2 - 1}\right)$.

(b) (i) Show that $\frac{1}{2}\sinh(2\ln x)\cosh(2\ln x) = \frac{1}{8x^4}(x^8 - 1)$.

(ii) Find the possible values of $\sinh x$ if $\begin{vmatrix} \cosh x & -\sinh x \\ \sinh x & \cosh x \end{vmatrix} = 2$ leaving the answer in surd form.

(c) Sketch the graph of $y = \sinh^{-1} x$ on the x-y plane and state its domain and range.

The question was attempted by 99.3 percent of the candidates of which 15.9 percent scored below 3 marks, 28.1 percent scored from 3 to 5.5 marks and 55.9 scored from 6 to 10 marks with 101 (1.1%) candidates scoring 10 marks. The analysis shows that this question had a good performance because the percentage of candidates who scored from 3 to 10 marks is 84.1.

The analysis of the candidates' responses indicated that candidates who scored high marks were able to; define the hyperbolic functions, converted the inverse hyperbolic functions into logarithmic functions correctly and sketched the graph of the inverse hyperbolic functions $y = \sinh^{-1} x$ accordingly. A sample of the work of one of the candidates who answered this question correctly is shown in Extract 2.1.

Extract 2.1

2 (a) $4\cosh\theta + 5\sinh\theta$ in form of $r\sinh(\theta + \alpha)$

Expansion.

$r[\sinh\theta\cosh\alpha + \cosh\theta\sinh\alpha]$.

comparing

$r\sinh\theta\cosh\alpha + r\sinh\alpha\cosh\theta = 4\cosh\theta + 5\sinh\theta$

$R^2\sinh^2\alpha = 4^2$ —(i)

$R^2\cosh^2\alpha = 5^2$ —(ii)

$(\cosh^2\theta - \sinh^2\theta)r^2 = 5^2 - 4^2$

$r^2 = 5^2 - 4^2$

$r = \sqrt{25 - 16}$

$r = 3.$

$r = 3$

Then from $r\sinh\alpha = 4$

$3\sinh\alpha = 4$

$\sinh\alpha = 4/3$

$\alpha = \sinh^{-1}(4/3)$

$\alpha = 1.0986$

$\therefore$ In form of $r\sinh(\theta + \alpha) = 3\sinh(\theta + 1.0986)$

$\tan\alpha = \left(\frac{4}{5}\right) = 0.8$

$\therefore$ $r = 3$ and $\tanh\alpha = 0.8$

(ii)

$\cosh^{-1}x = \ln(x + \sqrt{x^2 - 1})$

let $\cosh^{-1}x = y$

$\cosh y = x$

$\frac{e^y + e^{-y}}{2} = x$

$e^y + e^{-y} = 2x$

$e^{2y} + e^{-y+y} = 2xe^y$

$e^{2y} + 1 = 2xe^y$

$e^{2y} - 2x e^y + 1 = 0.$

$e^y = \frac{2x \pm \sqrt{4(x)^2 - 4}}{2}$

$e^y = \frac{2x \pm 2\sqrt{x^2 - 1}}{2}$

$e^y = \frac{2x + 2\sqrt{x^2 - 1}}{2}$

$e^y = x + \sqrt{x^2 - 1}$

Introduce ln

$y \ln e = \ln(x \pm \sqrt{x^2 - 1})$

But -ve of logarithm does not exist then

$y = \ln(x + \sqrt{x^2 - 1})$

$y = \ln(x + \sqrt{x^2 - 1})$

proved

(b) (i) $\frac{1}{2} \sinh(2\ln x) \cosh(2\ln x) = \frac{1}{8x^4}(x^8 - 1)$

$\frac{1}{2} \sinh(\ln x^2) \cosh(\ln x^2)$.

$\sinh x = \frac{e^x - e^{-x}}{2}$, $\cosh x = \frac{e^x + e^{-x}}{2}$.

$\frac{1}{2}\left[\frac{e^{\ln x^2} - e^{-\ln x^2}}{2}\right]\left[\frac{e^{\ln x^2} + e^{-\ln x^2}}{2}\right]$

$\frac{1}{8}\left[e^{\ln x^2} - e^{-\ln x^2}\right]\left[e^{\ln x^2} + e^{-\ln x^2}\right]$

$\frac{1}{8}\left[x^2 - \frac{1}{x^2}\right)\left(x^2 + \frac{1}{x^2}\right)$

$\frac{1}{8}\left[x^4 + 1 - 1 + (-\frac{1}{x^4})\right]$

$\frac{1}{8}\left[x^4 + (-\frac{1}{x^4})\right]$ factor out $\frac{1}{x^4}$.

$\frac{1}{8}\left[x^8 - 1\right]\frac{1}{x^4}$

$\frac{1}{8x^4}\left[x^8 - 1\right]$

$= \frac{1}{8x^4}\left[x^8 - 1\right]$

proved.

(ii) $\begin{vmatrix} \cosh x & -\sin x \\ \sinh x & \cosh x \end{vmatrix}$

$\cosh^2 x + \sinh^2 x = 2.$

but

$\cosh^2 x - \sinh^2 x = 1$

$\cosh^2 x = 1 + \sinh^2 x.$

$1 + \sinh^2 x + \sinh^2 x = 2$

$2\sinh^2 x = 2 - 1$

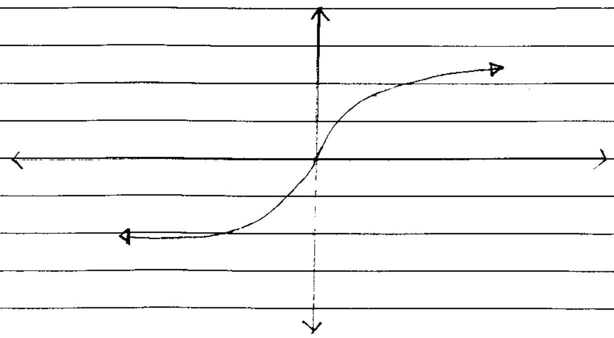
$2\sinh^2 x = 1$

$\sqrt{\sinh^2 x} = \sqrt{1/2}$

$\sinh x = \pm\sqrt{1/2}.$

$\sinh x = \sqrt{1/2}$

(c) A graph of $y = \sinh^{-1} x$.



Domain : {All real numbers}

Range : {All real numbers}.

Extract 2.1 shows the work of one of the best responses of the candidate who recalled and applied hyperbolic identities, performed computations and sketched the graph of hyperbolic inverse functions correctly. Moreover, he/she converted the inverse hyperbolic functions into logarithmic functions correctly.

Despite these strengths, there were few candidates who performed poorly in this question. The candidates lacked skills and knowledge to define hyperbolic functions. Other candidates lacked the technique of letting $y = \cosh^{-1} x$ which is equivalent to $x = \frac{1}{2}\left(e^{y} + e^{-y}\right)$, the base that would serve to prove that $\cosh^{-1} x = \ln\left(x + \sqrt{x^2 - 1}\right)$. Some candidates were noted to sketch the graph of $\sinh^{-1} x$ without showing arrows and also did not manage to expand the determinant $\begin{vmatrix} \cosh x & -\sinh x \\ \sinh x & \cosh x \end{vmatrix} = 2$. Other factors that contributed to the poor performance of the candidates include; failure to distinguish between hyperbolic functions and trigonometric functions, for instance using the Pythagoras' theorem $\left(a^2 + b^2 = c^2\right)$ in hyperbolic functions instead of making the slight amendment $\left(b^2 - a^2 = c^2\right)$, failure to distinguish between the graphs of hyperbolic sine function $y = \sinh x$ and the inverse hyperbolic sine function $y = \sinh^{-1} x$ and neglecting the negative signs in the hyperbolic functions, for example, some candidates wrote, $\cosh x = \frac{e^{x} + e^{x}}{2}$ instead of $\cosh x = \frac{e^{x} + e^{-x}}{2}$. Extract 2.2 depicts one of these weaknesses.

Extract 2.2

(b) (ii) Soln.

$$\frac{\cosh x - \sinh x}{\sinh x + \cosh x} = 2.$$

Rationalizing denominator.

$$\frac{(\cosh x - \sinh x)(\sinh x + \cosh x)}{\cancel{\sinh}\ \cosh^2 x - \sinh^2 x}.$$

$$\frac{\sinh x \cosh x + \cosh^2 x - \sinh^2 x + \sinh x \cosh x}{\sinh^2 x - \cosh^2 x}.$$

In Extract 2.2, the candidate failed to expand the determinant in order to obtain the required values of $\sinh x$ an indication of lack of understanding of the concepts of matrices/determinants and hyperbolic functions.

2.1.3 Question 3: Linear Programming

The question examined candidates' knowledge on understanding of the concepts of linear programming and transportation problem. The question consisted of parts (a) and (b). Part (a) stated as follow "A company owns two mines. Mine A produces 1 ton of high grade ore, 3 tons of medium grade ore and 5 tons of low grade ore each day; and mine B produces 2 tons of each of the three grades of ore each day. The company needs 80 tons of high grade ore, 160 tons of medium grade ore and 200 tons of low grade ore." The candidates were then required to find the number of days each mine should be operated if it costs shs 200,000/= per day to operate each mine.

Part (b) stated as follows: "A sugar company ships sugar from two origins S_1 and S_2 to three market centers M_1, M_2 and M_3. The table showing the available tons of sugar and the required tons together with the unit transportation cost in shillings is shown below:"

	M_1	M_2	M_3	Available
S_1	20	10	5	220
S_2	10	25	30	100
Requirement	120	80	120	

Candidates were then required to:

(i) Use the given information in the table to formulate the objective cost function (Z) to be minimized.

(ii) Write down all equalities and inequalities of the transportation problem.

(iii) Verify whether the transportation problem in 3 (b) is a balanced one or not.

Use Xij's to denote the amount transported from source i to destination j.

The question was attempted by 99.3 percent of the candidates whereby 58.9 percent scored below 3 marks, 17.5 percent scored from 3 to 5.5 marks and 23.6 percent scored from 6 to 10 marks. The analysis has also shown that there was only one candidate who scored all the 10 marks. It was noted that this question is among the three questions that had the average performance because the percentage of candidates who scored 30 percent or more of the marks that were allocated for this question was 41.1 percent. Further analysis has proved that only six candidates scored from 8.5 to 10 marks.

The analysis of candidates' responses shows that many candidates scored highly in part 3 (a) as compared to part 3 (b). The candidates who answered well part (a) were able to identify the required decision variable and formulated the correct objective functions, constraints, equalities as well as the inequalities. The candidates were also able to verify that the transportation problem is balanced. Other reasons that contributed to average performance in this question include candidates' ability to draw correct graphs for the inequalities that represented the constraints and the ability to identify the feasible region in order to calculate the corner points. Extract 3.1 portrays the sample responses from one of the candidates with best solutions.

Extract 3.1

3. a)

	Mine A	Mine B	Minimum
H·G O	1	2	80
M G O	3	2	160
L G O	5	2	200

Objective function

let x represent number of days for mine A and y for mine B

$f(x, y) = 200,000x + 200000y$ — (1)

Inequalities	Equations
$x + 2y \geq 80$	$x+2y=80$ (0,40) (80,0)
$3x + 2y \geq 160$	$3x+2y=160$ (0,80) ($\frac{160}{3}$, 0)
$5x + 2y \geq 200$	$5x+2y=200$ (0,100) (40, 0)
$x \geq 0$	$x=0$
$y \geq 0$	$y=0$

Corner points

A (0,100) $f(A) =$ 20,000,000/=

B (20,50) $f(B) =$ 14,000,000/=

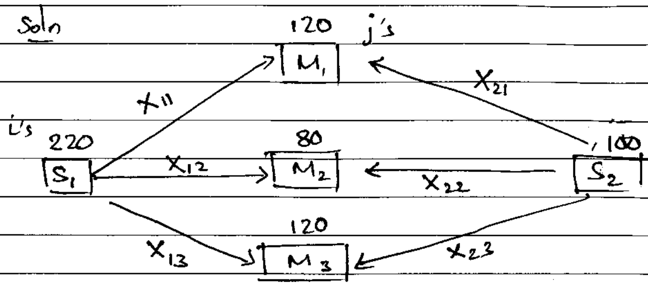
C (40,20) $f(C) =$ 12,000,000/=

D (80,0) $f(D) =$ 16,000,000/=

C is the optimal point $x = 40$ $y = 20$

3\. a) Mine A should be operated for 40 days and mine B for 20 days.

b) Soln



i) Objective cost function

$$f(x) = 20X_{11} + 10X_{12} + 5X_{13} + 10X_{21} + 25X_{22} + 30X_{23}$$

$$f(x) = Z$$

$$Z = 20X_{11} + 10X_{12} + 5X_{13} + 10X_{21} + 25X_{22} + 30X_{23}$$

ii) Equalities.

$$X_{11} + X_{12} + X_{13} = 220$$

$x_{11} + x_{21} = 120$

$x_{21} + x_{22} + x_{23} = 100$

$x_{12} + x_{22} = 80$ and $x_{13} + x_{23} = 120$

Inequalities

$x_{11} \leq 120$	$x_{11} \geq 0$
$x_{12} \leq 80$	$x_{12} \geq 0$
$x_{13} \leq 120$	$x_{13} \geq 0$
$x_{21} \leq 120$	$x_{21} \geq 0$
$x_{22} \leq 80$	$x_{22} \geq 0$
$x_{23} \leq 120$	$x_{23} \geq 0$

b) iii) for a balanced problem

Sum of supply (sugar available) = Sum of demand

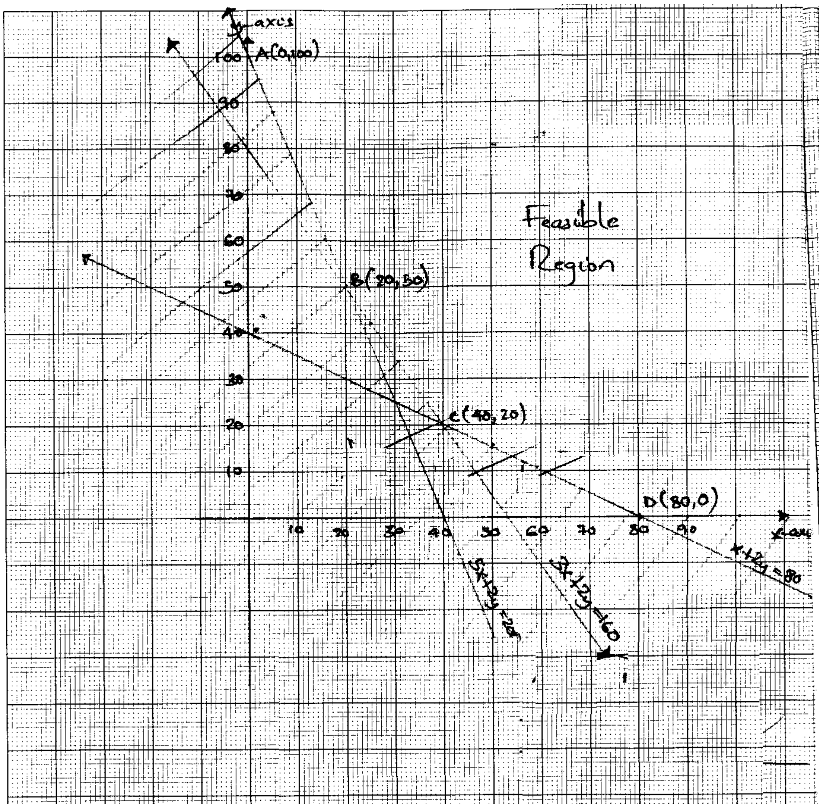
from the problem

Available sugar = 220 + 100 = 320 units

Demand (requirement) = 120 + 80 + 120 = 320 units

Hence the transportation problem is a balanced one.

3. a)



Extract 3.1 shows one of the best responses where the candidate was able to formulate the correct objective function, used the correct inequalities for the constraints, drew correct graphs, identified the feasible region and was able to solve the transportation problem using the notation that was given X_{ij} correctly.

The candidates with low marks used the symbol $\leq$ instead of $\geq$ in part 3 (a) to formulate the constraints. Such candidates scored $01^{1}/_{2}$ marks only of drawing the graphs of $x + 2y = 80$, $3x + 2y = 160$ and $5x + 2y = 200$. In part (b), the candidates used the variables x, y and z instead of the decision variable X_{ij} instructed in the question. Another surprising error that recurred frequently was failure to indicate whether the objective function was to be maximized or minimized and being unable to shade the required region. Extract 3.2 shows a sample response from a candidate with poor solution.

Extract 3.2

3. let Mine A be x
Mine B be y.

Mine A	high grade	low grade
Mine B		

	high grade	low grade
Mine A		
Mine B		

	high grade	medium grade	low grade
Mine A	1 ton	3 tons	5 tons
Mine B	2 tons	2 tons	2 tons
	80 tons	160 tons	200 tons

$x + 2y \leq 80$

$3x + 2y \leq 160$

$5x + 2y \leq 200$.

objective function

$f(x,y) = 200,000x + 200,000y$

Extract 3.2 illustrates the work from a candidate who used wrong inequality signs in formulating the linear constraints. He/she did not state whether the objective was to be maximized or minimized.

2.1.4 Question 4: Statistics

The candidates were given the information that, "Kamunonge cooperative farm has 20 branches, each recorded one among the following sales of wheat last month: 6.1, 11.0, 22.3, 34.6, 37.5, 34.3, 29.4, 10.9, 1.5, 5.4, 3.2, 15.6, 27.6, 21.7, 20.5, 31.3, 47.9, 46.3, 41.4 and 48.2" They were required to group the data into class intervals 0 – 10, 10 – 20, etc. and determine the following; (a) (i) Mode of the data correct to 4 significant figures, (ii) Median of the data, (iii) Standard deviation correct to 4 significant figures and (b) the lower and upper quartiles.

This question was attempted by 99.3 percent of candidates of which 26.9 percent scored below 3 marks and 76.4 percent scored from 3 to 10 out of 10 marks. The analysis has shown that 1348 (14.7%) candidates scored all the marks allocated to this question. The analysis shows that the performance of candidates in question 4 was good.

The analysis of the candidates' responses indicates that the candidates who scored high marks used the class intervals 0 – 10, 10 – 20, etc to make the frequency distribution table and used the correct formulae: $\text{Mode} = L_1 + \left(\frac{\Delta_1}{\Delta_1 + \Delta_2}\right)C$, $\text{Median} = L + \left(\frac{\frac{N}{2} - n_b}{n_w}\right) \times C$,

$S.D = \sqrt{\frac{1}{n} \times \sum X^2 f - \left(\frac{\sum fX}{n}\right)^2}$, $Q_1 = l + \frac{\left(\frac{N}{4} - C\right)h}{f}$ and $Q_3 = l + \frac{\left(\frac{iN}{4} - C\right)h}{f}$

to calculate the mode, median, standard deviation, lower and upper quartiles respectively. The candidates also presented the mode and standard deviation correctly to four significant figures. A sample answer from the script of one of the candidates who performed well is shown in Extract 4.1.

Extract 4.1

4.	Class interval	f	cf	x	$f(x)$	$x-\bar{x}$	$(x-\bar{x})^2$	$f(x-\bar{x})^2$
	0 – 10	4	4	5	20	−20.5	420.25	1681
	10 – 20	3	7	15	45	−10.5	110.25	330.75
	20 – 30	5	12	25	125	−0.5	0.25	1.25
	30 – 40	4	16	35	140	9.5	90.25	361
	40 – 50	4	20	45	180	19.5	380.25	1521
					$\Sigma fx = 510$		$\Sigma f(x-\bar{x})^2 = 3895$	

$\bar{x} = \frac{\Sigma fx}{N}$

$\bar{x} = \frac{510}{20}$

$= 25.5$

a)ii) Median $= L + \left(\frac{N/2 - nb}{nw}\right)c.$

Class interval is 20–30.

$L = 20 \quad nb = 7 \quad nw = 5 \quad c = 10. \ N = 20$

Median $= 20 + \left(\frac{20/2 - 7}{5}\right)10$

$= 20 + (10-7)2$

$= 20 + 6$

$= 26.$

i) Mode $= L + \left(\frac{t_1}{t_1 + t_2}\right)c$

Modal class (20–30)

$L = 20 \quad t_1 = 2 \quad t_2 = 1 \quad c = 10.$

Mode $= 20 + \left(\frac{2}{2+1}\right)10$

$= 20 + (2/3)\ 10$

$= 20 + 6.667$

4a)i) Mode $= 2.667 \times 10^1$

iii) Standard Deviation $= \sqrt{\frac{\Sigma f(x-\bar{x})^2}{N}}$

$= \sqrt{\frac{3895}{20}}$

$= \sqrt{194.75}$

$= 1.396 \times 10^1.$

b). lower quartiles (Q_1).

$Q_1 = L + \left(\frac{N/4 - nb}{nw}\right)c$

$Q_1 = \frac{N}{4} = \frac{20}{4} = 5$

Class interval (10–20)

~~Q~~ $L = 10 \quad c = 10 \quad nb = 4 \quad nw = 3 \quad N/4 = 5.$

$Q_1 = L + \left(\frac{N/4 - nb}{nw}\right)c$

$= 10 + \left(\frac{5-4}{3}\right) 10$

$= 10 + 3.33$

$= 13.33$

The upper quartile Q_3

$Q_3 = L + \left(\frac{3/4 N - nb}{nw}\right) c$

$Q_3 = \frac{3N}{4} = \frac{3 \times 20}{4} = 3 \times 5 = 15.$

4.b) Class interval of (30-40)

$L = 30$ $c = 10$ $nb = 12$ $nw = 4.$ $3N/4 = 15$

$Q_3 = L + \left(\frac{3N/4 - nb}{nw}\right) c$

$= 30 + \left(\frac{15 - 12}{4}\right) 10$

$= 30 + \frac{3}{4} \times 10$

$= 30 + 7.5$

$= 37.5.$

Extract 4.1 shows the work of one of the candidates who performed well in question 4. He/she used the appropriate formulas.

Despite these strengths, there were few candidates (17.8%) who performed poorly in this question. The candidates failed to apply the required formulae appropriately. For example, in the formula $\text{Mode} = L_1 + \left(\frac{\Delta_1}{\Delta_1 + \Delta_2}\right) C$, some of them used the values of Δ_1 in the position of Δ_2 and vice versa. The candidates also failed to identify the lower class limits from the continuous class intervals as they subtracted 0.5 from these limits. Extract 4.2 illustrates how the candidates performed poorly in part 4 (a) (i).

Extract 4.2

$$\text{Mode} = L + \left(\frac{d_1}{d_1+d_2}\right)i$$

$$\text{Mode} = 20 + \left(\frac{1}{1+2}\right)10$$

$$= 20 + \frac{1}{3}\times 10$$

$$= 23\cdot 333 = 23\cdot 33$$

(into four significant figures)

$$= \underline{\underline{23\cdot 33}}$$

Extract 4.2 depicts a solution from a candidate who substituted the figures for d_1 and d_2 in the formula incorrectly.

2.1.5 Question 5: Sets

The question aimed to test candidates' knowledge on Sets and had parts, (a), (b) and (c). Part (a) demanded the candidates to (i) use Venn diagram to show that $(A \cap B) \cup (A' \cap B) = B$ and (ii) find the members of set R where $R = \left\{x : \frac{x^2 - 9}{x^2 - 1} \le 0; x \in \Re\right\}$.

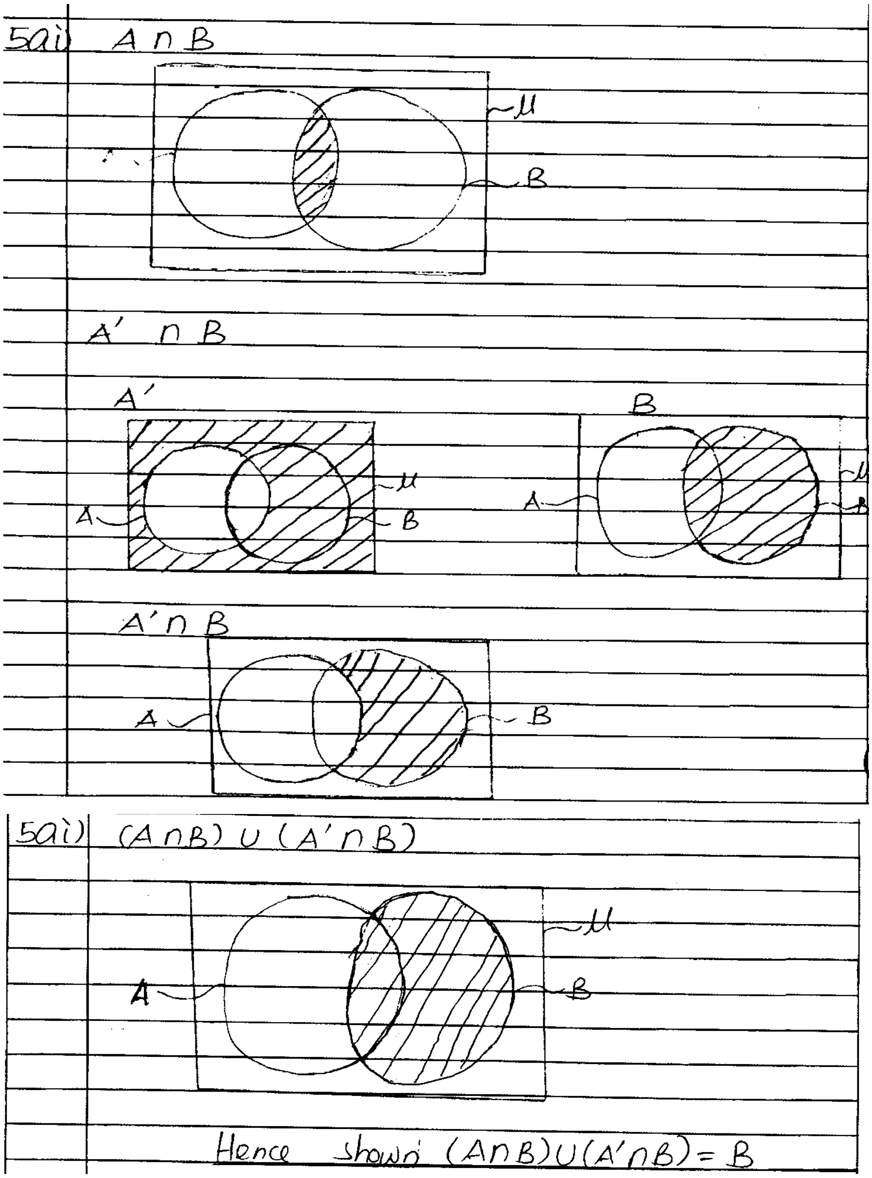
In part (b), candidates were required to use the basic properties of set operations to simplify (i) $(A \cap B) \cup (A - B)$ and (ii) $\left[(A \cup B)' \cap (A \cap B)'\right]'$.

Part (c) had the following information "In a bunch of twenty flowers, twelve of the flowers are yellow and nine of the flowers are red. If four of the flowers are neither yellow nor red", candidates were then required to find the number of the flowers that are both yellow and red by using Venn diagram.

The analysis of data shows that 9,146 (99.3%) candidates attempted this question whereas 13.2 percent scored below 3 marks, 24.4 percent scored from 3 to 4.5 marks and 62.4 percent scored from 5 to 10 marks with 23 (0.3%) candidates scoring full marks. This is the second best performed question in this examination.

The candidates who scored higher marks applied accordingly, the laws of algebra of sets to simplify set expressions and found the required members of set R. Moreover, they used Venn diagram correctly to show that $(A \cap B) \cup (A' \cap B) = B$. Similarly, they used Venn diagram to find the number of flowers that were both yellow and red as required. Extract 5.1 shows a sample answer from one of these candidates.

Extract 5.1



5ai) $R = \left\{ x : \frac{x^2-9}{x^2-1} \le 0 , x \in R \right\}$

$x^2 - 9 = (x-3)(x+3)$

$x = 3 \quad , x = {}^{-}3$

$x^2 - 1 = (x-1)(x+1)$

$x = 1 , \quad x = {}^{-}1 .$

	$-\infty \le x \le -3$	$-3 \le x < 1$	$-1 \le x < 0$	$0 \le x < 1$	$1 < x \le 3$	$3 \le x < \infty$
$f(x)$	$+$	$-$	$+$	$+$	$-$	$+$

The values which bring $\frac{x^2-9}{x^2-1} \le 0$ lie between ${}^{-}3 \le x < -1$ and $1 < x \le 3$

$\therefore R = \{ x : {}^{-}3 \le x < {}^{-}1 ; 1 < x \le 3 \}$

5bi) $(A \cap B) \cup (A - B)$

$(A \cap B) \cup (A \cap B')$ set difference.

$A \cap (B \cup B')$ Distributive law

$A \cap U$ Compliment law

A Identity law.

5bii) $[(A \cup B)' \cap (A \cap B)']'$

$(A \cup B) \cup (A \cap B)$ Demorgan's law

let $A \cup B = X$

$X \cup (A \cap B)$

$(X \cup A) \cap (X \cup B)$ Distributive law

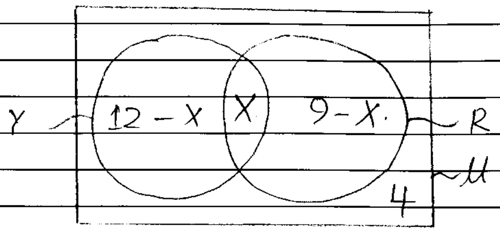
$(A \cup B \cup A) \cap (A \cup B \cup B)$

$((A \cup A) \cup B) \cap (A \cup (B \cup B))$ Commutative law.

$(A \cup B) \cap (A \cup B)$ Idempotent law.

$A \cup B$ Idempotent law

5C Let Yellow flowers be Y
Red flowers be R



$12 - X + X + 9 - X + 4 = 20$

$21 - X + 4 = 20$

$25 - X = 20$

$X = 25 - 20$

$X = 5$

$\therefore$ 5 FLOWERS

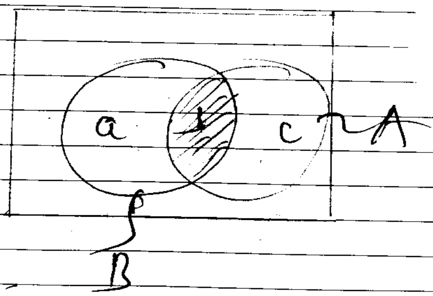
Extract 5.1 illustrates a sample solution from a candidate who was able to show that $(A \cap B) \cup (A' \cap B) = B$ using Venn diagram, find the members of set R by inspection method, simplified the given set expressions and evaluated the number of flowers with both colors using Venn diagram correctly.

Despite this general good performance, there were few candidates who performed poorly in this question. The candidates were unable to; apply the laws of algebra of sets in simplifying set expressions, failed to use Venn diagram to show that $(A \cap B) \cup (A' \cap B) = B$ and used a table of values to obtain the members of set R instead of the members which satisfy $\frac{x^2 - 9}{x^2 - 1} \le 0$. Additionally, some candidates ignored the sign "≤" in solving the inequality $\frac{x^2-9}{x^2-1} \leq 0$, instead they replaced it with either the symbol < or = while others multiplied both sides of the inequality $\frac{x^2-9}{x^2-1} \leq 0$ by x^2-1 which resulted into getting wrong members of set R. In part (c), the candidates used the formula instead of using the Venn diagram to find the number of flowers that are both yellow and red as was instructed. Extract 5.2 shows a sample answer from one of these candidates.

Extract 5.2

5) (a) (i) $(A\cap B) \cup (A'\cap B) = B$

let



$U = \{a, b, c\}$

$A = \{b, c\}$

$B = \{a, b\}$

$(A\cap B) = \{b\}$

$A' = \{c\}$, $(A\cap B)\cup(A'\cap B) = \{b\}$

$A'\cap B = \{\emptyset\}$ hence shown

Extract 5.2 shows the work of a candidate who failed to use Venn diagram to show that $(A\cap B)\cup(A'\cap B) = B$. He/she used numbers and shading approach which was wrong.

2.1.6 Question 6: Functions

This question had two parts (a) and (b). In part (a), the candidates were required to (i) determine the value of k for which $f\circ g(x) = g\circ f(x)$ and (ii) prove that $f\circ(f\circ f(x)) = 125x + 124$ if

$f : x \rightarrow 5x + 4$ and $g : x \rightarrow 6x - k$. Part (b) required the candidates to draw the graph of $\frac{2x^3}{x^2 - 9}$.

The question was attempted by 99.3 percent of the candidates of which the majority (93.7%) scored from 3 to 10 marks. Further analysis has shown that only 575 (6.3%) candidates scored below 3 marks. The analysis revealed that this was the best performed question in the examination.

The analysis of the responses of the candidates shows that, those who scored high marks managed to answer all parts correctly. In part (a), the candidates demonstrated sufficient knowledge and skills in the topic of composite functions while in part (b), most of them showed good understanding in drawing the graph of $\frac{2x^3}{x^2 - 9}$. Extract 6.1 shows a sample of a response from a script of a candidate who answered this question correctly.

Extract 6.1

6. (a) $f(x) = 5x + 4$

$g(x) = 6x - k$

i/ $fog(x) = f(6x - k)$

$= 5(6x - k) + 4$

$= \cancel{30x - 30k}$

$= 30x - 5k + 4$

$gof(x) = g(5x + 4)$

$= 6(5x + 4) - k$

$= 30x + 24 - k$

$fog(x) = gof(x)$

$\cancel{30x} - 5k + 4 = \cancel{30x} + 24 - k$

$-5k + k = 24 - 4$

$-4k = 20$

$k = -5$

$\therefore \quad k = -5$

ii) $fo(fof(x)) = 125x + 124$

$fof(x) = f(5x+4)$

$= 5(5x+4) + 4$

$= 25x + 20 + 4$

$= 25x + 24$

$fo(25x+24) = 5(25x+24) + 4$

$= 125x + 120 + 4$

$= 125x + 124$

$\therefore \quad fo(fof(x)) = 125x + 124$ hence proved

6. (b) $y = \dfrac{2x^3}{x^2 - 9}$

Vertical asymptotes

x	-5	-4	-3	-2	-1	0	1	2	3	4	5
y	-15.6	-18.3	α	3.2	0.25	0	-0.25	-3.2	α	18.3	15.6

$x^2 - 9 = 0$

$x^2 = 9$

$x = \pm 3$

$x = 3, \quad x = -3$

$$\begin{array}{r|l} & 2x \\ x^2 - 9 & 2x^3 \\ - & 2x^3 - 18x \\ \hline & 18x \end{array}$$

$= 2x + \dfrac{18x}{x^2 - 9}$

$\dfrac{18x}{x^2-9}$

$\dfrac{18/x}{1 - 9/x} \quad x \longrightarrow \alpha$

$= 0$

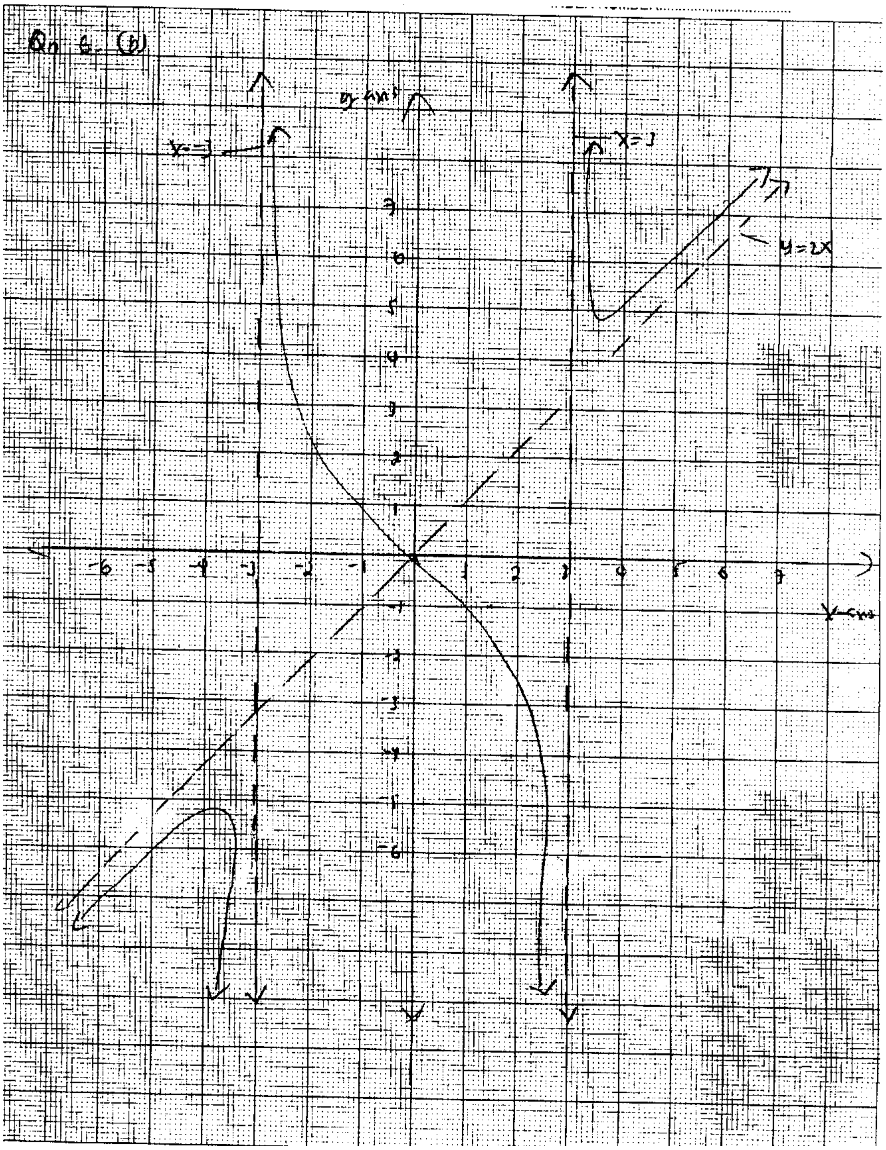
$y = 2x$

Slant asymptotes $y = 2x$

Y-intercepts $x = 0$, $y = 0$ $(0, 0)$

X-intercepts $y = 0$ $x = 0$ $(0, 0)$

⇒The graph is on the graph paper



Extract 6.1 shows the solution from a candidate who was able to determine the value of *k* from the given equation. The candidate

was also able to prove that $f \circ (f \circ f(x)) = 125x + 124$. In part (b), he/she drew the graph of the rational function as required.

Nevertheless, the candidates who scored low marks were unable to determine the asymptotes particularly the oblique asymptote. Some of the candidates were did not realize that they were supposed to calculate few points on the graph that was an essential step in showing the behavior of the graph of $\frac{2x^3}{x^2-9}$ in positive and negative regions. Other reasons that contributed to such poor performance include: failure to distinguish horizontal asymptotes from oblique asymptotes and using continuous lines to represent the asymptotes instead of dotted lines. A sample response from one of the candidates who faced the challenges of this kind is shown in Extract 6.2.

Extract 6.2

6 (b) $\sqrt{x^2} = \sqrt{9}$

$x = \pm 3.$

$\therefore V.A = \pm 3.$

~~For~~ Horizontal asymptotes

Take

$y = \frac{2x^3}{x^2-9}$ divide by x^3 both sides

$y = \frac{\frac{2x^3}{x^3}}{\frac{x^2}{x^3} - \frac{9}{x^3}}$

$y = \frac{2}{\frac{1}{x} - \frac{9}{x^3}}$

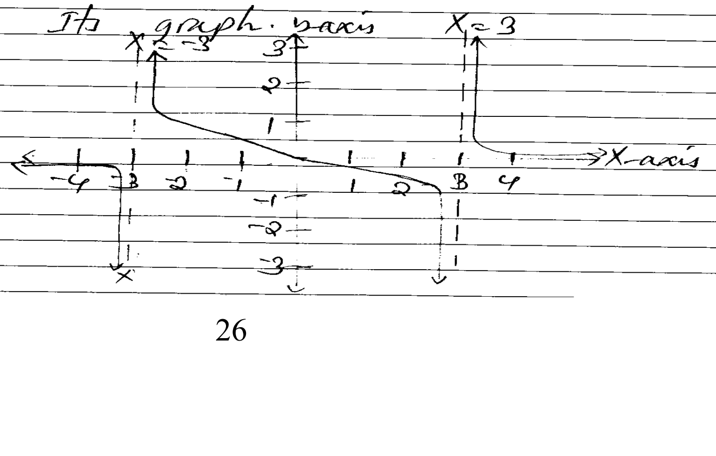
As $\frac{1}{x}$ ~~and~~ $\to 0$

$y = \frac{2}{0 - 9/0^3}$

$y = \frac{2}{0}$

$\therefore$ There is no Horizontal asymptote.

Its graph. y-axis



In Extract 6.2, the candidate divided the numerator and denominator of $\frac{2x^3}{x^2-9}$ by x^3 instead of x^2 as a result could not obtain the required oblique asymptote.

2.1.7 Question 7: Numerical Methods

The question had two parts namely (a) and (b) and the candidates were asked to (a) start with $x_0 = -1$ to approximate the root of $f(x) = x + e^x$ in four iterations using the Newton-Raphson method, presenting all the iterations in five significant figures. In part (b), the candidates were required to (i) apply both Simpson's and Trapezium rule with eleven ordinates to find an approximate value of $\int_0^2 \sin(1+\sqrt{x})dx$ correct to four decimal places and (ii) explain why the Simpson's rule is said to be more efficient than the trapezium rule.

The analysis of data shows that 9,146 (99.3%) candidates attempted this question. Majority of the candidates (52.4%) scored below 3 marks with 9 percent of them scoring a 0 mark. It was revealed that 21.1 percent scored from 3 to 4.5 marks and 2,427 (26.5%) candidates scored from 5 to 10 marks. This indicates that the question was averagely performed as 47.6 percent of candidates scored 3 marks or above.

In part (a), the candidates who performed poorly were using the formula $x_{n+1} = x_n + \frac{f(x_n)}{f'(x_n)}$ or $x_{n+1} = x_n - \frac{f'(x_n)}{f(x_n)}$ instead of $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$. In part (b), the candidates were using the non-programmable calculators while in degree mode instead of radian mode which resulted into getting wrong figures of ordinates. Some candidates did not follow the instructions as described in this question, for instance presenting iterations in four decimal places instead of five significant figures. Extract 7.1 illustrates these mistakes.

Extract 7.1

7 (b) i)
$n = 11 - 1$
$n = 10$

$$\int_0^2 \sin\left(1+\sqrt{x}\right)dx$$

~~by trapezium rule.~~

$A = \frac{h}{2}\left[y_n + y_0 + 2\sum \text{middle value}\right]$

$h = \frac{a-b}{n} = \frac{2-0}{10} = 0.2.$

x	0	0.2	0.4	0.6	0.8	1
f(x)	0.01749	0.02525	0.028487	0.03096	0.03306	0.034899
	y_0	y_1	y_2	y_3	y_4	y_5

1.2	1.4	1.6	1.8	2
0.03656	0.03809	0.03952	0.04085	0.04212
y_6	y_7	y_8	y_9	y_{10}

$A = \frac{0.2}{2}\left[(0.01745 + 0.04212) + 2\left[0.02525 + 0.028487\right.\right.$
$+ 0.03096 + 0.03306 + 0.034899 +$
$\left.\left.0.03656 + 0.03809 + 0.03952 + 0.04085\right]\right.$

$A = 0.0674922$
$A = 0.0675$ (4 decimal place)

~~from simpson rule.~~

$A = \frac{h}{3}\left[y_n + y_0 + 4\sum \text{odd} + 2\sum \text{even value}.\right]$

$A = \frac{0.2}{3}\left[0.017625 + 0.042212\right] + 4\left[0.02525\right.$
$+ 0.03096 + 0.036899 + 0.03809 +$
$\left.0.04085\right] + 2\left[0.028487 + 0.03306 +\right.$
$\left.0.03656 + 0.03952\right]$

$A = 0.067668$

$A = 0.0677$ (by simpson rule).

(b) (ii) Simpson's rule is said to be more efficience than trapezium because the value of the solution of simpson is nearly to the actual value of the function

Actual value $= 0.0678$

Simpson value $= 0.0677$

Trapezium value $= 0.0675$

Extract 7.1 shows that in filling the table of values in part (a), the candidate forgot to change the mode of the calculator from degree to radian. Also the candidate gave the comment on the efficiency of Simpson's and Trapezium rules by relying on the answers obtained in part (i) instead of using an understanding of the Simpson's and trapezium rules.

Conversely, the candidates who scored high marks carried the iterative process correctly to approximate the root of $f(x) = x + e^x$. Moreover, they applied correctly both the Simpson's and Trapezium rules to approximate the value of $\int_0^2 \sin\left(1+\sqrt{x}\right)dx$ and gave explanations that Simpson's rule is more efficient because it uses parabolas to approximate area under a curve while Trapezium rule uses straight lines to form rectangles with the curve assumed straight. Extract 7.2 serves as an example of detailed calculations which were presented by one of the candidates in this question.

Extract 7.2

7a) N-R's formula;

$$X_{n+1} = X_n - \frac{f(X_n)}{f'(X_n)}$$

$$f(x) = x + e^x$$

$$f(X_n) = X_n + e^{X_n}$$

$$f'(X_n) = 1 + e^{X_n}$$

$$X_{n+1} = X_n - \left(\frac{X_n + e^{X_n}}{1 + e^{X_n}}\right)$$

$$= \frac{X_n + X_n e^{X_n} - X_n - e^{X_n}}{1 + e^{X_n}}$$

$$X_{n+1} = \frac{(X_n - 1)e^{X_n}}{1 + e^{X_n}}$$

7a) When $x_0 = -1$

for 1st iteration $n = 0$

$$X_1 = \frac{(X_0 - 1)e^{X_0}}{1 + e^{X_0}}$$

$$= \frac{(-1-1)e^{-1}}{1 + e^{-1}}$$

$$\therefore X_1 = -0.53788$$

2nd iteration $n = 1$

$$X_2 = \frac{(X_1 - 1)e^{X_1}}{1 + e^{X_1}}$$

$$X_2 = \frac{(-0.53788 - 1)e^{-0.53788}}{1 + e^{-0.53788}}$$

$$X_2 = -0.56699$$

3rd iteration $n = 2$

$$X_3 = \frac{(X_2 - 1)e^{X_2}}{1 + e^{X_2}}$$

$$= \frac{(-0.56699 - 1)e^{-0.56699}}{1 + e^{-0.56699}}$$

$$X_3 = -0.56714$$

4th iteration $n=3$

$$x_4 = \frac{(x_3-1)e^{x_3}}{1+e^{x_3}}$$

$$= \frac{(-0.56714-1)e^{-0.56714}}{1+e^{-0.56714}}$$

$$x_4 \doteq -0.56714$$

$\therefore$ approximate root of $f(x) = x + e^x$

$= -0.56714$

7b)

i) $N = 11$

$n = 11 - 1$

$n = 10$

$a = 0,\ b = 2$

$$h = \frac{b-a}{n}$$

$$= \frac{2-0}{10}$$

$$= 2/10$$

$\therefore h = 0.2$

	y_0	y_1	y_2	y_3	y_4	y_5	y_6	y_7
x	0	0.2	0.4	0.6	0.8	1.0	1.2	1.4
$y = \sin(1+\sqrt{x})$	0.8415	0.9924	0.9981	0.9793	0.9481	0.9093	0.8655	0.8183

1.6	1.8	2.0
0.7686	0.7173	0.6649
y_8	y_9	y_n

Using Simpson's rule

$$\int f(x) = \frac{h}{3}\left(y_0 + y_n + 4\Sigma\text{odd} + 2\Sigma\text{even}\right)$$

$$= \frac{h}{3}\left(y_0 + y_n + 4(y_1+y_3+y_5+y_7+y_9) + 2(y_2+y_4+y_6+y_8)\right)$$

$$= \frac{0.2}{3}\left(0.8415 + 0.6649 + 4(0.9924 + 0.9793 + 0.9093 + 0.8183 + 0.7173) + 2(0.9981 + 0.9481 + 0.8655 + 0.7686)\right)$$

$= 1.75556$

$= 1.7556$ (4 d.p)

Using Trapezoidal

$$\int f(x) = \frac{h}{2}\left(y_0 + y_n + 2\Sigma\text{ other ordinates}\right)$$

$$= \frac{0.2}{2}\left(0.8415 + 0.6649 + 2(0.9924 + 0.9981 + 0.9793 + 0.9481 + 0.9093 + 0.8655 + 0.818\right.$$

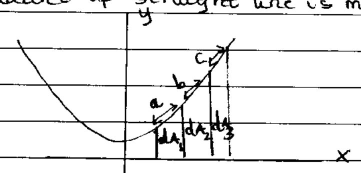
7b) $+ 0.7686 + 0.7173))$

i)

$\int f(x) = 1.75002.$

$= 1.7500$ (4 d.p)

(ii) Because it uses more divisions of the interval or ordinates and since the functions are curves the closer the intervals the more accurate the area below it, as gradient of straight line is more accurate)



Slope of $a \approx b \approx c$

$\therefore$ the addition becomes more accurate as a straight line is found between 2 closer points.

Extract 7.2 shows a sample response from a script of a candidate who scored full marks. This candidate demonstrated competence and high level of skills in the topic of Numerical Methods.

2.1.8 Question 8: Coordinate Geometry I

The question comprised of two parts, (a) and (b). In part (a), the candidates were required to; (i) sketch the diagram of the equation of locus of points which move such that they are equidistant from two intersecting lines, (ii) find the equations of bisectors to two intersecting lines whose equations are $6x - 8y = -7$ and $4x + 3y = 12$ and (iii) find the equation of locus of points which is equidistant from the lines $y = 2x$ and $2x + 4y - 3 = 0$. Part (b) required the candidates to determine the distance of the point $(8, -6)$ from the line $2x + 5y + 34 = 0$.

A total of 9,146 (99.3%) candidates responded to the question of which 38.7 percent scored below 3 marks and among them 15.6 percent scored a 0 mark. It was also noted that 18.4 percent of the candidates scored from 3 to 4.5 marks and 42.9 percent scored 5 marks or above. Thus candidates' general performance on this question was good as 61.2 percent of candidates scored 3 marks and above.

The analysis of candidates' responses shows that, the candidates who performed highly used the formula $\frac{A_1x + B_1y + C_1}{\sqrt{A_1^2 + B_1^2}} = \pm\frac{A_2x + B_2y + C_2}{\sqrt{A_2^2 + B_2^2}}$ correctly to form the equations of the bisectors of two intersecting lines. The candidates also sketched correctly the required diagram of the locus of points. Moreover, they realized that the distance d of the point (x_1, y_1) from the line $ax + by + c = 0$ was given by the formula $d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$ and used it correctly to get the required distance $\frac{20\sqrt{29}}{29} = 3.714$ units. Extract 8.1 is a sample answer from one of the candidates who abide by the question demands.

Extract 8.1

8 a) ii) let the equations be

$$a_1x + b_1y + c_1 = 6x - 8y + 7$$
$$a_2x + b_2y + c_2 = 4x + 3y - 12$$

Formula for bisectors of lines.

$$\frac{a_1x + b_1y + c_1}{\sqrt{a_1^2 + b_1^2}} = \left|\frac{a_2x + b_2y + c_2}{\sqrt{a_2^2 + b_2^2}}\right|$$

$$\frac{6x - 8y + 7}{\sqrt{6^2 + (-8)^2}} = \pm\left(\frac{4x + 3y - 12}{\sqrt{4^2 + 3^2}}\right)$$

$$\frac{6x - 8y + 7}{10} = \pm\left(\frac{4x + 3y - 12}{5}\right)$$

8 a) ii) $6x - 8y + 7 = \pm 2(4x + 3y - 12)$

$$6x - 8y + 7 = \pm(8x + 6y - 24)$$

either

$$6x - 8y + 7 = 8x + 6y - 24 \quad \text{or}$$

$$6x - 8y + 7 = -8x - 6y + 24$$

for

$$6x - 8y + 7 = 8x + 6y - 24$$

$$8x - 6x + 6y + 8y - 24 - 7 = 0$$

$$2x + 14y - 31 = 0$$

for

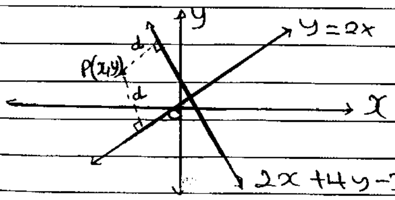
$$6x - 8y + 7 = -8x - 6y + 24$$

$$6x - 8y + 8x + 6y + 7 - 24 = 0$$

$$14x - 2y - 17 = 0$$

$\therefore$ The equations of bisectors are $2x + 14y - 31 = 0$ and $14x - 2y - 17 = 0$

8 (a) iii)



8 | a) iii) from distance formula:

$$d = \left| \frac{ax + by + c}{\sqrt{a^2 + b^2}} \right|$$

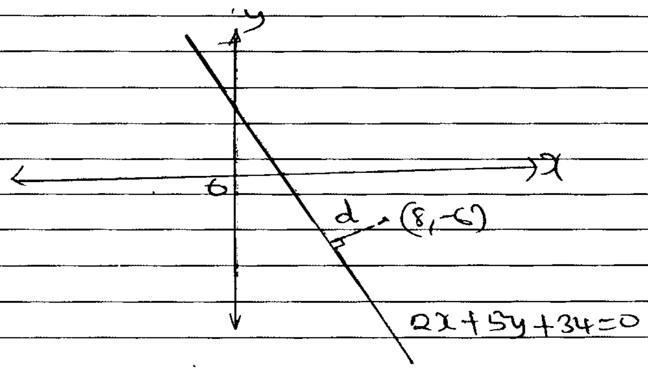
for $y = 2x$
$2x - y = 0$

$$d = \frac{2x - y}{\sqrt{2^2 + (-1)^2}}$$

$$d = \frac{2x - y}{\sqrt{5}}$$

for $2x + 4y - 3$

	$d' = \frac{2x+4y-3}{\sqrt{2^2+4^2}}$
	$d' = \frac{2x+4y-3}{\sqrt{20}}$
	$d' = \frac{2x+4y-3}{2\sqrt{5}}$
	but the locus of a point is such that
	$d = d'$
	$\frac{2x-y}{\sqrt{5}} = \frac{2x+4y-3}{2\sqrt{5}}$

8	a) iii) $2(2x-y) = 2x+4y-3$
	$4x-2y = 2x+4y-3.$
	$4x-2x-2y-4y+3=0$
	$2x-6y+3=0$
	∴ The equation of locus of points is $2x-6y+3=0$
8	b)
	

from distance formula:

x_1:

$$d = \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}$$

$(x_1, y_1) = (8, -6)$.

$$d = \frac{2(8) + 5(-6) + 34}{\sqrt{2^2 + 5^2}}$$

8 b) $$d = \frac{16 - 30 + 34}{\sqrt{29}}$$

$$d = \frac{20}{\sqrt{29}} \text{ units}$$

$$d = 3.714 \text{ units}$$

$\therefore$ distance of a point is 3.714 units

Extract 8.1 shows a candidate's work in which he /she was able to sketch the diagram of the locus and determined correctly the equations of bisectors of two intersecting lines as well as using the distance formula accurately.

On the other hand, the candidates who scored low marks were unable to differentiate between a locus and a circle. They thought that any locus is a circle, thus sketching wrong diagrams for the locus. It was further observed that, candidates were unable to find the equation of bisectors of two intersecting lines. Some candidates solved the equations $6x - 8y = -7$ and $4x + 3y = 12$ simultaneously. Other candidates were using the formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ instead of $d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$ to find the distance of a point from the line. Such candidates were not awarded any mark. Extract 8.2 is a part of the response of a candidate which illustrates a poor answer of part (a) (ii).

Extract 8.2

8 (a) (ii) Given:

$6x - 8y = -7$

$4x + 3y = 12.$

Required: Equation of bisectors:

From given equations

$8y = 6x + 7$

$y = \frac{3}{4}x + \frac{7}{8}$

$\therefore M_1 = 3/4$

And: $4x + 3y = 12$

$3y = -4x + 12$

$y = \frac{-4}{3}x + 4$

$\therefore M_2 = \frac{-4}{3}$

Then:

Recall:

$\tan\theta = \frac{M_1 - M_2}{1 + M_1M_2}$

Where θ is the Angle between the lines.

$\tan\theta = \frac{3/4 - -4/3}{1 + \left(\frac{3}{4}\right)\left(\frac{-4}{3}\right)}$

In Extract 8.2, the candidate found the angle between two intersecting lines instead of the required distance indicating that he/she did not understand the requirement of the question.

2.1.9 Question 9: Integration

This question had three parts (a), (b) and (c). In part (a), the candidates were required to integrate $\int \sec^3 x\,dx$. Part (b) required the candidates to evaluate $\int_{-1}^{0}\left(\frac{2x+3}{x^2+2x+4}\right)dx$ and in part (c), the candidates were required to find the area in surd form of the region bounded by the graphs of $y = \sin x$ and $y = \cos x$ between $x = 0$ and $x = \frac{\pi}{2}$.

This question was attempted by 99.3 percent of the candidates. Most candidates (74.7%) scored from 0 to 2.5 out of 10 marks and minority (25.3%) scored 3 marks or above with only 10 (0.1%) candidates scoring all 10 marks. The analysis of the data has also shown that, 52 percent of candidates scored a 0 mark indicating that the topic on Integration was not clear to majority of the candidates. Hence, the general performance of the topic was poor.

The analysis of candidates' responses revealed that, the candidates who performed poorly failed to use the techniques of integration by parts to integrate $\int \sec^3 x\,dx$. For example, some candidates were using the substitution $u = \cos x$ as an appropriate substitution to evaluate this integral. Other reasons that contributed to poor performance include failure to split the denominator of $\frac{2x+3}{x^2+2x+4}$ and using wrong techniques of integration. It was also noted that, in the process of calculating the area of the region bounded by the given curves, most candidates faced the challenge of sketching the graph of $y = \sin x$ and $y = \cos x$ between $x = 0$ and $x = \frac{\pi}{2}$ which could help them to identify the limits of the integration. Extract 9.1 is a sample answer taken from the script of one of the candidates who faced the challenges of that kind.

Extract 9.1

$$9\ (a)\quad \int \sec^3 x\, dx \qquad \sec x = \frac{1}{\cos x} \qquad \int \frac{1}{\cos^3 x},$$

$$\text{let } u = \cos x \qquad \int \frac{1}{u^3}\,\frac{du}{-\sin x}$$

$$du/dx = -\sin x$$

$$\frac{-1}{\sin x} \ln u^3 + c.$$

$$= -\frac{1}{\sin x}\left(\ln \cos^3 x\right) + c$$

$$= -\frac{1}{\sin x}\left(\ln \cos^3 x\right) + c$$

In Extract 9.1 the candidate failed to use the proper techniques of integration in part 9 (a). He/she used substitution method while was supposed to use integration by parts or reduction method indicating lack of skills in evaluating integrals.

On the other hand, the few candidates who scored highly, recognized the need of integrating $\int \sec^3 x\, dx$ by parts or by using the reduction formula $I_n = \frac{1}{n-1}\tan x \sec^{n-2} x + \frac{n-2}{n-1} I_{n-2}\ \ n \geq 2$. The candidates were also able to split the numerator of $\frac{2x+3}{x^2+2x+4}$ into $\frac{2x+2}{x^2+2x+4} + \frac{1}{x^2+2x+4}$ and thus obtained the required value of $\int_{-1}^{0}\left(\frac{2x+3}{x^2+2x+4}\right)dx$ as $\frac{\sqrt{3}}{18}\pi + \ln\frac{4}{3}$. In addition, these candidates were able to draw the graphs of $y = \sin x$ and $y = \cos x$ between $x = 0$ and $x = \frac{\pi}{2}$ where they found the area bounded by two curves correctly. Extract 9.2 shows a sample response from one of the candidates who performed well.

Extract 9.2

9(a) $\int \sec^3 x\,dx = \int \sec^2 x \sec x\, dx$

$u = \sec x \longrightarrow u' = \sec x \tan x$

$v' = \sec^2 x \longrightarrow v = \tan x$.

fm $\int uv' = uv - \int u'v$.

$\int \sec^3 x\, dx = \sec x \tan x - \int \sec x \tan^2 x\, dx$

$\int \sec^3 x\, dx = \sec x \tan x - \int \sec x(\cancel{1+}\sec^2 x)\, dx$.

$\int \sec^3 x\, dx = \sec x \tan x - \int \sec^3 x\, dx + \int \sec x\, dx$

$2\int \sec^3 x\, dx = \sec x \tan x + \int \frac{\sec x\,(\sec x + \tan x)}{\sec x + \tan x}\, dx$

$\therefore \int \sec^3 x\, dx = \frac{\sec x \tan x + \ln(\sec x + \tan x)}{2} + c$

(b) $\int_{-1}^{0} \frac{2x+3}{x^2+2x+4}\, dx$

let $2x+3 = A(2x+2) + B$.

$2x+3 = 2Ax + 2A + B$.

$\therefore 2A = 2$

$A = 1$.

$3 = 2A + B$

$\therefore B = 1$.

$$\int_{-1}^{0} \frac{2x+3}{x^2+2x+4}\,dx = \int_{-1}^{0} \frac{2x+2+1}{x^2+2x+4}\, dx$$

$$= \int_{-1}^{0} \frac{2x+2}{x^2+2x+4}\,dx + \int_{-1}^{0} \frac{dx}{x^2+2x+4}$$

$$= \left[\ln(x^2+2x+4)\right]_{-1}^{0} + \int_{-1}^{0} \frac{dx}{x^2+2x+1+3}$$

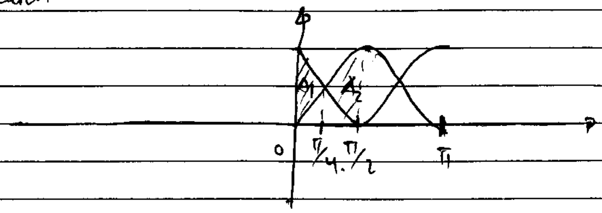
$$= \left[\ln(x^2+2x+4)\right]_{-1}^{0} + \frac{1}{3}\int_{-1}^{0} \frac{dx}{1+\left(\frac{x+1}{\sqrt{3}}\right)^2}$$

$$= \left[\ln(x^2+2x+4) + \frac{\sqrt{3}}{3}\tan^{-1}\left(\frac{x+1}{\sqrt{3}}\right)\right]_{-1}^{0}$$

$$\therefore \int_{-1}^{0} \frac{2x+3}{x^2+2x+4} = \ln(0+4) + \frac{\sqrt{3}}{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) - \ln(-1^2-2+4) - \frac{\sqrt{3}}{3}\tan^{-1}\left(\frac{0}{\sqrt{3}}\right)$$

$= \underline{0.58998}$

9(c) Sketch



Area = $A_1 + A_2$.

$$\text{Area} = \left|\int_0^{\pi/4} (\sin x - \cos x)\right| + \left|\int_{\pi/4}^{\pi/2} (\sin x - \cos x)\right|$$

$$= \left|\left[-\cos x - \sin x\right]_0^{\pi/4}\right| + \left|\left[-\cos x - \sin x\right]_{\pi/4}^{\pi/2}\right|$$

$$= \left|\left(-\cos \tfrac{\pi}{4} - \sin \tfrac{\pi}{4} + \cos 0 - \sin 0\right)\right| + \left(-\cos \tfrac{\pi}{2} - \sin \tfrac{\pi}{2} + \cos \tfrac{\pi}{4} + \sin \tfrac{\pi}{4}\right)$$

$$= \left| -\tfrac{\sqrt{2}}{2} - \tfrac{\sqrt{2}}{2} + 1 \right| + |$$

$$= |1 - \sqrt{2}| + |\sqrt{2} - 1|$$

$$= \sqrt{2} - 1 + \sqrt{2} - 1$$

Area = $2\sqrt{2} - 2$.

$\therefore$ Area bounded = $2\sqrt{2} - 2$

Extract 9.2 shows how the candidate had applied correctly the techniques of integration. The candidate also managed to evaluate the area bounded by the given curves obtained after drawing the graphs of $y = \sin x$ and $y = \cos x$ between $x = 0$ and $x = \frac{\pi}{2}$.

2.1.10 Question 10: Differentiation

The question consisted of three parts (a), (b) and (c). It required the candidates to:

(a) Find $\frac{dy}{dx}$ if $y = (1+2t)^3$ and $x = t^3$.

(b) Find $\frac{d}{dx}\left(\tan\sqrt{6x^3+2}\right)$.

(c) (i) Find dU if $U = x^2 e^{\frac{y}{x}}$.

(ii) Show that $(3x^2y - 2y^2)dx + (x^3 - 4xy + 6y^2)dy$ can be written as an exact differential equation of a function $\phi(x, y)$ and then find this function.

This question was attempted by 99.3 percent of the candidates, of which 39.4 percent scored below 3 marks , 21.8 percent scored from 3 to 4.5 marks and 38.8 percent of the candidates scored 5 or above with 43 (0.5%) candidates scoring 10 marks while 17.2 percent scored a 0 mark. The analysis implies that the performance of candidates in question 10 was good.

The candidates who performed highly demonstrated a good understanding on how to differentiate parametric equations as they used correctly the chain rule to find $\frac{dy}{dx}$ in part (a) as well as $\frac{d}{dx}\left(\tan\sqrt{6x^3+2}\right)$ in part (b). It was also observed that the candidates had good understanding of the techniques of implicit differentiation and skills of solving differential equations therefore managed to get

$$dU = \left(2xe^{\frac{y}{x}} - ye^{\frac{y}{x}}\right)dx + xe^{\frac{y}{x}}dy \quad \text{and} \quad \phi = x^3y - 2xy^2 + 2y^3 + c$$

as the correct answers of (c) (i) and (ii). The illustration in Extract 10.1 is an example of an answer that was provided by such candidates.

Extract 10.1

10.a) $y = (1+2t)^3$

$\frac{dy}{dt} = 3(1+2t)^2(2)$

$\frac{dy}{dt} = 6(1+2t)^2$

$x = t^3$

$\frac{dx}{dt} = 3t^2$

$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$

10a, $\frac{dy}{dx} = 6(1+2t)^2 \times \frac{1}{3t^2}$

$\frac{dy}{dx} = \frac{2(1+2t)^2}{t^2}$

10b). $\frac{d}{dx}\left(\tan\sqrt{6x^3+2}\right)$

let $y = \tan\sqrt{6x^3+2}$

let $u = \sqrt{6x^3+2}$

$u^2 = 6x^3+2$

$2u\frac{du}{dx} = 18x^2$

$\frac{du}{dx} = \frac{9x^2}{u} = \frac{9x^2}{\sqrt{6x^3+2}}$

$y = \tan u$

$\frac{dy}{dx} = \sec^2 u \frac{du}{dx}$

$\frac{dy}{dx} = \sec^2\left(\sqrt{6x^3+2}\right) \cdot \frac{9x^2}{\sqrt{6x^3+2}}$

$\therefore \frac{dy}{dx} = \frac{9x^2\sec^2\left(\sqrt{6x^3+2}\right)}{\sqrt{6x^3+2}}$

10c) i). $u = x^2 e^{y/x}$

let $t = e^{y/x}$

$\ln t = y/x = yx^{-1}$

$\frac{1}{t}\frac{dt}{dx} = y\left(\frac{-1}{x^2}\right) + \frac{1}{x}\frac{dy}{dx}$

(10c)i)	$\frac{dt}{dx} = \left(\frac{-y}{x^2} + \frac{1}{x}\frac{dy}{dx}\right) e^{y/x}$
	$u = x^2 t$
	$\frac{du}{dx} = x^2 \frac{dt}{dx} + 2xt$
	$\frac{du}{dx} = x^2\left(\frac{-y}{x^2} + \frac{1}{x}\frac{dy}{dx}\right) e^{y/x} + 2xe^{y/x}$
	$\therefore du = \left(x^2 e^{y/x}\left(\frac{-y}{x^2} + \frac{1}{x}\frac{dy}{dx}\right) + 2xe^{y/x}\right) dx$
(10c)ii)	$(3x^2y - 2y^2)dx + (x^3 - 4xy + 6y^2)dy$
	$\frac{\delta M}{\delta y} = \cancel{\delta x}\ 3x^2 - 4y \qquad \frac{\delta N}{\delta x} = 3x^2 - 4y$
	$\frac{dP}{dx} = 3x^2y - 2y^2 \qquad \frac{dP}{dy} = x^3 - 4xy + 6y^2$
	$\int dP = \int 3x^2y - 2y^2\, dx \qquad \int dP = \int x^3 - 4xy + 6y^2\, dy$
	$P(x,y) = \frac{3x^3y}{3} - 2xy^2 + A(y) \qquad P(x,y) = x^3y - \frac{4xy^2}{2} + \frac{6y^3}{3} + B(x)$
	$\frac{\cancel{3}x^3y}{\cancel{3}} - 2xy^2 + A(y) = x^3y - 2xy^2 + 2y^3 + B(x)$
	$\therefore A(y) = 2y^3$
	$B(x) = 0$
	$\therefore x^3y - 2xy^2 + 2y^3 = c$
	$\therefore$ Since $\frac{\delta M}{\delta y} = \frac{\delta N}{\delta x} = 3x^2 - 4y$, therefore the equation
	can be written as an exact differential equation.
	$\therefore$ the function is $x^3y - 2xy^2 + 2y^3 = c$

In Extract 10.1 the candidate demonstrated competence in using appropriate techniques of differentiation; that is substitution and the chain rule.

Despite the good performance, there were few candidates who performed poorly. Some of them failed to use various techniques of differentiation while others used wrong approaches to deal with the partial derivative problem in part (c). An example of the work of one of the candidates who failed to apply these techniques is shown in Extract 10.2.

Extract 10.2

$$\frac{dy}{dx} = -\cancel{x^2}\frac{y}{\cancel{x^2}} e^{y/x}) + e^{y/x}(2x)$$

$$\frac{dy}{dx} = 2xe^{y/x} - ye^{y/x}.$$

$$dy = [(2x-y)e^{y/x}]\,dx.$$

In Extract 10.2, the candidate could not apply the knowledge of partial derivatives to obtain the required expression for dU.

2.2 142/2 ADVANCED MATHEMATICS 2

2.2.1 Question 1: Complex Numbers

The question had four parts which are (a), (b), (c) and (d). In part (a), the candidates were required to show that, $\frac{1}{2}\left(z+\frac{1}{z}\right)=1-\frac{\theta^2}{2}-\frac{i\theta^3}{6}+\frac{\theta^4}{4}+\ldots$ by using the Euler formula $z=e^{i\theta}$ for exponentials. In part (b), the candidates were given that, one root of $z^4+z^3+3z^2+z+2=0$ is i and were required to find the other roots. In part (c) (i), the candidates were given that $z_1=1+i\sqrt{3}$, $z_2=\sqrt{3}+i$ and were required to find the modulus and the principle argument of z_1z_2. Part (c) (ii) required the candidates to find the locus represented by the equation $|z-(2-i)|=|z-(3+2i)|$ if z is a complex number. Finally, part (d) required the candidates to express w in modulus argument form if

$$z=\sqrt{3}\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right) \text{ and } z^2w=21\left[\cos\left(-\frac{2\pi}{3}\right)+i\sin\left(-\frac{2\pi}{3}\right)\right].$$

This question was attempted by 9,143 (99.3%) candidates, out of which 64 (0.7%) candidates scored all 15 marks allocated to this question. However, 34.5 percent of the candidates had their scores below 4.5 marks. The analysis shows that the general performance of the question is good because the candidates who scored from 4.5 to 15 marks is 65.5 percent.

The candidates who performed well realized that $e^{i\theta} = 1 + i\theta - \frac{\theta^2}{2} - \frac{i\theta^3}{6} + \frac{\theta^4}{24} + ...$ and used this knowledge to show that $\frac{1}{2}\left(z + \frac{1}{z}\right) = 1 - \frac{\theta^2}{2} + \frac{\theta^4}{24} + ...$ and not $1 - \frac{\theta^2}{2} - \frac{i\theta^3}{6} + \frac{\theta^4}{4} + ...$ The candidates also noted that if one root of $z^4 + z^3 + 3z^2 + z + 2 = 0$ is i then its conjugate $-i$ is also a root and therefore they were able to find the remaining roots of this equation. Moreover, they were able to determine the expression for $z_1 z_2$ and thus got the values for the modulus and the principle argument. Finally, the candidates found that, to express w in modulus argument form appropriately, they should apply both the Demoivres theorem and the additional formula for the cosine to $7\left[\cos\left(-\frac{2\pi}{3}\right) + i\sin\left(-\frac{2\pi}{3}\right)\right] \times \left[\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right]^{-1}$. The sample answer from the script of one of the candidates who answered this question accordingly is shown in Extract 11.1.

Extract 11.1

1(a) Given $z = e^{i\theta} \Rightarrow \frac{1}{z} = \bar{z} = e^{-i\theta}$.

$\therefore e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}$ $\quad \therefore i^2 = -1, i^3 = -i, i^4 = 1$

$\therefore z = e^{i\theta} = 1 + i\theta + \frac{(i\theta)^2}{2!} + \frac{(i\theta)^3}{3!} + \frac{(i\theta)^4}{4!} + .$

$\therefore z = e^{i\theta} = 1 + i\theta - \frac{\theta^2}{2} - \frac{i\theta^3}{6} + \frac{\theta^4}{24} + -,$

$\bar{z} = \frac{1}{z} = e^{-i\theta} = 1 - i\theta + \frac{(-i\theta)^2}{2!} + \frac{(-i\theta)^3}{3!} + \frac{(-i\theta)^4}{4!}$

$\therefore \frac{1}{z} = e^{-i\theta} = 1 - i\theta - \frac{\theta^2}{2} + \frac{i\theta^3}{6} + \frac{\theta^4}{24} + \ldots$

$\therefore \frac{1}{2}\left(z + \frac{1}{z}\right) = \frac{1}{2}\left(1 + \cancel{i\theta} - \theta^2/2 - \cancel{i\theta^3/6} + \theta^4/24 + \ldots + 1 - \cancel{i\theta} - \theta^2/2 + \cancel{i\theta^3/6} + \frac{\theta^4}{24} + \ldots\right)$

$\therefore \frac{1}{2}\left(z + \frac{1}{z}\right) = \frac{1}{2}\left(2 - \frac{2\theta^2}{2} + \frac{2\theta^4}{24} + \ldots\right)$

$\therefore \frac{1}{2}\left(z + \frac{1}{z}\right) = 1 - \frac{\theta^2}{2} + \frac{\theta^4}{24} + \ldots$

(hence shown)

1(b)

$$\begin{array}{r} z^2 + z + 2 \\ z^2+1 \overline{\smash{)}\, z^4 + z^3 + 3z^2 + z + 2} \\ -(z^4 + z^2) \\ \hline z^3 + 2z^2 + z + 2 \\ -(z^3 + z) \\ \hline 2z^2 + 2 \\ -(2z^2 + 2) \\ \hline - \; - \end{array}$$

$\therefore z^2 + z + 2$ is also a factor

Let $z^2 + z + 2 = 0$

$z = \frac{-1 \pm \sqrt{1^2 - 4(2)}}{2}$

$z = \frac{-1 \pm \sqrt{-7}}{2}$

$z = \frac{-1 \pm i\sqrt{7}}{2}$

$\therefore$ The other roots are $-i$, $\frac{-1 + i\sqrt{7}}{2}$ and $\frac{-1 - i\sqrt{7}}{2}$.

1(c)(i) $z_1 z_2 = (1 + i\sqrt{3})(\sqrt{3} + i)$.

$z_1 = 1 + i\sqrt{3}$ ~~argu~~

$\Rightarrow \arg(z_1) = \theta_1 = \tan^{-1}(\sqrt{3}) = 60^\circ$

modulus $z_1 = r_1 = \sqrt{1 + (\sqrt{3})^2} = 2$.

$z_2 = \sqrt{3}+i$

$\theta_2 = \arg(z_2) = \tan^{-1}(1/\sqrt{3}) = 30°$

$|z_2| = \sqrt{\sqrt{3}^2+1} = 2$

1(c) $z_1 z_2 = 4(\cos 90° + i\sin 90°)$

$\therefore |z_1 z_2| = 4$ (modulus of z_1z_2)

and $\arg(z_1z_2) =$ principal argument of $z_1z_2 = \underline{90°}$.

1(c) (ii) Let $z = x+iy$,

$|z-(2-i)| = |z-(3+2i)|$

$\Rightarrow |x+iy-2+i| = |x+iy-3-2i|$

$\Rightarrow |(x-2)+i(y+1)| = |(x-3)+i(y-2)|$.

$\Rightarrow \sqrt{(x-2)^2+(y+1)^2} = \sqrt{(x-3)^2+(y-2)^2}$

$\Rightarrow (x-2)^2+(y+1)^2 = (x-3)^2+(y-2)^2$

$\Rightarrow \cancel{x^2}$

$(x-2)^2-(x-3)^2 = (y-2)^2-(y+1)^2$

$\Rightarrow (x-2+x-3)(\cancel{x}-2-\cancel{x}+3) = (y-2+y+1)(\cancel{y}-2-\cancel{y}-1)$

$\Rightarrow (2x-5)(1) = (2y-1)(-3)$

$\Rightarrow 2x-5 = -6y+3$

$-6y = 2x-8$

$y = -\frac{x}{3}+\frac{4}{3}$.

$\therefore$ The locus is a straight line with slope, $-\frac{1}{3}$ and y-intercept at $= \frac{4}{3}$.

1(d) Given

$z = \sqrt{3}\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

$$z^2 w = 21\left[\cos\left(-\frac{2\pi}{3}\right) + i\sin\left(-\frac{2\pi}{3}\right)\right]$$

$$\because z^2 = \left(\sqrt{3}\left(\cos\pi/3 + i\sin\pi/3\right)\right)^2$$

$$z^2 = 3\left(\cos\left(\frac{2\pi}{3}\right) + i\sin\left(2\pi/3\right)\right)$$

$$\therefore w = \frac{21\left[\cos\left(-\frac{2\pi}{3}\right) + i\sin\left(-2\pi/3\right)\right]}{3\left(\cos\left(2\pi/3\right) + i\sin 2\pi/3\right)}$$

$$\cancel{\therefore w =}$$

$$\therefore w = 7\left[\cos\left(-2\pi/3 - \frac{2\pi}{3}\right) + i\sin\left(-2\pi/3 - 2\pi/3\right)\right]$$

$$\therefore w = 7\left[\cos\left(-4\pi/3\right) + i\sin\left(-4\pi/3\right)\right]$$

Extract 11.1 illustrates a candidate's solution in which he/she was able to apply the Euler's formula to deal with series involving complex numbers, found the roots of the given complex polynomial function and determined the modulus and the principle argument of the complex equation. In part (d), the candidate managed to express correctly the complex numbers in modulus – argument form.

However, there were some candidates who performed poorly in this question. Many of these candidates failed to recall and apply the Euler's formula in verifying complex series. The candidates also ignored the negative sign, for instance, writing $(-i\theta)^2$ as θ^2 instead of $-\theta^2$ and were unable to calculate the principal modulus and argument in part (c) (i). Moreover, they lacked the skills on how to find the equation of locus represented by the equation $|z-(2-i)| = |z-(3+2i)|$ and failed to substitute $z = \sqrt{3}\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$ into $z^2 w = 21\left[\cos\left(-\frac{2\pi}{3}\right) + i\sin\left(-\frac{2\pi}{3}\right)\right]$ to get w in modulus – argument form. An example of the work of one of the candidates who scored low marks is shown in Extract 11.2.

Extract 11.2

(b) Soln

Given one root of the equation

$z^4+z^3+3z^2+z+2=0$ is i, Required other root

Then the root will be $z+i$

$(z+i)(z+i)(z+i)(z+i)$

$(z+i)(z+i) = z^2+zi+zi-1$

$(z^2+2zi-1)(z^2+2zi-1)$

In Extract 11.2, the candidate failed to understand that if one root of the equation $z^4 + z^3 + 3z^2 + z + 2 = 0$ is i then the other root must also be a complex number which is the complex conjugate of the first root and is $-i$ and not $z + i$.

2.2.2 Question 2: Logic

The question consisted of three parts namely (a), (b) and (c). In part (a) (i), the candidates were required to prepare a truth table for the proposition $((q \rightarrow \sim p) \wedge ((p \vee r) \wedge q)) \rightarrow r$ and in part (a) (ii), the candidates were required to determine the truth value and comment on the validity of the following argument using a truth table:

$\text{p} \rightarrow (q \vee \sim r)$

$\underline{q \rightarrow (p \wedge r)}$

$[(q \vee \sim r) \wedge (p \wedge r)] \rightarrow r.$

In part (b), the candidates were required to use laws of algebra in logic to (i) determine the validity of the argument "If there is rain, the crops will grow well. If crops grow well, there is no famine. But there is famine. Therefore there is no rain" and (ii) simplify the proposition $\sim ((p \vee q) \vee (\sim \text{p} \wedge q))$. In part (c), the candidates were required to translate the following compound statements in symbolic notation using letters P, Q and R to stand for the statements: (i) Either the manufactured drug is not fault and accepted by the Tanzania Food and Drug Authority (TFDA) or the manufactured drug is fault and is not accepted by the TFDA and (ii) If Kapirima is a member of a social committee then the committee is strong. The committee is strong if and only if Kaprima's argument is accepted by other members. Therefore, Kapirima's argument is not accepted and the committee is not strong.

This question was attempted by 9,143 (99.3%) candidates, out of which 40 (0.4%) candidates scored all 15 marks. On the other hand, 48.4 percent of the candidates had their scores below 4.5 marks in the question. The analysis shows that, the general performance of the question is good because the candidates who scored from 4.5 to 15 marks is 51.6 percent.

The analysis of responses of candidates shows that, candidates who performed well prepared the truth tables with correct entries of T and F and used the laws of proposition of algebra accordingly to determine and simplify the validity of the given arguments and propositions respectively. The candidates also translated appropriately the given compound statements into symbolic notations of P, Q and R. Extract 12.1 shows the response of one of the candidates who adhered to the requirements of the questions.

Extract 12.1

2 (a) i/ Soln.

$$[(q \to \sim p) \wedge ((p \vee r) \wedge q)] \to r$$

p	q	r	$\sim p$	(a) $q \to \sim p$	$p \vee r$	(b) $(p \vee r) \wedge q$	$a \wedge b$	$(a \wedge b) \to r$
T	T	T	F	F	T	T	F	T
T	T	F	F	F	T	T	F	T
T	F	T	F	T	T	F	F	T
T	F	F	F	T	T	F	F	T
F	T	T	T	T	T	T	T	T
F	T	F	T	T	F	F	F	T
F	F	T	T	T	T	F	F	T
F	F	F	T	T	F	F	F	T

Where a is $q \to \sim p$
b is $(p \vee r) \wedge q$

ii/ Soln.

Truth table.

$$\left[p \to (q \vee \sim r) \wedge (q \to p \wedge r)\right] \to \left(\left[(q \vee \sim r) \wedge (p \wedge r)\right] \to r\right)$$

p	q	r	(a) $q \vee \sim r$	(c) $p \to a$	(b) $p \wedge r$	(d) $q \to b$	$c \wedge d$	(e) $a \wedge b$	$e \to r$	$f \to g$
T	T	T	T	T	T	T	T	T	T	T
T	T	F	T	T	F	F	F	F	T	T
T	F	T	F	F	T	T	F	F	T	T
T	F	F	T	T	F	T	T	F	T	T
F	T	T	T	T	F	F	F	F	T	T
F	T	F	T	T	F	F	F	F	T	T
F	F	T	F	T	F	T	T	F	T	T
F	F	F	T	T	F	T	T	F	T	T

-2- (a) ii/

∴ Since the proposition is tautology therefore the argument is valid.

(b) ii/ $\sim\left[(p \vee q) \vee (\sim p \wedge q)\right]$

Soln

$(\sim p \wedge \sim q) \wedge (p \vee \sim q)$	Demorgans law
$(\sim p \wedge \sim q \wedge p) \vee (\sim p \wedge \sim q \wedge \sim q)$	Distributive law
$(F \wedge \sim q) \vee (\sim p \wedge \sim q)$	Complement law
$F \vee \sim p \wedge \sim q$	Identity law
$\sim p \wedge \sim q$	Identity law

i/ Soln.

let p be there is rain

q be crops will grow well

r be there is famine

Argument

$(p \to q), (q \to \sim r), r \vdash \sim p$

Proposition

$\left[(p \rightarrow q) \wedge (q \rightarrow \sim r) \wedge r\right] \longrightarrow \sim p$

$\sim\left[(\sim p \vee q) \wedge (\sim q \vee \sim r) \wedge r\right] \vee \sim p$ Definition $p \rightarrow q \equiv \sim p \vee q$

-2- (b) i/

$\left[(\sim r \vee q) \wedge T\right] \vee \left[T \wedge (\sim p \vee \sim q)\right]$ Complement law

$(\sim r \vee q) \vee (\sim p \vee \sim q)$ Identity law

$(\sim r \vee \sim p) \vee (q \vee \sim q)$ Commutative law

$(\sim r \vee \sim p) \vee T$ Complement law

T Identity law

$\therefore$ Since the proposition is tautology, therefore the argument is valid.

(c) i/ Soln.

Let p be Manufactured drug is fault

Q be accepted by TFDA

$\cancel{\sim p \vee} \left([\sim p \wedge Q] \vee [p \wedge \sim Q]\right)$

$\therefore$ In Symbolic $\left[(\sim p \wedge Q) \vee (p \wedge \sim Q)\right]$

ii/ Soln

Let p kapirima is a member of Social community

Q be community is strong

R kapirima's argument is accepted by other member.

In Symbolic

$(P \rightarrow Q),\ Q \leftrightarrow R \vdash \sim R \wedge \sim Q$

$\therefore \left[(p \rightarrow Q) \wedge (Q \leftrightarrow R)\right] \longrightarrow (\sim R \wedge \sim Q)$

Extract 12.1 shows a sample solution of a candidate who was able to answer the given question correctly. He/she prepared a truth table using the appropriate laws of propositions of algebra, simplified the given statements, tested the validity of the given argument and expressed the given statements in symbolic form.

Nevertheless, there were few candidates who performed poorly in this question. The reasons for poor performance include failure to follow instructions, that is, instead of using the laws of algebra in logic to simplify or test the validity, they used the truth tables. Similarly, failure to simplify the compound statements; failure to determine the validity of the arguments and failure to translate the given statements into symbolic notations contributed to the poor performance of the candidates. Extract 12.2 shows the procedures of one of the candidates who performed poorly.

Extract 12.2

(b)(i) let p = if there is rain
q = crop grow well
r = Famine
$[(p \to q) \wedge ((q \to \sim r) \wedge r)] \to \sim p$

2. (b)(i)

				A	B	C	D		
p	q	r	~r	(p→q)	(q→~r)	B∧r	C∧A	D→~p	~p
T	T	T	F	T	F	F	F	T	F
T	T	F	T	T	T	F	F	T	F
T	F	T	F	F	T	T	F	T	F
T	F	F	T	F	T	F	T	T	F
F	T	T	F	T	F	F	F	T	T
F	T	F	T	T	T	F	F	T	T
F	F	T	F	T	T	T	T	T	T
F	F	F	T	T	T	F	F	T	T

Extract 12.2 illustrates the work of a candidate who did not meet the requirements of question 2 (b) (i) because he/she used the truth table instead of the laws of propositions of algebra to determine the validity of the given argument. However, he/she obtained $[(p \to q) \wedge (q \to \sim r) \wedge r] \to \sim p$ correctly and was awarded few marks.

2.2.3 Question 3: Vectors

The question had three parts, (a), (b) and (c). In part (a) (i), the candidates were required to find the expression for the work done used to move a 15kg baby against gravity from $\underline{r} = u_1\underline{i} + u_2\underline{j}$

to $\underline{p} = v_1\underline{i} + v_2\underline{j}$ and hence deduce the actual work done when $(u_1, u_2)$and (v_1, v_2) are $(-1, 7)$ and $(2, 3)$ respectively. In part (a) (ii), the candidates were required to find the magnitude and direction relative to the 60N of the resultant force, if two forces of 40N and 60N act on a point in a plane and the angle between the force vectors is 30^o. The candidates were required to express the magnitude of the resultant force in two significant figures and the angle to the nearest degree. In part (b), the candidates were given A (1, 1, 2), B (3, 2, -1) and C (-4, 1, 3 as vertices of a triangle and were required to use the knowledge on vectors to find the area of the triangle ABC. In part (c), the candidates were required to (i) determine the velocity and acceleration of a particle at any time *t* and (ii) find the magnitude of the velocity and acceleration at time $t = 0$ when the particle moves along a curve whose parametric equations are $x = e^{-t}$, $y = 2\cos 3\text{t}$ and $z = 2\sin 3\text{t}$ where *t* is the time and its position vector is $\underline{r} = x\underline{i} + y\underline{j} + z\underline{k}$.

The question was attempted by 9,143 (99.3%) candidates, out of which 52 percent scored below 4.5 out of 15 marks with 18.6 percent of them scoring a 0 mark while 13 (0.1%) candidates scored all 15 marks. The analysis shows that the general performance of the question is average because the candidates who scored from 4.5 to 15 marks is 48 percent.

The analysis of candidates' responses shows that the candidates who performed well recognized the need of utilizing the definition $\text{Workdone} = \text{Force} \times \text{distance}$ to get an expression for the work done $147\sqrt{(v_1 - u_1)^2 + (v_2 - u_2)^2}$. Furthermore, they used the triangular and parallelogram laws of forces to evaluate the magnitude and direction of the resultant force. They also made the proper application of cross product of vectors to find the area of triangle ABC and expressed velocity and acceleration as derivatives of displacement $\underline{r} = x\underline{i} + y\underline{j} + z\underline{k}$. Extract 3.1 shows a sample solution of one of the candidates who performed well.

Extract 13.1

3 a) i) Consider vector distance, $\vec{d}$

$$\underset{\sim}{d} = \underset{\sim}{r} - \underset{\sim}{r}$$

$$= v_1 i + v_2 j - (u_1 i + u_2 j)$$

$$= (v_1 - u_1)i + (v_2 - u_2)j$$

Work done $= |F|\,|d| \cos\theta$

put $\theta = 0$

$$|F| = 15 \times 9.8\ N$$
$$= 147N$$

$$|d| = \sqrt{(V_1 - U_1)^2 + (V_2 - U_2)^2}$$

$\therefore$ Work done $= 147\sqrt{(V_1-U_1)^2+(V_2-U_1)^2}$ Joules

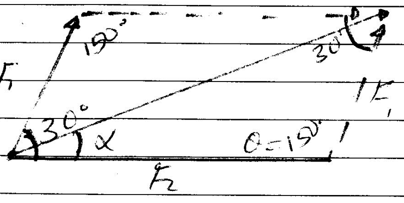
$\Rightarrow (U_1, U_2) = (-1, 7)$

$(V_1, V_2) = (2, 3)$

Work done $= 147\ \sqrt{(2--1)^2+(3-7)^2}$

$= 735$ Joules

3 a) ii).



$\alpha = ?$

$\theta = 150°$

3 a) iii) By using cosine rule

Resultant force, $F = \sqrt{F_1^2 + F_2^2 - 2F_1F_2\cos\theta}$

$= \sqrt{40^2 + 60^2 - 2(40)(60)\cos 150}$

$= 97\ N$

direction, by using sine rule.

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin x}{F_1} = \frac{\sin \theta}{F}$$

$$\sin x = \frac{F_1 \sin \theta}{F}$$

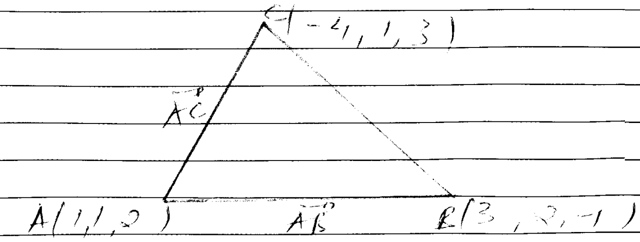
$$x = \sin^{-1}\left(\frac{F_1 \sin \theta}{F}\right)$$

$$= \sin^{-1}\left(\frac{40 \sin 150}{97}\right)$$

$$= 12^\circ$$

$\therefore$ Magnitude is 97N and direction is 12° relative to 60N force

3 b/



3 b/ Consider $\vec{AC} = (-4-1)i + (1-1)j + (3-2)k$

$= -5i + k$

also $\vec{AB} = (3-1)i + (2-1)j + (-1-2)k$

$= 2i + j - 3k$

$\Rightarrow$ area $= \frac{1}{2}|\vec{AB} \times \vec{AC}|$

Consider $\vec{AB} \times \vec{AC} = \begin{vmatrix} i & j & k \\ -5 & 0 & 1 \\ 2 & 1 & -3 \end{vmatrix}$

$= i\begin{vmatrix} 0 & 1 \\ 1 & -3 \end{vmatrix} - j\begin{vmatrix} -5 & 1 \\ 2 & -3 \end{vmatrix} + k\begin{vmatrix} -5 & 0 \\ 2 & 1 \end{vmatrix}$

$= -i - j(15-2) + k(-5-0)$

$= -i - 13j - 5k$

$|\vec{AB} \times \vec{AC}| = \sqrt{(-1)^2 + (-13)^2 + (-5)^2}$

$= 13.964$

$\text{Area} = \frac{1}{2} |\vec{AB} \times \vec{AC}| = \frac{1}{2} \times 13.964$

$= 6.982$

$\therefore$ Area = 6.982 square units

3 d.

Consider

$x = e^{-t}$

$\frac{dx}{dt} = -e^{-t}$

3 d. also $y = 2\cos 3t$

$\frac{dy}{dt} = -6\sin 3t$

also

$z = 2\sin 3t$

$\frac{dz}{dt} = 6\cos 3t$

ii. $r = xi + yj + zk$

Velocity, $\frac{dr}{dt} = i\frac{dx}{dt} + j\frac{dy}{dt} + k\frac{dz}{dt}$

Velocity, $v = \underset{\sim}{i}(-e^{-t}) + \underset{\sim}{j}(-6\sin 3t) + \underset{\sim}{k}(6\cos 3t)$

$v = -e^{-t}\underset{\sim}{i} - 6\sin 3t\,\underset{\sim}{j} + 6\cos 3t\,\underset{\sim}{k}$

also

acceleration $a = \frac{dv}{dt}$

$\frac{dv}{dt} = e^{-t}i - 18\cos 3t\,\underset{\sim}{j} - 18\sin 3t\,\underset{\sim}{k}$

$\therefore$ Velocity $= -e^{-t}i - 6\sin 3t\,j + 6\cos 3t\,k$

acceleration $= e^{-t}\underline{i} - 18\cos 3t\,\underline{j} - 18\sin 3t\,\underline{k}$

3 c. iii)

$|V| = \sqrt{(-e^{-t})^2 + (-6\sin 3t)^2 + (6\cos 3t)^2}$

$t = 0$

$|V| = \sqrt{(-e^{0})^2 + ((6\sin 3(0))^2 + (6\cos 3(0))^2}$

3 c. iii)

$|V| = \sqrt{37}$

$= 6.083$ units

Magnitude of accelerat

$|a| = \sqrt{(e^{-t})^2 + (-18\cos 3t)^2 + (-18\sin 3t)^2}$

at $t = 0$

$|a| = \sqrt{(e^{-0})^2 + (-18\cos 3(0))^2 + (-18\sin 3(0))^2}$

$|a| = 18.028$ units

$\therefore$ Magnitude for velocity is 6.083 units,

acceleration is 18.028 units

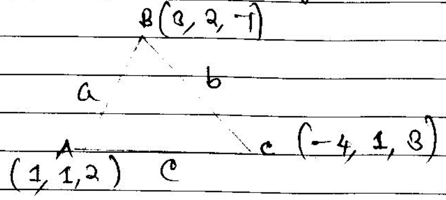
Extract 13.1 is a sample solution from one of the candidates who was able to answer the given question correctly. He/she applied the concept of vector and differentiation rules to find the required work done, acceleration and velocity.

However, the candidates who scored low marks were unable to apply the formula for work done = Force × Distance = mass × acceleration × absolute value of displacement vector. Also, some candidates used incorrect formulae to find the area of triangle ABC, see Extract 13.2. Other candidates used the concept of dot product instead of cross product in finding the area of triangle ABC. Moreover the candidates failed to differentiate the position vector $\underline{r} = x\underline{i} + y\underline{j} + z\underline{k}$ in order to obtain the velocity and acceleration. Extract 13.2 exposes some of these factors.

Extract 13.2

3 (b) Soln

Consider the triangle



$\underset{\sim}{A} = \underset{\sim}{i} + \underset{\sim}{j} + 2\underset{\sim}{k}$

$\underset{\sim}{b} = 3\underset{\sim}{i} + 2\underset{\sim}{j} - \underset{\sim}{k}$

$\text{Area} = |\underset{\sim}{a} \wedge \underset{\sim}{b}|$

$$= \begin{vmatrix} \hat{i} & \hat{j} & k \\ 1 & 1 & 2 \\ 3 & 2 & -1 \end{vmatrix}$$

$= |-1-4| + |-1-6| + |2-3|$

$= 5 + 7 + 1$

$= 13$

$\therefore$ Area of the triangle is 13 square units

Extract 13.2 shows a sample of a poor solution. In part (b), the candidate used a wrong formula to find the area of triangle ABC. He/she wrote Area $= |\underline{a} \times \underline{b}|$ instead of $\text{Area} = \frac{1}{2}|\underline{a} \times \underline{b}|$.

2.2.4 Question 4: Algebra

The question comprised of four parts, (a), (b), (c) and (d). In part (a), the candidates were required to solve the system of equations $2x + y - z = 3$, $x - y + z = 0$ and $x + 2y + z = -3$ by using Cramer's rule. In part (b), the candidates were required to (i) use binomial expansion of $\left(1 - \frac{1}{50}\right)^{\frac{1}{2}}$ to find the value of $\sqrt{2}$ correct to seven significant figures and (ii) decompose $\frac{2}{4n^2 - 1}$ into partial fractions and thereafter find $\sum_{n=1}^{n} \frac{2}{4n^2 - 1}$. In part (c), the candidates were given the information that "one of the zeros of the polynomial function $f(x) = x^4 - (2+h)x^3 + (2h-5)x^2 + (5h+6)x - 6h$ is obtained when $h = 1$". The candidates were required to find the value of the constants p, q and r when $f(x) = (1 - 2x + x^2)(px^2 - qx - r)$. In part (d), the candidates were given the simultaneous equations $\begin{cases} 3^x - 2^y = 0 \\ x + y - 1 = 0 \end{cases}$ and they were asked to show that $y = \log_6 3$.

This question was attempted by 9,140 (99.3%) candidates, out of which 76 (0.8%) candidates scored all 15 marks. However, 36.4 % of the candidates had their scores below 4.5 marks. The analysis shows that the general performance of the question is good because the percentage of candidates who scored from 4.5 to 15 marks is 63.6 percent.

The analysis of candidates' responses indicated good performance on which the majority of the candidates exhibited sufficient ability in applying Cramer's rule to solve the system of simultaneous equations and in breaking $\frac{2}{4\mathrm{n}^2 - 1}$ into partial fractions, specifically $\frac{2}{4\mathrm{n}^2 - 1} = \frac{1}{2\mathrm{n} - 1} - \frac{1}{2\mathrm{n} + 1}$. The candidates also applied the binomial theorem to expand the expression $\left(1 - \frac{1}{50}\right)^{\frac{1}{2}}$ as $1 + (0.5)(-0.02) + \frac{(0.5)(-0.5)}{2!}(-0.02)^2 + \frac{(0.5)(-0.5)(-1.5)}{3!}(-0.02)^3$ which was used to find the value of $\sqrt{2}$. In addition, they applied correctly the knowledge of exponents and logarithm to show that $y = \log_6 3$

from the simultaneous equations $\begin{cases} 3^x - 2^y = 0 \\ x + y - 1 = 0 \end{cases}$. Extract 14.1 represents one of the best responses from such candidates.

Extract 14.1

4 a) $\begin{pmatrix} 2 & 1 & -1 \\ 1 & -1 & 1 \\ 1 & 2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \\ -3 \end{pmatrix}$

let $A = \begin{pmatrix} 2 & 1 & -1 \\ 1 & -1 & 1 \\ 1 & 2 & 1 \end{pmatrix}$

$|A| = 2\begin{vmatrix} -1 & 1 \\ 2 & 1 \end{vmatrix} - \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} - 1\begin{vmatrix} 1 & -1 \\ 1 & 2 \end{vmatrix}$

$|A| = -6 + -3$

$|A| = -9$

deleting column of x

$\begin{pmatrix} 3 & 1 & -1 \\ 0 & -1 & 1 \\ -3 & 2 & 1 \end{pmatrix} \begin{pmatrix} \cancel{x} \\ \cancel{y} \\ \cancel{z} \end{pmatrix} = A_x$

$|A_x| = 3\begin{vmatrix} -1 & 1 \\ 2 & 1 \end{vmatrix} - \begin{vmatrix} 0 & 1 \\ -3 & 1 \end{vmatrix} + -\begin{vmatrix} 0 & -1 \\ -3 & 2 \end{vmatrix}$

4 a) $|A_x| = -9$ $\quad x = \dfrac{|A_x|}{|A|} = \dfrac{-9}{-9}$

$x = 1$

Deleting column of y

$\begin{pmatrix} 2 & 3 & -1 \\ 1 & 0 & 1 \\ 1 & -3 & 1 \end{pmatrix} = A_y$

$|A_y| = 2\begin{vmatrix} 0 & 1 \\ -3 & 1 \end{vmatrix} - 3\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} - 1\begin{vmatrix} 1 & 0 \\ 1 & -3 \end{vmatrix}$

$|A_y| = 9$ $\quad y = \dfrac{|A_y|}{|A|}$

$y = \dfrac{9}{-9} = -1$

Deleting column of z.

$$\begin{pmatrix} 2 & 1 & 3 \\ 1 & -1 & 0 \\ 1 & 2 & -3 \end{pmatrix} = A_z$$

$$|A_z| = 2\begin{vmatrix} -1 & 0 \\ 2 & -3 \end{vmatrix} - \begin{vmatrix} 1 & 0 \\ 1 & -3 \end{vmatrix} + 3\begin{vmatrix} 1 & -1 \\ 1 & 2 \end{vmatrix}$$

$$|A_z| = 18$$

$$z = \frac{|A_z|}{|A|} = \frac{18}{-9} = -2$$

$\therefore X = 1, \quad y = -1 \text{ and } z = -2$

b(i) From;

$$(1+x)^n = 1 + nx + \frac{n(n-1)x^2}{2!} + \ldots$$

let $x = -1/50$ & $n = 1/2$

4b(i) $$(1+(-x))^{1/2} = 1 + \frac{1}{2}(-x) + \frac{\frac{1}{2}\left(\frac{-1}{2}\right)(-x)^2}{2!} + \frac{\frac{1}{2}\left(\frac{-1}{2}\right)\left(\frac{-3}{2}\right)(-x)^3}{3!}$$

$$(1-x)^{1/2} = 1 - \frac{x}{2} - \frac{x^2}{8} - \frac{3}{48}x^3$$

$$(1-x)^{1/2} = 1 - x/2 - x^2/8 - 1/16\, x^3$$

Putting $x = 1/50$

$$\left(1 - 1/50\right)^{1/2} = 0.9899495$$

$$\sqrt{\frac{49}{50}} = 0.9899495$$

$$\frac{7}{\sqrt{25 \times 2}} = 0.9899495$$

$$\frac{7}{5\sqrt{2}} = 0.9899495 \qquad \sqrt{2} = \frac{7}{5 \times 0.9899495}$$

$$\sqrt{2} \simeq 1.414214$$

4b (ii) $$\frac{2}{(2n)^2 - 1^2} = \frac{2}{(2n-1)(2n+1)}$$

$$\frac{2}{(2n-1)(2n+1)} = \frac{A}{2n-1} + \frac{B}{2n+1}$$

$$\frac{2}{(2n-1)(2n+1)} = \frac{(2n+1)A + (2n-1)B}{(2n-1)(2n+1)}$$

$$2 = (2n+1)A + (2n-1)B$$

If $n = 1/2$

$2 = 2A \qquad A = 1$

If $n = -1/2$

$2 = -2B \qquad B = -1$

4b(ii) $\therefore \frac{2}{4n^2-1} = \frac{1}{2n-1} - \frac{1}{2n+1}$ (In partial fraction)

Now;

$$\sum_{1}^{n} \left(\frac{2}{4n^2-1}\right)$$

Suppose if we find $\sum_{m=1}^{m=n} \frac{2}{4m^2-1}$

$$\frac{1}{2m-1} - \frac{1}{2m+1}$$

$m = 1 \qquad \frac{1}{1} - \frac{1}{3}$

$m = 2 \qquad \frac{1}{3} - \frac{1}{5}$

$m = 3 \qquad \frac{1}{5} - \frac{1}{7}$

$\vdots$

$m = n-1 \qquad \frac{1}{2n-3} - \frac{1}{2n-1}$ +

$m = n \qquad \frac{1}{2n-1} - \frac{1}{2n+1}$

$$S_n = 1 - \frac{1}{2n+1}$$

$$S_n = \frac{2n+1-1}{2n+1}$$

$$S_n = \frac{2n}{2n+1}$$

n

$$\therefore \sum_{n=1}^{n} \frac{2}{4n^2-1} = \frac{2n}{2n+1}$$

4 c) $f(x) = x^4 - (2+h)x^3 + (2h-5)x^2 + (5h+6)x - 6h$

$h = 1 \quad f(x) = 0$

$0 = x^4 - 3x^3 - 3x^2 + 11x - 6$

factors; $x = 1$

$x = 3$

$(x-1)(x-3) = x^2 - 3x - x + 3$

$x^2 - 4x + 3$

$$\begin{array}{r} x^2 + x - 2 \\ x^2-4x+3 \overline{\big) x^4 - 3x^3 - 3x^2 + 11x - 6} \\ \underline{x^4 - 4x^3 + 3x^2} \\ x^3 - 6x^2 + 11x - 6 \\ \underline{x^3 - 4x^2 + 3x} \\ -2x^2 + 8x - 6 \\ \underline{-2x^2 + 8x - 6} \\ 0 + 0 \end{array}$$

$f(x) = (x^2 - 4x + 3)(x^2 + x - 2)$

Expanding $(1 - 2x + x^2)(px^2 - qx - r) = f(x)$

$f(x) = (1 - 2x + x^2)px^2 - qx(1 - 2x + x^2) - r(1 - 2x + x^2)$

$f(x) = px^2 - 2px^3 + px^4 - qx + 2qx^2 - qx^3 - r + 2rx - rx^2$

$f(x) = px^4 - (2p+q)x^3 + ((p-r)+2q)x^2 + (2r-q)x - r$

Equating with

$f(x) = x^4 - 3x^3 - 3x^2 + 11x - 6$

$-r = -6 \qquad r = 6$

$2r - q = 11 \qquad q = 2r - 11 = 1$

$px^4 = x^4 \qquad p = 1$

$\therefore p = 1, \quad q = 1$ and $r = 6$

4 d) $3^x - 2^y = 0$

$x + y = 1$

$3^x = 2^y$

$x \log 3 = y \log 2$

$x = \frac{y \log 2}{\log 3}$

$x + y = 1$

$\frac{y \log 2}{\log 3} + y = 1$

$$y\left(\frac{\log 2+\log 3}{\log 3}\right)=1$$

$$y=\frac{\log 3}{\log 2+\log 3}$$

$$y=\frac{\log 3}{\log(2\times 3)}=\frac{\log 3}{\log 6}$$

$$y=\log_6 3$$

Hence Shown.

Extract 14.1 is a good solution of a candidate who applied correctly knowledge about determinants to get the solution of the given system of equations, used the binomial expansion to get the correct value of $\sqrt{2}$ and applied correctly knowledge of algebra to compute the required polynomial.

On the other hand, there were few candidates who performed poorly in this question. The candidates who performed poorly failed to follow instructions of the question. For instance, some candidates used the inverse method instead of Cramer's rule to solve the given system of equations while others did not understand how to apply the determinants in solving the system of simultaneous equations in part (a). It was noted that a number of candidates solved the given equations in part (d) for x and y instead of showing that $y = \log_6 3$. It was also noted that some candidates failed even to decompose a rational function $\frac{2}{4n^2-1}$ into the sum of its partial fractions $\frac{1}{2n-1}$ and $-\frac{1}{2n+1}$. Extract 14.2 shows some of these weaknesses.

Extract 14.2

4. b) ii)

$$\frac{2}{4n^2-1}=\frac{A}{4n^2-1}+\frac{Ax+B}{4n^2-1}.$$

$$\frac{2}{4n^2-1}=\frac{A+Ax+B}{4n^2-1}$$

$2 = A+B+Ax$.

$A+B=2$.

$Ax=0$.

$A=0$.

$A+B=2$.

$0+B=2$.

$B=2$.

Extract 14.2 illustrates that the candidate expressed $\frac{2}{4n^2-1}$ as $\frac{A}{4n^2-1}+\frac{Ax+B}{4n^2-1}$ instead of $\frac{A}{2n-1}+\frac{B}{2n+1}$, an indication of lack of skills on how to write $4n^2-1$ as a difference of two squares.

2.2.5 Question 5: Trigonometry

The question had four parts; (a), (b), (c) and (d). In part (a), the candidates were required to solve the trigonometric equation $\sec^2\theta+\tan\theta-1=0$ for $0°\le\theta\le 360°$. Part (b), required the candidates to factorize completely the trigonometric expression $\cos\alpha-\cos 3\alpha-\cos 5\alpha+\cos 7\alpha$. In part (c), the candidates were required to (i) verify that $\frac{\cos^2 t-3\cos t+2}{\sin^2 t}=\frac{2-\cos t}{1+\cos t}$ and (ii) prove that $\frac{\sin 3A\sin 6A+\sin A\sin 2A}{\sin 3A\cos 6A+\sin A\cos 2A}=\tan 5A$. Finally, part (d) required the candidates to show that $\frac{\sin^2 x+2\sin x+1}{\cos^2 x}=\frac{\cos^2 x}{1-2\sin x+\sin^2 x}$.

This question was attempted by 5,848 (63.5%) candidates, out of which 328 (3.6%) candidates scored all the 20 marks allocated for this question. However, 35.4% of candidates had their scores below 6 marks. The analysis shows that the general performance of the question was good because the percentage of candidates who scored from 6 to 20 marks is 64.6.

The candidates who scored high marks recognized the need of using the formula $180n+\alpha$ to find angles that satisfy the equations

$\tan\theta = 0$ and $\tan\theta = -1$ in the interval $0° \le \theta \le 360°$. The candidates also used the factor formula to factorize $\cos\alpha - \cos 3\alpha - \cos 5\alpha + \cos 7\alpha$ into $-4\cos 4\alpha \sin 2\alpha \sin\alpha$ and used the additional theorem for cosine and sine correctly and applied $2\sin A\cos B = \sin(A+B) + \sin(A-B)$ and $2\cos A\cos B = \cos(A+B) - \cos(A-B)$ to show that $\frac{\sin 3A\sin 6A + \sin A\sin 2A}{\sin 3A\cos 6A + \sin A\cos 2A} = \tan 5A$. Moreover, the candidates managed to factorize $\cos^2 t - 3\cos t + 2$ as $(\cos t - 2)(\cos t - 1)$ then applied the identity $\sin^2 t = (1-\cos t)(1+\cos t)$ to show that $\frac{\cos^2 t - 3\cos t + 2}{\sin^2 t} = \frac{2-\cos t}{1+\cos t}$. Similarly, candidates recognized the need to factorize $\sin^2 x + 2\sin x + 1$ into $(\sin x + 1)^2$ and thereafter it was easy to show that $\frac{\sin^2 x + 2\sin x + 1}{\cos^2 x} = \frac{\cos^2 x}{1 - 2\sin x + \sin^2 x}$. An example of the work of one of the candidates who answered this question clearly and correctly is shown in Extract 15.1.

Extract 15.1

5a) $\cancel{1} + \tan^2\theta + \tan\theta - \cancel{1} = 0$

$\tan\theta(\tan\theta + 1) = 0$

$\tan\theta = 0$, $\tan\theta = -1$

$\theta = 180°n$, $\theta = 180°n - 45°$

for $n = 0, 1, 2 \ldots$

$\theta = 0°, 180°, 135°, 315°, 360°$

$\therefore \theta = 0°, 135°, 180°, 315°$ and $360°$

b) $= \cos\alpha - \cos 3\alpha - \cos 5\alpha + \cos 7\alpha$

$= \cos 7\alpha + \cos\alpha - (\cos 5\alpha + \cos 3\alpha)$

$= \cancel{2\cos 4\alpha \sin 3\alpha - (2\sin}$

$= 2\cos 4\alpha \cos 3\alpha - (2\cos 4\alpha \cos\alpha)$

$= 2\cos 4\alpha [\cos 3\alpha - \cos\alpha]$

$= 2\cos 4\alpha [-2\sin 2\alpha \sin\alpha]$

$\therefore = -4\cos 4\alpha \sin 2\alpha \sin\alpha$

5 c i) Consider the Left hand side

$$= \frac{(\cos t - 2)(\cos t - 1)}{\sin^2 t}$$

$$= \frac{(\cos t - 2)(\cos t - 1)}{1 - \cos^2 t}$$

$$= \frac{(\cos t - 2)(\cos t - 1)}{(1 - \cos t)(1 + \cos t)}$$

$$= \frac{-(\cos t - 2)\cancel{(1 - \cos t)}}{\cancel{(1 - \cos t)}(1 + \cos t)}$$

$$= \frac{-\cos t + 2}{1 + \cos t}$$

$$= \frac{2 - \cos t}{1 + \cos t}$$ hence verified

ii) Consider the Left hand side

$$= \frac{-\frac{1}{2}(\cos 9A - \cos 3A) + (-\frac{1}{2})(\cos 3A - \cos A)}{\frac{1}{2}[\sin 9A + \sin(-3A)] + \frac{1}{2}(\sin 3A - \sin A)}$$

$$= \frac{-\frac{1}{2}[\cos 9A - \cancel{\cos 3A} + \cancel{\cos 3A} - \cos A]}{\frac{1}{2}[\sin 9A - \cancel{\sin 3A} + \cancel{\sin 3A} - \sin A]}$$

$$= \frac{-[\cos 9A - \cos A]}{\sin 9A - \sin A}$$

$$= \frac{+[+\cancel{2}\sin 5A \cancel{\sin 4A}]}{\cancel{2}\cos 5A \cancel{\sin 4A}}$$

$$= \frac{\sin 5A}{\cos 5A}$$

$$= \tan 5A$$ hence proved

ii) Consider the Left hand side

$$= \frac{-\frac{1}{2}(\cos 9A - \cos 3A) + (-\frac{1}{2})(\cos 3A - \cos A)}{\frac{1}{2}[\sin 9A + \sin(-3A)] + \frac{1}{2}(\sin 3A - \sin A)}$$

$$= \frac{-\frac{1}{2}[\cos 9A - \cancel{\cos 3A} + \cancel{\cos 3A} - \cos A]}{\frac{1}{2}[\sin 9A - \cancel{\sin 3A} + \cancel{\sin 3A} - \sin A]}$$

$$= \frac{-[\cos 9A - \cos A]}{\sin 9A - \sin A}$$

$$= \frac{+[+\cancel{2}\sin 5A \cancel{\sin 4A}]}{\cancel{2}\cos 5A \cancel{\sin 4A}}$$

$$= \frac{\sin 5A}{\cos 5A}$$

$$= \tan 5A \quad \text{hence proved}$$

5 d) Consider the left hand side

$$= \frac{\sin^2 x + 2\sin x + 1}{\cos^2 x}$$

$$= \frac{(\sin x + 1)(\sin x + 1)}{\cos^2 x}$$

$$= \frac{(\sin x + 1)(\sin x + 1)}{1 - \sin^2 x}$$

$$= \frac{\cancel{(\sin x + 1)}(\sin x + 1)}{\cancel{(1 + \sin x)}(1 - \sin x)}$$

$$= \frac{\sin x + 1}{1 - \sin x} \cdot \frac{1 - \sin x}{1 - \sin x}$$

$$= \frac{1 - \sin^2 x}{(1 - \sin x)^2}$$

$$= \frac{\cos^2 x}{1 - 2\sin x + \sin^2 x}$$

hence shown.

Extract 15.1 shows the solution of a candidate who managed to answer the given question correctly. He/she was able to apply trigonometric identities to find the angles and used the factor formulas to factorize expressions and proved the equations.

Conversely, the candidates who performed poorly failed to use correct trigonometric definitions in doing proper substitutions and calculations. For instance, some candidates used incorrect substitution of t – formula instead of $1 + \tan^2 \theta = \sec^2 \theta$. In addition, other candidates did not keep in mind that they should borrow the knowledge of factorization in solving part c (i) and (d). Extract 15.2 shows one among the solutions of the candidates who performed poorly.

Extract 15.2

5(c). Soln

$$\sec^2\theta + \tan\theta - 1 = 0$$

$$0^\circ \leq \theta \leq 360^\circ$$

$$\sec^2\theta + \tan\theta - 1 = 0$$

From T formula

$$\& \cos\frac{\theta}{2} = \frac{1-t^2}{1+t^2}$$

$$\tan\frac{\theta}{2} = \frac{2t}{1+t^2}$$

$$\sec^2\theta + \tan\theta - 1 = 0$$

$$\frac{1}{\cos^2\theta} + \tan\theta - 1 = 0$$

$$\left(\frac{1-t^2}{1+t^2}\right)^2 + \frac{2t}{1+t^2} - 1 = 0$$

$$\frac{1-2t^2+t^4}{1+2t^2+t^4} + \frac{2t}{1+t^2} - 1 = 0$$

$$\frac{1-2t^2+t^4}{(1+t^2)^2} + \frac{2t}{1+t^2} = 1$$

$$\frac{1-2t^2+t^4+2t(1+t^2)}{(1+t^2)^2} = 1$$

$$\cancel{1+t^2}\ \frac{1-2t^2+t^4+2t+3t^3}{(1+t^2)^2} = 1$$

Extract 15.2 indicates the sample solution that was performed poorly. The candidate used t-formula instead of using the trigonometric identity $1+\tan^2\theta = \sec^2\theta$.

2.2.6 Question 6: Probability

The question comprised of four parts; (a), (b), (c) and (d). Part (a) stated that, "A school needs 10 prefects out of which 5 are supposed to be girls and 5 are to be boys". The candidates were required to find in how many different ways can the 10 prefects be selected if 5 boys are to be selected from a group of 8 boys and 5 girls from 9 girls.

Part (b) stated that, "Three athletes from Tanzania will participate in an International Coca Cola marathon race next year". The candidates were required to find the probability that at least two of them will complete the marathon if the probabilities to complete the marathon are 0.9, 0.7 and 0.6 respectively. In part (c), the candidates were given a random variable X that has probability density function $f(x) = \begin{cases} \frac{|x|}{8} & \text{for } -2 \le x \le 4, \\ 0 & \text{otherwise} \end{cases}$ and then were required to calculate the (i) expected value of X, (ii) standard deviation of X and (iii) variance of X. Finally, part (d) required the candidates to use the information that "if $P(A \cup B) = 80\%$ and $P(A \cup B') = 70\%$" to determine P(A).

This question was attempted by few candidates (16.5%) with only one candidate scoring 20 marks. The analysis further indicates that 80.3 percent of candidates had their scores below 6 marks. In this context, the general performance of the question was poor. It was also noted that it was the poorest performed question because the percentage of candidates who scored from 4.5 to 20 marks is 19.7. This justifies that, very few candidates had the required knowledge and skills on the topic of probability.

The analysis of the responses indicates that, many candidates scored low marks. The common mistakes made by these candidates include: using the principle of permutation ${}^{8}P_{5} \times {}^{9}P_{5}$ instead of the principle of combination ${}^{8}C_{5} \times {}^{9}C_{5}$ to obtain different ways of selecting 10 prefects. Also the candidates interpreted the integral

$\int_{-2}^{4} \frac{|x|}{8} dx$ as "$\int_{-2}^{4} \frac{x}{8} dx$" instead of $-\int_{-2}^{0} \frac{x}{8} dx + \int_{0}^{4} \frac{x}{8} dx$ an indication of lack of knowledge of absolute value functions. Other candidates used wrong concepts to compute the probability that at least two athletes will complete the marathon. For example some candidates were noted to use the concept discrete probability in part 6 (b). Extract 16.1 shows a sample answer of a candidate who failed to apply the rules of probability to solve the given problems.

Extract 16.1

6(b) let x be Values of Athlets

X	1	2	3
$P(x)$	0.9	0.7	0.6

Probability that atleast two will complete,

$P(X \geq 2) = P(2) + P(3)$

$= 0.7 + 0.6$

$= 1.3$

$\therefore$ Probability that atleast two will complete marathon is 1.3.

In Extract 16.1, the candidate used the concept of discrete probability distribution where he/she considered X to assume the values x_1, x_2 and x_3 which was wrong.

On the other hand, the few candidates who performed well realized that the process of obtaining different ways of selecting 10 prefects does not take into account of the order, so they used the principle of combination ${}^{8}C_{5} \times {}^{9}C_{5}$ correctly and got 7056 as the required answer in part (a). In part (b), the candidates understood the requirements of the question and therefore, used the probability diagram to find the probability that at least two athletes will complete the marathon. In part (c), they also managed to interpret the probability density function $f(x)=\begin{cases}\dfrac{|x|}{8} & \text{for } -2 \le x \le 4 \\ 0 & \text{otherwise}\end{cases}$ into $f(x)=\begin{cases}\dfrac{-x}{8} & -2 \le x \le 0 \\ \dfrac{x}{8} & 0 \le x \le 4\end{cases}$

and hence proceeded smoothly in getting the expectance value of X, standard deviation of X as well as the variance of X. Furthermore, these candidates used correctly the formulae $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ and $P(A \cup B') = P(A) + P(B') - P(A \cap B')$ in part (d) to determine P(A) given that $P(A \cup B) = 80\%$ and $P(A \cup B') = 70\%$. Extract 16.2 shows one among the best solutions of this question.

Extract 16.2

6a Solution.

By concept of combination.

	Requirements	Available
Boys	5	8
Girls.	5	9.

No of ways of selecting 5 boys is given by

$^8C_5$

No of ways of selecting 5 girls is given by

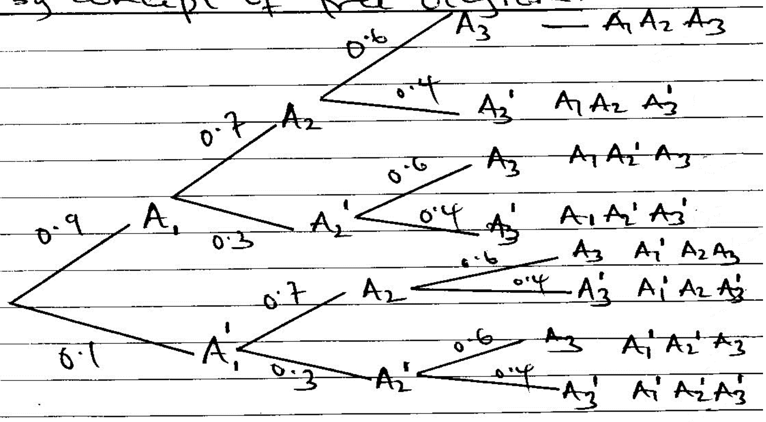
$^9C_5$.

Total no of ways.

$^8C_5 \times {}^9C_5$

$= \underline{\underline{7056 \text{ ways.}}}$

(b) By concept of tree diagram.



where
A_1 – athlete one will compete
A_1' – athlete one wont compete
A_2 – athlete two will compete
A_2' – athlete two wont compete ~~where~~
A_3 – athlete 3 to compete
A_3' – athlete not to compete

(i) Probability of atleast two.

$= P(A_1A_2A_3) + P(A_1A_2A_3') + P(A_1A_2'A_3) + P(A_1'A_2A_3)$

$= 0.9 \times 0.7 \times 0.6 + 0.9 \times 0.7 \times 0.4 + 0.9 \times 0.3 \times 0.6 + 0.1 \times 0.7 \times 0.6$

$= \cancel{0.384}\ \ 0.834$

(c) $f(x) = \begin{cases} \frac{|x|}{8} & \text{for } -2 \le x \le 4 \\ 0 & \text{otherwise} \end{cases}$

~~This can be~~
solution.
This can be rewritten as.

$f(x) = \begin{cases} -x/8 & \text{for } -2 \le x \le 0 \\ x/8 & \text{for } 0 \le x \le 4 \\ 0 & \text{otherwise} \end{cases}$

was to satisfy.

(i) Expectation E(x)

$$E(x) = \int_a^b x f(x)\, dx$$

$$E(x) = \int_{-2}^{0} \frac{-x^2}{8}\, dx + \int_0^4 \frac{x^2}{8}\, dx$$

$$= \underline{\underline{2.33333.}}$$

(ii) Standard deviation of x

$$S.D = \sqrt{var(x)}$$

$$var(x) = \int x^2 f(x)\, dx - (\bar{x})^2$$

$$= \int_{-2}^{0} \frac{-x \cdot x^2}{8}\, dx + \int_0^4 \frac{x^2 \cdot x}{8} - (2.3333)^2$$

$$= 0.5 + 8 - (2.3333)^2$$

$$var\,x = 3.0557$$

$$S.D = \sqrt{var(x)}$$

$$S.D = \underline{\underline{1.748.}}$$

INDEX NUMBER.....................................

(iii) var(x)

$$var(x) = \int x^2 f(x)\, dx - (E(x))^2$$

$$= \int_{-2}^{0} \frac{-x^3}{8}\, dx + \int_0^4 \frac{x^3}{8}\, dx - (2.3333)^2$$

$$= 0.5 + 8 - (2.3333)^2$$

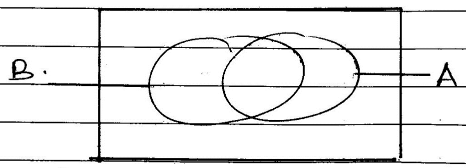
$$= 3.0555571111$$

d) $P(A \cup B) = 80\%$

$P(A \cup B') = 70\%$

From probability table derived from venn diagram.

	~~A~~	~~A'~~
~~B~~	~~$A \cap B$~~	~~$A' \cap B$~~
~~B'~~		



	A	A'
B	$A \cap B$	$A' \cap B$
B'	$A \cap B'$	$A' \cap B'$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$+\ P(A \cup B') = P(A) + P(B') - P(A \cap B')$

$P(A \cup B) + P(A \cup B') = 2(P(A) + 100 - P(A \cap B) - P(A \cap B')$

$80\% + 70\% = 2P(A) + 100\% - P(A \cap B) - P(A \cap B')$

Now

$P(A) = P(A \cap B) + P(A \cap B')$

$80\% + 70\% = 2P(A) + 100\% - [P(A \cap B) + P(A \cap B')]$

$80\% + 70\% = 2P(A) + 100\% - P(A)$

$150\% = 2P(A) + 100\% - P(A)$

$P(A) = 50\%$

In Extract 16.2, the candidate demonstrated high level of understanding of the basic concepts of probability.

2.2.7 Question 7: Differential Equations

This question had four parts; (a), (b), (c) and (d). In part (a) (i), the candidates were required to form the first order differential equations representing the family of the curve $x^2 + y^2 - 2kx = 0$. In part (a) (ii), the candidates were required to find the particular solution of the differential equation $x\frac{dy}{dx} = x + y$, given that $y = -1$ when $x = 1$. Part (b), required the candidates to solve the initial value problem $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = xe^x$, $y(0) = 4$ and $\frac{dy}{dx} = -5$ when $x = 0$. In part (c), the candidates were given the information that, "the rate of decrease of temperature of water is direct proportional to the difference between temperature of water and that of the medium". If water at a temperature of $100°C$ cools in 10 minutes to $80°C$ in a room temperature of $25°C$, they were then required to find (i) temperature of water after 20 minutes and (ii) the time when the temperature is $40°C$ given that, any approximation in calculations must be presented in 5 decimal places. Lastly, part (d) required the candidates to find the equation of the curve which passes through (2, 3) given that $\frac{dy}{dx} = \frac{x^2 y}{x^3 + 1}$.

This question was attempted by 5,701 (61.9%) candidates, out of which 10 (0.2%) candidates scored all 20 marks that were allocated for this question. However, 25.7 % of candidates had their scores below 6 marks. The analysis shows that the general performance of the question is good because the percentage of candidates who scored from 6 to 20 marks is 74.3. It was the best performed question in paper 2.

The candidates who scored high marks in part (a) recognized the need to eliminate k in the equation $x^2 + y^2 - 2kx = 0$ by differentiating this equation once with respect to x. In part (a) (ii) the candidates realized that, the equation $x\frac{dy}{dx} = x + y$ is homogenous and so they used the substitution $y = vx$ to reduce it to a variable separable equation $\frac{dv}{dx} = \frac{1}{x}$ which was easy to work on. In part (b), the candidates managed to reduce the homogeneous equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$ into the characteristic equation $m^2 - 2m + 1 = 0$ which was essential in obtaining the complementary solution $y = e^x\left(Ax + B\right)$. The candidates were also able to substitute $y = px^3e^x$ into the original equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = xe^x$ and be able to obtain the particular solution, $y = \frac{1}{6}x^3e^x$ which was added with $y = e^x\left(Ax + B\right)$ to get the general solution $y = e^x\left(-9x + 4\right) + \frac{1}{6}x^3e^x$. In part (c), the candidates used the knowledge of first order differential equation to solve the question about Newton's law of cooling to get the temperature of water after 20 minutes as well as the time when the temperature is $40°C$. Similarly, in part (d), the knowledge of first order differential equation was used by the candidates to find the equation of the solution curve which passes through (2, 3) given that $\frac{dy}{dx} = \frac{x^2y}{x^3 + 1}$. Extract 17.1 shows a sample answer from one of the candidates who did well in this question.

Extract 17.1

7a)i)

$x^2 + y^2 - 2kx = 0$

$2x + 2y\frac{dy}{dx} - 2k = 0$

$2y\frac{dy}{dx} = 2k - 2x$

$\frac{dy}{dx} = \frac{k - x}{y}$

	but $k = \frac{x^2+y^2}{2x}$
	$\frac{dy}{dx} = \frac{x^2+y^2}{2x} - x$
	y
	$y\,\frac{dy}{dx} = \frac{x^2+y^2-2x^2}{2x}$

7a) i)	$y\,\frac{dy}{dx} = \frac{y^2-x^2}{2x}$
	$\frac{dy}{dx} = \frac{y^2-x^2}{2xy}$
7a) ii)	$x\frac{dy}{dx} = x+y$
	$\frac{dy}{dx} = \frac{x+y}{x} = 1 + \frac{y}{x}$
	let $u = y/x$
	$y = ux$
	$\frac{dy}{dx} = u + x\frac{du}{dx}$
	$u + x\frac{du}{dx} = 1 + u$
	$x\,du/dx = 1$
	$\int du = \int \frac{dx}{x}$
	$u = \ln x + c.$
	$\frac{y}{x} = \ln x + c$
	$y = x\ln x + cx$
	$x = 1,\ y = -1$
	$-1 = 1(\ln 1) + (+1)c$
	$-1 = c$
	$\therefore$ The particular solution is $y = x\ln x - x$

7b)	$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = xe^x$
	$m^2 - 2m + 1 = 0$
	$m = \frac{2 \pm \sqrt{4-4}}{2}$
	$m = \frac{2 \pm 0}{2}$

$m = 1$

$y_c = e^x(Ax + B)$

Let $y_p = ax^3e^x$

$y'_p = ax^3e^x + ae^x(3x^2)$

$= ax^3e^x + 3ax^2e^x$

7b).

$y''_p = 3ax^2e^x + ax^3 + 3ax^2e^x + 3ae^x(2x)$

$= 3ax^2e^x + ax^3 + 3ax^2e^x + 6axe^x$

$= 6ax^2e^x + ax^3 + 6axe^x$

$6a\cancel{x^2}e^x + a\cancel{x^3} + 6axe^x - 2ax^3e^x - 6a\cancel{x^2}e^x + ax^3e^x = xe^x$

$6ax = x$

$6a = 1$

$a = 1/6$

$\therefore y_p = ax^3e^x$

$= \frac{1}{6}x^3e^x$

$\therefore$ The ~~p~~ general solution is $\underline{y = e^x(Ax + B) + \frac{1}{6}x^3e^x}$.

$y = e^x(Ax + B) + \frac{1}{6}x^3e^x$

$y(0) = 4$

$4 = B$

$dy/dx = e^x(A) + e^x(Ax + B) + \frac{1}{6}x^3e^x + \frac{3}{6}x^2e^x$

$-5 = A + B$

$-5 = A + 4$

$A = -5 - 4$

$A = -9$

$\therefore$ The particular solution is

$$\underline{y = e^x(-9x + 4) + \frac{1}{6}x^3e^x}$$

7c).

$-\frac{d\theta}{dt} \propto \theta - \theta_e$

where θ_e - temperature of the medium

$\frac{d\theta}{dt} = -k(\theta - \theta_e)$

$\int \frac{d\theta}{\theta - \theta_e} = \int -k \, dt$

$\ln|\theta - \theta_0| = -kt + c$

$\theta - \theta_e = Ae^{-kt}$

at $t = 0$, $\theta = 100°C$, $\theta_e = 25°C$

$100 - 25 = Ae^{0}$

$75 = A$

$\theta - \theta_e = 75e^{-kt}$

At $t = 10$, $\theta = 80$

$80 - 25 = 75e^{-10k}$

$\frac{55}{75} = e^{-10k}$

$-10k = \ln\left(\frac{55}{75}\right)$

$k = -\frac{1}{10}\ln\left(\frac{55}{75}\right)$

At $t = 20$, $\theta = ?$

$\theta - 25 = 75e^{-\left(-\frac{1}{10}\ln\left(\frac{55}{75}\right) \times 20\right)}$

$\theta - 25 = 75e^{2\ln\left(\frac{55}{75}\right)}$

i) $\underline{\theta = 65.33333\ °C.}$

ii) At $t = ?$

$\theta - \theta_e = 75e^{-\left(-\frac{1}{10}\ln\left(\frac{55}{75}\right) \times t\right)}$

$40 - 25 = 75e^{-t \times 0.031015492}$

$\frac{1}{5} = e^{-t \times 0.031015492}$

$\ln\frac{1}{5} = -0.031015492t$

$t = 51.89142$

$\therefore$ The time will be $\underline{51.89142}$ minutes

7d) $\frac{dy}{dx} = \frac{x^2y}{x^3+1}$

$(x^3+1)\,dy = x^2y\,dx.$

$\int \frac{dy}{y} = \int \frac{x^2}{x^3+1}\,dx$

$\int \frac{dy}{y} = \int \frac{x^2}{t} \cdot \frac{dt}{3x^2}$ let $x^3+1 = t$, $3x^2\,dx = dt$

$\int \frac{dy}{y} = \frac{1}{3}\int \frac{1}{t}\,dt$

$\ln y = \frac{1}{3}\ln t + c.$

$\ln y = \frac{1}{3}\ln(x^3+1) + c$

$\ln y = \frac{1}{3}\ln(x^3+1) + c$

$3 \ln y = \ln A(x^3+1)$

$y^3 = A(x^3+1)$

at $(2, 3)$

$27 = A(9)$

$A = 3.$

$\therefore$ The ~~ge~~ particular solution is $y^3 = 3(x^3+1)$

Extract 17.1 illustrates responses of a candidate who answered the question correctly. He/she used the basic concepts of differential equations to formulate and solve the given equations.

However, there were few candidates who performed poorly in this question. These candidates failed to recognize that, they were supposed to substitute the equation $2k = \dfrac{x^2 + y^2}{x}$ into $2x + 2y\dfrac{dy}{dx} - 2k = 0$ to obtain the differential equation $2xy\dfrac{dy}{dx} = y^2 - x^2$. Some candidates used wrong substitution, for example $y = px^2e^x$ to find the particular equation while others did not know how to find the particular solution. Furthermore, the candidates did not manage to formulate differential equations from word problems and thus failing to find the time and temperature in part (c). There were also candidates who failed to identify that in part (d), the equation $\dfrac{dy}{dx} = \dfrac{x^2 y}{x^3 + 1}$ could be integrated directly, provided it was written in a separable form $\dfrac{dy}{y} = \dfrac{x^2}{x^3 + 1}dx$. Extract 17.2 shows sample response of the candidates who performed poorly in this question.

Extract 17.2

7. (a) (i) $x^2 + y^2 - 2kx = 0$

$x^2 + y^2 = 2kx$ —— (1)

$2x + 2y\, dy/dx = 2kx + 0$

$2y\, dy/dx = 2kx - 2x$

$dy/dx = \dfrac{x^2 + y^2 - 2x}{2y}$

$$\therefore dy/dx = \frac{x^2+y^2-2x}{2y}$$

7. (d) $dy/dx = \frac{x^2y}{x^3+1}$

$dy/dx = \frac{(2)^2(3)}{2^3+1} = 4/3$

$dy/dx = \cancel{7}\ 4/3$

From

$m = \frac{y_2-y_1}{x_2-x_1}$

$4/3 = \frac{y-3}{x-2}$

$4(x-2) = 3(y-3)$

$4x-8 = 3y-9$

$4x-3y+1=0$.

$\therefore$ The equation of a curve $= 4x-3y+1=0$

Extract 17.2 shows a sample solution from one of the candidates who performed poorly. In part (a) (i), the candidate failed to eliminate k in formulating the first order differential equation. In part (d), he/she substituted the x and y values in the equation $\frac{dy}{dx} = \frac{x^2y}{x^3+1}$ instead of solving the differential equation.

2.2.8 Question 8: Coordinate Geometry II

This question comprised of three parts; (a), (b) and (c). In part (a), the candidates were required to; (i) find the equation of the tangent to the ellipse $9x^2+25y^2-18x-100y-116=0$ at $(1, 5)$ and (ii) show that the locus of the midpoint of AB is an ellipse with the same eccentricity as $\frac{x^2}{16}+\frac{y^2}{9}=1$ provided that the normal to the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ at the point $p(4\cos\theta, 3\sin\theta)$ meets the x-axis and y-axis at A and B respectively. In part (b), the candidates were required to (i) determine the polar equation of $x^2+y^2-2x-3y=0$ and (ii) draw the graph of the polar equation obtained in (b) (i). In part (c), the candidates were required to (i) show that the line $3x-y+1=0$ touches the parabola $y^2=12x$ and (ii) find an equation connecting a, b, and c, if $ax+by+c=0$ touches the parabola $x^2-8y=0$.

This question was attempted by 4,784 (52%) candidates, out of which 32 (0.7%) candidates scored 20 marks. However, 35.7 % of candidates had their scores below 6 marks. The analysis shows that the general performance of the question is good because the percentage of candidates who scored from 6 to 20 marks is 64.3.

Most candidates managed to answer correctly parts (a) (i), (b) (i) and (c). They managed to apply the techniques of differentiation to find slope of the given equation of the ellipse at the given point and finally determined the equation of the tangent. They were also able to show that, the locus of the midpoint of AB is an ellipse with the same eccentricity as that of the given ellipse and managed to convert the given Cartesian equation into polar equation and finally drew its graph. In part (c), they realized that solving the equations $y=3x+1$ and $y^2=12x$ gives a quadratic equation $9x^2-6x+1=0$ with a repeated root $x=\frac{1}{3}$. Thus, the given line touches the parabola. Extract 18.1 shows one of the good responses in this question.

Extract 18.1

(c) (i) to show that; $3x-y+1=0$ touches the parabola $y^2=12x$.— ⊛

8(c)(i) $y = 3x+1$ — (1) insert ~~(*)~~ equation(1) into equation(*)

$(3x+1)^2 = 12x$

$\Rightarrow 9x^2 + 6x + 1 = 12x$

$9x^2 - 6x + 1 = 0.$

~~for~~ $b^2 - 4ac = ?$

$= (-6)^2 - 4(9)(1)$

$= 36 - 36$

$= 0$

$\therefore$ Since $b^2 - 4ac = 0$; then the line $3x - y + 1 = 0$ touches the parabola $y^2 = 12x$

8(c)(ii) Given that $ax + by + c = 0$ touches the parabola $x^2 - 8y = 0$

$\Rightarrow x^2 = 8y$ — (1)

also $by = -ax - c$

$y = -\frac{a}{b}x - \frac{c}{b}$ (2)

insert equation (2) into equation (1)

$x^2 = 8\left(\frac{-ax - c}{b}\right)$

$\Rightarrow bx^2 = -8ax - 8c$

$bx^2 + 8ax + 8c = 0$

Because it touches the parabola

$b^2 - 4ac = 0 \Rightarrow b^2 = 4ac$

$(8a)^2 = 4(b)(8c) \Rightarrow \frac{64a^2}{32} = \frac{32bc}{32}$

$\Rightarrow 2a^2 = bc$ (is the required equation)

Extract 18.1 portrays response of a candidate who answered the question 8 (c) correctly. He/she verified that the linear equation touches the parabola and managed to find the equation connecting the constants a, b and c.

On the other hand, there were few candidates who failed to answer this question. The candidates failed because they had insufficient knowledge to differentiate the equation of the ellipse implicitly which was the necessary step to find the equation of the tangent to the ellipse. Similarly, others had inadequate knowledge and skills to manipulate the concept of a normal at a point to the ellipse while others faced difficulties to draw the graph of the polar equation. Therefore, lack of knowledge on the concepts of tangents and normal to a parabola and ellipse and also failure to convert the mode of the scientific calculator into radian when evaluating the angles in graph drawing of the polar equation, were main weakness in this question. Extract 18.2 illustrates some of these weakness.

Extract 18.2

from $m = \frac{y_2 - y_1}{x_2 - x_1}$

$\frac{2}{5} = \frac{y-5}{x-1}$

$5(y-5) = 2(x-1)$

$5y - 25 = 2x - 2.$

$5y - 25 + 2 = 2x.$

$5y - 23 = 2x.$

$2x - 5y + 23 = c.$

$\therefore$ The equation is $2x - 5y + 23 = 0.$

8 (b) (ii) $\frac{x^2}{16} + \frac{y^2}{9} = 1$

~~Equation of a~~ tan ~~normal can~~ be obtainede by Slope of a tangent

$\frac{2x}{16} + \frac{2y}{9}\frac{dy}{dx} = 0$

$\frac{2y}{9}\frac{dy}{dx} = \frac{2x}{16}$

$\frac{2y}{2y}\frac{dy}{dx} = \frac{18x}{16 \times 2y}$

$\frac{dy}{dx} = \frac{18x}{32y}$ $\quad$ $\frac{dy}{dx} = \frac{x}{2y}$

Slope of a normal

$M_T \cdot M_N = -1$

$\frac{x}{2y} \times M_N = -1$

$M_N = \frac{-2y}{x}$

at point $P(4\cos\varphi, 3\sin\varphi)$

$M_N = -\frac{2 \times 3\sin\varphi}{4\cos\varphi} = \frac{-6\sin\varphi}{4\cos\varphi}$

Extract 18.2 shows a poor solution from one of the candidates who failed to differentiate the equation of an ellipse in part (a) (i). He/she got $18x + 50\frac{dy}{dx} - 18 - 100 = 0$ instead of $18x + 50\frac{dy}{dx} - 18 - 100\frac{dy}{dx} = 0$. Moreover, in part (a) (ii), he/she calculated the slope instead of showing that the locus of the midpoint of AB is an ellipse with the same eccentricity as that of the given ellipse.

3.0 PERFORMANCE OF CANDIDATES IN VARIOUS TOPICS

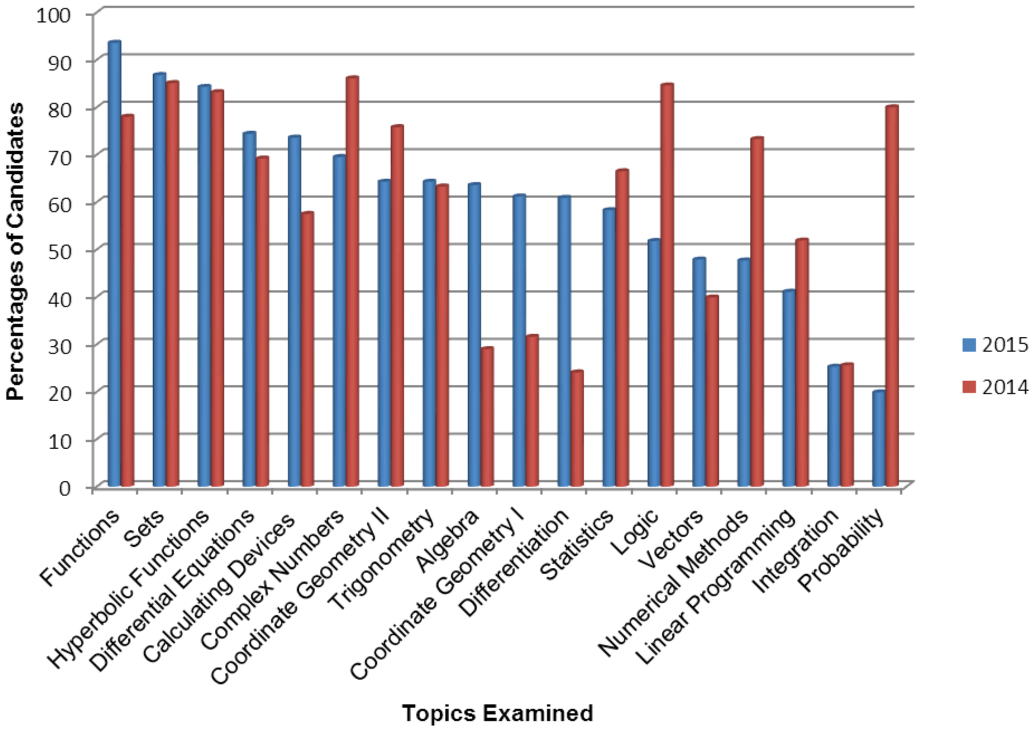
The analysis of the candidates' performance in each topic in both papers indicated that they had good performance in Functions, Sets, Hyperbolic Functions, Differential Equations, Calculating Devices, Complex Numbers, Coordinate Geometry II, Trigonometry, Algebra, Coordinate Geometry I, Differentiation, Statistics and Logic. The candidates' good performance of the examination was attributed by the ability to follow the instructions of questions, use of the correct formula, and application of right principles to make mathematical manipulations. Also, the candidates realized the required logics, theorems, rules and mathematical concepts for solving various questions. Therefore, the candidates' ability to use their knowledge and skills in solving examination questions enabled them to have good performance.

The analysis also showed that, candidates had an average performance in the topics of Vectors, Numerical Methods and Linear Programming. The candidates' scores in these topics ranged from an average of 30 percent to 49 percent. The reasons that caused candidates' average performance include lack of knowledge and skills to use mathematical signs, formulas, laws and theorems. For instance in linear programming candidates used wrong inequality signs in formulating linear constraints. Some of the candidates lacked the competence to show, derive and determine the required steps in the process of solving the questions. Therefore, these candidates lose the allocated marks on the given item.

Further analysis showed that candidates performed poorly in the topics of Integration and Probability. The scores of these candidates in these topics ranged from average of zero percent to 29 percent. The reasons that have contributed to the poor performance include wrong interpretations of formulae and using wrong concepts and principles. Other factors include insufficient skills to sketch graphs and using wrong techniques of integrations.

The comparison of the performance of candidates in the topics that were examined in ACSEE 2014 and 2015 Advanced Mathematics is summarized in the following figure. However, tabular form presentation is shown in the appendix.

Figure: The percentages of candidates who scored an average of 30 or more of ACSEE 2014 and 2015 Advanced Mathematics



The information shown in the figure above indicates the comparison of performance in each topic that was examined in ACSEE 2015 and ACSEE 2014 Advanced Mathematics. The bars in the figure show the rise and fall in performance of candidates in the tested topics. For instance, the increase in performance is noted in ten topics of Functions, Sets, Hyperbolic Functions, Differential Equations, Calculating Devices, Coordinate Geometry I, Differention and Vectors. However, the decrease is noted in seven topics which are Complex Numbers, Coordinate Geometry II, Statistics, Logic, Numerical Methods, Linear Programming and Probability. In addition, the performance of candidates in the topic of integration has remained relatively weak as it was noted in the ACSEE 2014 Advanced Mathematics Examiners' report. Persistence of poor performance in thopic could be due the reasons given in the previous paragraphs.

4.0 CONCLUSION AND RECOMMENDATIONS

4.1 Conclusion

The analysis of the candidates' performance in each question indicated that the performance was good because many candidates scored 30 percent or above of the marks allocated to individual questions. It was discovered that, among 18 topics which were examined, 13 topics were performed well, 3 topics were performed averagely and 2 topics were performed poorly. Good performance in many topics was contributed by the ability of candidates to use their knowledge and skills in solving examination questions.

Although the general performance in Advanced Mathematics in 2015 is good, there is a slight drop of performance as compared to previous year. The candidates who passed in 2015 is 85.02 percent as compared to 89.4 percent of candidates who passed in 2014. This might have been caused by the nature of the cohort and the rise and fall in performance for the tested topics.

4.2 Recommendations

In order to improve the candidates' performance in Advanced Mathematics it is recommended that;

(a) The Ministry of Education and Vocational Training (MoEVT) should establish a mathematics program in each district that will mobilize and motivate teachers and students to meet and share experience in the topics that had poor performance.

(b) The Ministry of Education and Vocational Training (MoEVT) should provide the teaching and learning materials to improve quality of learning mathematics.

(c) The teachers should cover the whole syllabus to enhance leaners to acquire variety of knowledge, skills and attitude from different topics.

(d) Learners should do a lot of exercises so as to improve their competences in answering mathematics question.

Appendix

Summary of Candidates' Performance per Topic in 2014 and 2015

S/N	Topic	Number of questions	2015		2014	
			The % of candidates who scored an average of 30 % and above.	Remarks	The % of candidates who scored an average of 30 % and above..	Remarks
1	Functions	1	93.6	Good	78	Good
2	Sets	1	86.8	Good	85.1	Good
3	Hyperbolic Functions	1	84.3	Good	83.2	Good
4	Differential Equations	1	74.4	Good	69.2	Good
5	Calculating Devices	1	73.6	Good	57.5	Good
6	Complex Numbers	1	69.5	Good	86.1	Good
7	Coordinate Geometry II	1	64.3	Good	75.8	Good
8	Trigonometry	1	64.3	Good	63.3	Good
9	Algebra	1	63.6	Good	29	Weak
10	Coordinate Geometry I	1	61.2	Good	31.6	Average
11	Differentiation	1	60.9	Good	24.1	Weak
12	Statistics	1	58.3	Good	66.5	Good
13	Logic	1	51.8	Good	84.6	Good
14	Vectors	1	47.9	Average	39.9	Average
15	Numerical Methods	1	47.7	Average	73.3	Good
16	Linear Programming	1	41.1	Average	51.9	Good
17	Integration	1	25.3	Weak	25.6	Weak
18	Probability	1	19.9	Weak	80	Good