THE UNITED REPUBLIC OF TANZANIA
MINISTRY OF EDUCATION, SCIENCE AND TECHNOLOGY
NATIONAL EXAMINATIONS COUNCIL OF TANZANIA

CANDIDATES' ITEM RESPONSE ANALYSIS REPORT ON THE ADVANCED CERTIFICATE OF SECONDARY EDUCATION EXAMINATION (ACSEE) 2022

BASIC APPLIED MATHEMATICS

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141 BASIC APPLIED MATHEMATICS

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FOREWORD

This report provides an analysis of the candidates' performance in the Basic Applied Mathematics paper of the Advanced Certificate of Secondary Education Examination (ACSEE) 2022. The report reveals strengths and weaknesses observed in the candidates' responses.

The analysis shows that the candidates performed well in the topics of *Algebra* and *Linear Programming* but they had average performance in the topics of *Calculating Devices* and *Functions*. The good performance was due to the ability of the candidates to formulate equations/inequalities from word problems, solve the equations, draw graphs of linear inequalities, and apply the formula for determining the general term of an arithmetic progression.

The overall performance of candidates in other topics was weak. The topics with weak performance included *Exponential and Logarithmic Functions*, *Integration*, *Trigonometry*, *Differentiation*, *Probability* and *Statistics*. The weak performance resulted from candidates' failure to draw graphs of exponential and logarithmic functions, apply integration techniques, calculate the area between two curves, and apply trigonometric identities and rules. Other factors for weak performance included inability of the candidates to use the first principles of differentiation; apply the knowledge of differentiation in solving optimization problems and recall the condition for mutually exclusive events and formulae for calculating the mean, median and variance.

The National Examinations Council of Tanzania is thankful to everyone who participated in the preparation of this report.

Athumani S. Amasi

EXECUTIVE SECRETARY

1.0 INTRODUCTION

In 2022 the Basic Applied Mathematics paper was set following the 2019 Examination Format and the 2010 Basic Applied Mathematics Syllabus for Advanced Secondary Education. This paper consisted of 10 compulsory questions, each carrying ten (10) marks. A total of 34,549 candidates sat for this paper, and 20,184 (58.66%) of them passed. Compared to the performance in 2021, the performance in 2022 has decreased by 2.19 per cent. Figure 1 shows the percentage of the candidates for each grade of performance in 2022.

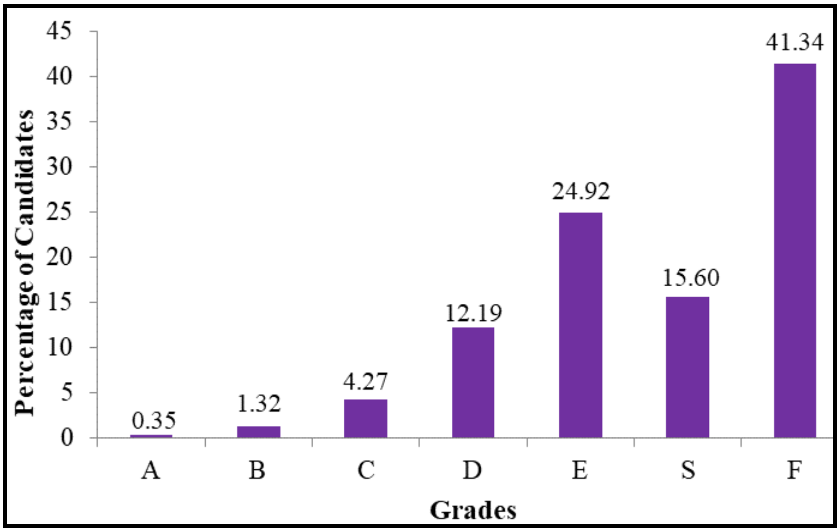


Figure 1: *Performance of candidates by grades in 2022*

In Section 2, the report analyses the performance of candidates in each question and the descriptions of the strengths and weaknesses observed in candidates' responses. This report includes extracts of the candidates' responses that illustrate their strengths or weaknesses. The overall candidates' performance in each question is categorized based on the percentage of candidates who scored 3.5 marks or more. The categories are 60 - 100, 35 - 59, and 0 - 34 per cent for good, average and weak performance respectively. In graphs or charts, the performance is coloured green, yellow and red for good, average and weak performance.

Section 3 of the report analyses the performance of the candidates by topics, while Section 4 gives a conclusion and recommendations. Finally, this report includes **Appendix I** which shows the performance on the examined topic in 2022 and **Appendix II** which shows the candidates' performance in 2021 and 2022.

2.0 ANALYSIS OF CANDIDATES' RESPONSES IN EACH QUESTION

2.1 Question 1: Calculating Devices

The question instructed the candidates to use a non-programmable scientific calculator to:

(a) *(i)* *compute* $\dfrac{3254\times 3.14\sqrt{417}}{10^5\times\log\sqrt[3]{278}}$ *(correct to* 2 *decimal places).*

(ii) *evaluate* $\tan^{-1}\left[\dfrac{\ln\left(\dfrac{4\times 10^2}{\pi}\right)}{\log\sqrt{2}}\right]$ *correct to* 2 *significant figures.*

(iii) *find the value of* $\sum_{x=1}^{4} e^x\sqrt{1+x^2}$ *correct to* 4 *decimal places.*

(b) *Given that* $P(x=r)=\binom{n}{r}P^r(1-P)^{n-r}$ *where* $n=10$ *and* $P=0.45$*, find the numerical value of* $P(x=1)$ *correct to* 4 *significant figures.*

A total of 20,244 (58.60%) candidates who answered this question scored from 3.5 to 10 marks. Thus, the overall performance of the candidates in this question was average. Figure 2 shows the percentage of candidates who got low, average and high marks.

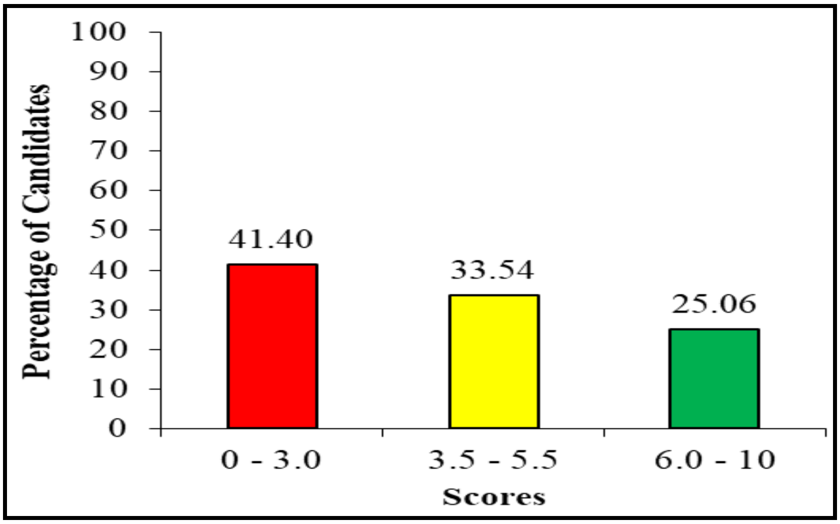


Figure 2: *The candidates' performance in question 1*

The data also reveal that 1,041 (3.01%) candidates correctly responded to all parts of the question, indicating that they were competent in using non-programmable scientific calculators. Extract 1.1 is the sample of a correct solution from one of the candidates.

01. (a) (i) $\frac{3254 \times 3.14\sqrt{417}}{10^5 \times \log\sqrt[3]{278}} = \underline{\underline{2.56}}$

(ii) $\tan^{-1}\left[\frac{\ln\left(\frac{4 \times 10^2}{\pi}\right)}{\log\sqrt{2}}\right] = \underline{\underline{88^\circ}}$

(iii) $\sum_{x=1}^{4} e^x\sqrt{1+x^2} = \underline{\underline{308.9966}}$

(b) Solution:

$P(x=r) = \binom{n}{r}p^r(1-p)^{n-r}$

Given

$n = 10$

$p = 0.45$

$= \binom{10}{r}0.45^r(1-0.45)^{10-r}$

$x = r$

$x = 1$

01. $= \binom{10}{1}0.45(0.55)^9$

$p = 0.02072.$

Extract 1.1: A sample of correct response to question 1

On the other hand, other candidates lost some or all marks allotted to this question. In all parts of the question, the majority failed to fix the calculator to the specific number of decimal places or significant figures. In part (a) (i), many candidates committed errors in applying the brackets when executing the values of expressions using the calculator. For instance, some candidates got 1.6998 in part (a) (i) after typing $3254 \times 3.14 \times \sqrt{417} \div 10^5 \times \log \sqrt[3]{278}$ in the calculator. In addition to errors observed in parts (a) (i) and (a) (ii), the candidates also replaced π with 180^0. Consequently, they got an incorrect answer because they considered both numbers and angles as real numbers.

In part (a) (iii), most candidates correctly calculated the value of the terms and wrote each term in four decimal places before performing addition. These candidates wrote $3.8442 + 16.5524 + 63.5160 + 225.1139$ which led to incorrect response 308.9965. Other candidates incorrectly interpreted the sigma notation. Some of the candidates only substituted $x = 1$ in the definition of the terms which resulted in an incorrect value of 3.8442 while other candidates who only substituted $x = 4$ and got 225.139. Moreover, some candidates wrongly interpreted the notation as integration and therefore, resulted in 172.3298.

In part (b), some candidates failed to realize that the question involves combination and permutation. Instead, they related the question to matrices (Extract 1.2).

1 (b)

$p(x=r) = \binom{n}{r} p^r (1-p)^{n-r}$

$n = 10$

$p = 0.45$

$p(x=r)$

from,

$p(x=r) = \binom{n}{r} pr (1-p)^{n-r}$

$p(x=1) = \binom{n}{1} p(1) (1-p)^{n-1}$

But

$n = 10$ and $p = 0.45$.

$p(x=1) = (n) p (1-p)^{n-1}$

$$P(x=1) = 10\begin{pmatrix}10\\1\end{pmatrix}0.45\,(1-0.45)^{10-1}$$

$$P(x=1) = \frac{4.5}{0/4}\begin{pmatrix}4.5\\0.45\end{pmatrix}\cdot(0.55)^{9}$$

$$P(x=1) = \begin{pmatrix}4.5\\0.45\end{pmatrix}\cdot 4.6\times10^{-3}$$

$$P(x=1) = 0.0207$$
$$2.07\times10^{-3}$$

$$P(x=1) = \begin{pmatrix}0.0207\\0.0021\end{pmatrix}$$

Extract 1.2: A sample of an incorrect response to part (b) of question 1

In Extract 1.2, the candidate wrote the steps of computing the expression instead of directly evaluating the expression using a calculator. The steps shown in the solution imply that the candidate wrongly considered $\begin{pmatrix}10\\1\end{pmatrix}$ as a matrix instead of a combination.

2.2 Question 2: Functions

The question tested the competence of candidates in drawing the graph of a rational function and interpreting a function with more than one definition. The question required the candidates to:

(a) *(i)* *sketch the graph of* $f(x)=\dfrac{2x+5}{x^2-x-6}$.

(ii) *write down the value(s) of* x *for which* $f(x)$ *does not exist.*

(b) *find the value of* $\dfrac{f(2)\,f(1)}{f(-1)}$ *given that* $f(x)=\begin{cases} x & \text{for } -1\le x<0 \\ x^2 & \text{for } 0<x<2 \\ x+2 & \text{for } 2\le x \end{cases}$.

A total of 34,548 candidates attempted this question, including 19,601 (56.74%) who got marks ranging from 3.5 to 10, which is an average performance. Figure 3 presents a summary of candidates' performance in this question.

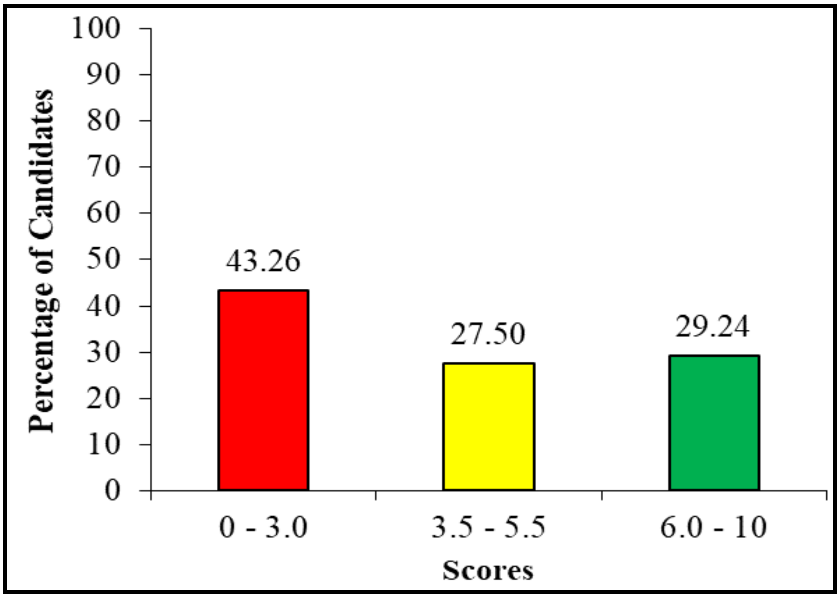
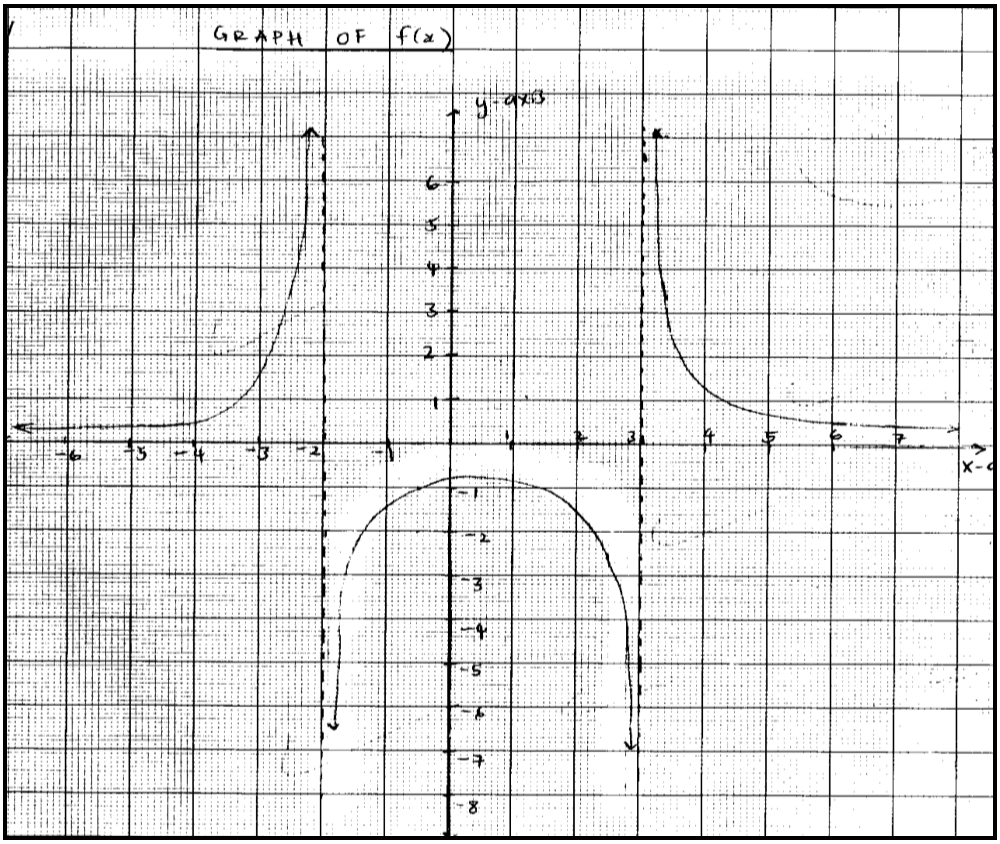


Figure 3: *The candidates' performance in question 2*

The data also reveals that 423 (1.22%) candidates scored all marks. In part (a), the candidates equated the denominator expression to zero resulting in the equation $x^2 - x - 6 = 0$ and solved it to get vertical asymptotes $x = 3$ or $x = -2$. The candidates also determined that the horizontal asymptote is $y = 0$ after correctly evaluating the value of $f(x)$ as x is a very large number, that is $\lim_{x \to \infty} \left(\frac{2x+5}{x^2 - x - 6} \right)$. These candidates further computed x and y intercepts by firstly assuming $y = 0$ and $x = 0$ in $y = \frac{2x+5}{x^2 - x - 6}$ respectively and then solving the resulting equations to get $x = -\frac{5}{3}$ and $y = -\frac{5}{6}$. Finally, these candidates used the information to draw the graph of $f(x)$ (Extract 2.1). By studying the graph, they realized that *f(x)* does not exist at both $x = -2$ and $x = 3$. In part (b), the candidates identified the appropriate definitions of $f(x)$ for $x = -1$, $x = 1$ and $x = 2$ as $f(x) = x$, $f(x) = x^2$ and $f(x) = x + 2$ respectively. Thus, they correctly got $f(-1) = -1$, $f(1) = 1$ and $f(2) = 4$ that lead to $\frac{f(2)f(1)}{f(-1)} = -4$.



Extract 2.1: A sample of correct response to part (a) of question 2

In Extract 2.1, the candidate correctly traced the points on *xy* - plane to obtain the graph of the given function.

On the otherhand, 14,947 (43.26%) candidates scored 3.0 marks or less. The responses of these candidates had several misconceptions in all or some parts of the question. In part (a)(i), many candidates presented incorrect graphs due to failure to determine the correct values of some or all necessary information. For instance, some candidates incorrectly computed the vertical asymptotes by considering the expression in the numerator $(2x+5)$ instead of the expression in the denominator (x^2-x-6). They came up with an equation $2x+5=0$ which yields $x=\frac{5}{2}$. Further, some candidates correctly considered the denominator, but they failed to solve the equation $x^2-x-6=0$. For example, some of them wrote $x(x-1)=6$ and therefore, $x=6$ or $x=1$. In determining the horizontal asymptotes, many candidates lacked knowledge of evaluating the limit of a function (Extract 5.2). In addition, other candidates stated that the horizontal asymptotes is $y=-\frac{5}{6}$ after calculating the y-intercept from $y=\frac{2x+5}{x^2-x-6}$. The incorrect values for vertical asymptotes also led to incorrect responses in part (a)(ii), particularly -3, -4, 2 and 4.

In part (b), a significant number of candidates got an incorrect answer for $f(2)$, $f(1)$ or $f(-1)$. Most candidates failed to identify the appropriate domain and consequently the definition for a particular value of *x*. For instance, some candidates wrote $f(2)=2$ implying that at $x=2$, $f(x)=x$. Further, some candidates worked on the expressions for the definition of $f(2)$, $f(1)$ and $f(-1)$, that is $x+2$, x^2 and x respectively and finally got $\frac{f(2)f(1)}{f(-1)}=x^2+2x$. Moreover, few candidates only struggled to draw the graph of $f(x)$, the approach which would not give the intended answer.

Horizontal asymptote is -

$$f(x) = \frac{\frac{2x}{x} + \frac{5}{x}}{\frac{x^2}{x} - \frac{x}{x} - \frac{6}{x}}$$

$$f(x) = \frac{2 + \frac{5}{x}}{x - 1 - \frac{6}{x}} \quad \text{but } x \to 0$$

$$f(x) = \frac{2+0}{0-1-0}$$

$$f(x) = -2.$$

~~Hort~~ Horizontal asymptote is -2.

Extract 2.2: A sample of an incorrect response to part (a) of question 2

In Extract 2.2, the candidate divided both numerator and denominator by x instead of x^2.

2.3 Question 3: Algebra

The question tested the candidates' competence in solving simultaneous equations with two variables and developing the general term of an arithmetic progression. The question comprised the following parts:

(a) *The total number of pencils and pens in a box is 47 and the product of the number of the pencils and the number of the pens is 370. Find the number of pencils present in the box.*

(b) *The 8^{th} and 15^{th} terms of an arithmetic progression are 11 and 21 respectively. Find the n^{th} term.*

The question was attempted by 34,548 candidates and 26,147 (75.68%) of them obtained 3.5 marks or more. Therefore, the candidates' performance in this question was good. Figure 4 shows the percentage of candidates who scored low, average and high marks.

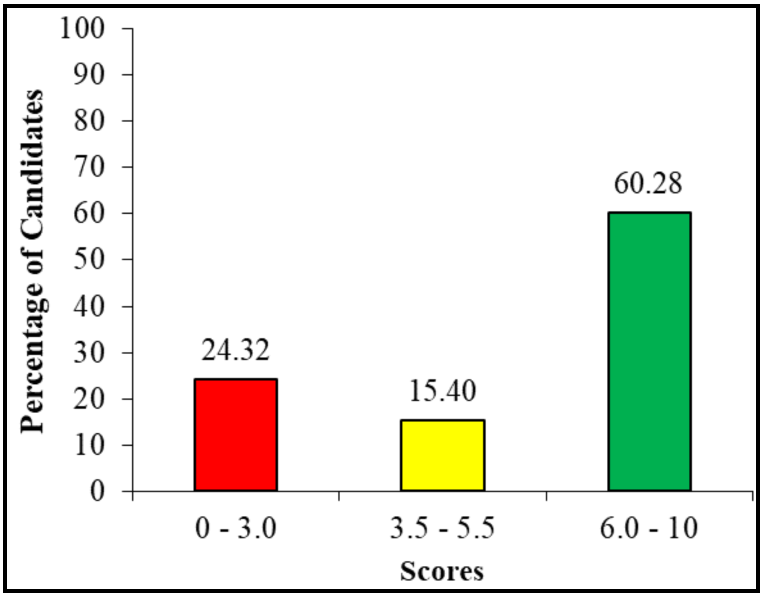


Figure 4: *The candidates' performance in question 3*

In this question, 6,201 (17.95%) candidates responded to both parts correctly. In part (a), the candidates translated the word problem into the equations $x+y=47$ and $xy=370$ where x and y represent the number of pencils and pens respectively (or vice versa). With these equations, the candidates correctly formulated and solved the equation $x^2-47x+370=0$ and got $x=10$ or $x=37$. Thus, they concluded that the number of pencils is 10 or 37 (Extract 3.1).

Similarly, in part (b), the candidates represented the first term and the common difference of the arithmetic progression by using letters, particularly a_1 and d respectively. They also applied the formula for developing the general term of the arithmetic progression $(A_n = A_1 + (n-1)d)$ to formulate the equations $A_1 + 7d = 11$ and $A_1 + 14d = 21$. Then, they solved the equations to obtain $A_1 = 1$ and $d = \frac{10}{7}$. Therefore, they replaced A_1 and d in the formula $A_n = A_1 + (n-1)d$ with 1 and $\frac{10}{7}$ respectively and simplified the terms to get $A_n = \frac{10}{7}n - \frac{3}{7}$.

$x + y = 47$ --- i)

$xy = 370$ --- ii)

Consider equation ii)

$xy = 370$

$y = \frac{370}{x}$ --- *

Substitute equation * into equation i)

$x + y = 47$

$x + \frac{370}{x} = 47$

$x^2 + 370 = 47x$

$x^2 - 47x + 370 = 0$

$a = 1, \quad b = -47 \quad c = 370$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{47 \pm \sqrt{(-47)^2 - 4 \times 1 \times 370}}{2(1)}$

$x = \frac{47 \pm \sqrt{729}}{2}$

$x = \frac{47 + 27}{2}$ or $\frac{47 - 27}{2}$

$x = 37$ or 10

$\therefore$ The number of pencils is either 37 or 10

Extract 3.1: A sample of correct response to part (a) of question 3

In Extract 3.1, the candidate correctly solved the equation $x^2 - 47x + 370 = 0$ by the general quadratic formula.

In spite of good performance, 8,401 (24.32%) candidates got low marks and among them 4,214 (12.20%) got zero. In part (a), most of these candidates wrongly interpreted the word problem, as a result they wrote incorrect equations $x - y = 47$ and $x^2 + y^2 = 370$. Further, a considerable number of candidates formulated correct equations $x + \frac{370}{x} = 47$ or $y + \frac{370}{y} = 47$, but failed to solve them. These candidates had insufficient skills for simplifying algebraic terms and solving quadratic equations.

In part (b), some candidates used incorrect formulae for determining the general term (A_n) of arithmetic progression. For instance, some candidates wrote $A_n = A_1 + (n-2)d$ and thus got incorrect expressions for the 8th and 15th terms as $A_1 + 6d$ and $A_1 + 13d$ respectively. Therefore, they obtained incorrect equations $A_1 + 6d = 11$ and $A_1 + 13d = 21$ that led to incorrect answers $d = \frac{11}{7}$, $A_1 = \frac{11}{7}$ and $A_n = \frac{11}{7}(n-1)$. The analysis also revealed that some candidates wrote the correct formula, but they wrongly interpreted its components. For instance, some candidates wrote $8 + 7d = 11$ and $15 + 14d = 21$ (Extract 3.2). Further, few candidates formulated correct equations $A_1 + 7d = 11$ and $A_1 + 14d = 21$, however, they failed to solve for A_1 and d. For example, some of these candidates got $d = 1.4$ and $A_1 = 1.2$.

From the formula:-

$A_n = A_1 + (n-1)d$

$A_8 = 8 + (8-1)d$

$= 8 + 8d - d$

$A_8 = 8 + 7d$

But $A_8 = 11$

$11 = 8 + 7d$

$11 - 8 = 7d$

$3 = 7d$

7 7

03b) $d = 0.43$

Asaeni:-

$A_n = A_1 + (n-1)d$

$A_{15} = 15 + (15-1)d$

$= 15 + (15d - d)$

$A_{15} = 15 + 14d$

But $A_{15} = 21$

$21 = 15 + 14d$

$21 - 15 = 14d$

$6 = 14d$

$\frac{6}{14} = \frac{14d}{14}$

$d = 0.43$

Extract 3.2: A sample of an incorrect response for part (b) of question 3

In Extract 3.2, the candidate wrongly interpreted "8th term" as $A_1 = 8$ and "15th term" as $A_1 = 15$. It should be known that A_1 in the formula $A_n = A_1 + (n-1)d$ represents the first term of an arithmetic progression.

2.4 Question 4: Differentiation

The question tested the skills in using the first principles and rules of differentiation and the application of differentiation in solving maximum and minimum problems. The question included the following parts:

(a) *(i)* *Differentiate* $y = \frac{1}{1+x}$ *from the first principles.*

(ii) *Find the first derivative of* $g(x) = \sqrt{x^2 + 2x}$.

(b) *The total length of the diameter and height of a cylinder is 3 metres. Show that the cylinder has the maximum volume when both height and radius measure 1 metre.*

A total of 34,548 candidates responded to this question, including 7,419 (21.47%) candidates who scored 3.5 marks or more. Therefore, the performance of the candidates in this question was generally weak. Figure 5 gives the percentages of candidates who scored low, average and high marks.

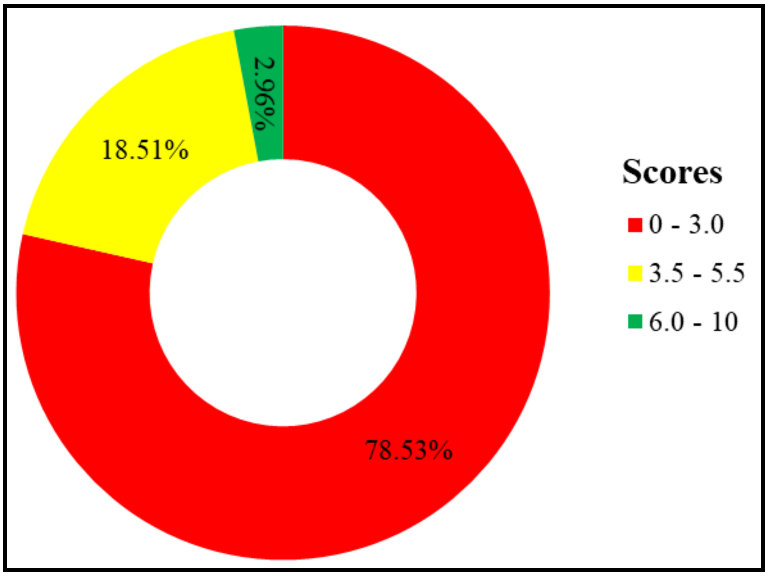


Figure 5: *The candidates' performance in question 4*

Figure 5 shows that 78.53 per cent, equivalent to 27,129 candidates scored 3.0 marks or less. In part (a) (i) many candidates correctly wrote $\frac{dy}{dx} = \lim_{h \to 0}\left(\frac{1}{h}\left(\frac{1}{1+x+h} - \frac{1}{1+x}\right)\right)$. However, most of them failed to transform the expression $\frac{1}{1+x+h} - \frac{1}{1+x}$ into $-\frac{h}{(1+x+h)(1+x)}$. Most of these candidates did not consider the brackets as they wrote $\frac{1+x+h-1+x}{(1+x+h)(1+x)}$ which leads to $\frac{2x+h}{(1+x+h)(1+x)}$. Other candidates were not conversant with the concept of common multiples. For instance, some candidates wrongly considered $1+x+h$ as the common multiple to both $1+x+h$ and $1+x$. These candidates wrote $\frac{(1+x)-(1+x+h)}{1+x+h}$ and therefore, they got $-\frac{h}{1+x+h}$. Also, some candidates lacked knowledge of evaluating limits. For example, a significant number of candidates replaced h in

$-\dfrac{h}{h(1+x+h)(1+x)}$ with 0 resulting in the undefined term. They were supposed to simplify the term $-\dfrac{h}{h(1+x+h)(1+x)}$ into $-\dfrac{1}{(1+x+h)(1+x)}$ before substituting $h=0$. Moreover, some candidates directly substituted $h=0$ in $\dfrac{1}{h}\left(\dfrac{1}{1+x+h}-\dfrac{1}{1+x}\right)$ which also resulted in undefined term. In part (a)(ii), many candidates correctly applied the chain rule, but they faced difficulties in determining the derivative of the expression involving radicals. Most of these candidates wrongly perceived that the derivative of $\sqrt{u}$ is $\dfrac{1}{\sqrt{u}}$ instead of $\dfrac{1}{2\sqrt{u}}$. The candidates let $y=\sqrt{x^2+2x}$ and $u=x^2+2x$ resulting in $\dfrac{du}{dx}=2x+2$ and $\dfrac{dy}{du}=\dfrac{1}{\sqrt{u}}$. Then, they applied the chain rule and obtained $\dfrac{dy}{dx}=\dfrac{2x+2}{\sqrt{x^2+2}}$ instead of $\dfrac{dy}{dx}=\dfrac{x+1}{\sqrt{x^2+2}}$. Similarly, some candidates got $\dfrac{du}{dx}=2x+2$ and $\dfrac{dy}{du}=\sqrt{u}$ ending up with $\dfrac{dy}{dx}=(2x+2)\sqrt{x^2+2x}$. These candidates wrongly considered that the derivative of $\sqrt{u}$ is $\sqrt{u}$.

In part (b), many candidates did not include the equation relating radius and height of a cylinder. Instead, they only applied the formula for calculating the volume of the cylinder. However, most of them used inappropriate formulae (Extract 4.1). Other inappropriate formulae included $V=\dfrac{1}{3}\pi r^2 h$ and $V=2\pi r^2+2\pi rh$ instead of $V=\pi r^2 h$ where V, r and h represent volume, radius and height respectively. Other candidates worked out to determine the diameter of the cylinder (not to verify the descriptions). These candidates recalled the fact that diameter is twice the radius and thus, they wrote diameter equals to 2 metres.

(b)	From $V = \frac{4}{3}\pi r^3$.
	$\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$
	$\frac{dV}{dr} = 4\pi r^2$
	$\frac{dV}{dt} = 4\pi r^2 \times 3$
	where as $r = 1$
	$\frac{dV}{dt} = 4\pi 3$
	$= 12\pi$.

Extract 4.1: A sample of an incorrect response to part (b) of question 4

In Extract 4.1, the candidate used the formula for calculating the volume of a sphere instead of a cylinder. Also, the candidate computed the rate of change in volume instead of verifying that at the maximum volume, both height and radius measure 1 metre.

The analysis reveals further that 311 (0.90%) candidates responded to all parts of the question correctly. In part (a) (i), the candidates correctly wrote $f(x+\delta x) = \frac{1}{1+(x+\delta x)}$ from $f(x) = \frac{1}{1+x}$ and applied the formula $\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{f(x+\delta x) - f(x)}{\delta x}$, and finally simplified the expression to obtain $\frac{dy}{dx} = -\frac{1}{(1+x)^2}$ (Extract 4.2). In part (a) (ii) these candidates rewrote $\sqrt{x^2+2x}$ in the form $(x^2+2x)^{\frac{1}{2}}$. Thereafter, they applied rules of differentiation particularly the chain rule and the fact that the derivative of x^n for real numbers n is nx^{n-1} to get $g'(x) = \frac{x+1}{\sqrt{x^2+2x}}$. In part (b), the candidates developed the formula $2r + h = 3$ from the statement *"total length of the diameter and height of a cylinder is 3 metres"* where r is the radius and h is the height of the cylinder. The candidates also realized that the task needs the formula for calculating the volume (V) of the given cylinder, that is $V = \pi r^2 h$. Therefore, they used the two formulae and reduced them to the formula $V = 3\pi r^2 - 2\pi r^3$. Thereafter, they performed differentiation of V with respect to r and got $\frac{dV}{dr} = 6\pi r - 6\pi r^2$. Further, these candidates were conversant with the fact that at the maximum value, the derivative of a particular function is zero. Therefore, they equated $\frac{dV}{dr}$ to zero and resulted in an equation $6\pi r - 6\pi r^2 = 0$ which was correctly solved to obtain $r = 0$ or $r = 1$. Furthermore, these candidates were aware of the fact that the radius of a cylinder is always greater than zero. Thus, they ignored $r = 0$ and dealt with $r = 1$ to determine the appropriate value of h and they replaced r in $2r + h = 3$ with 1 and ended up with $h = 1$ (Extract 4.3).

4 (a) (i) $y = \frac{1}{1+x}$

Let $y = f(x)$

$$f(x) = \frac{1}{1+x}$$

$$f(x+h) = \frac{1}{1+x+h}$$

From first principles

$$f'(x) = \frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{\frac{1}{1+(x+h)} - \frac{1}{1+x}}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{1 + x - 1 - x - h}{(1+x+h)(1+x)h}$$

$$f'(x) = \frac{-h}{(1+x+h)(1+x)h} \quad \lim h \to 0$$

$$f'(x) = \lim_{h \to 0} \frac{-1}{(1+x+h)(1+x)}$$

$$f'(x) = \frac{-1}{(1+x+0)(1+x)}$$

$$f'(x) = \frac{dy}{dx} = \frac{-1}{(1+x)^2}$$

$$\frac{dy}{dx} = \frac{-1}{(1+x)^2}$$

Extract 4.2: A sample of correct response to part (a) of question 4

As Extract 4.2 shows, the candidates correctly produced $f(x+\delta x)=\dfrac{1}{1+(x+\delta x)}$ from $f(x)=\dfrac{1}{1+x}$ and used the first principles to obtain $\dfrac{dy}{dx}=-\dfrac{1}{(1+x)^2}$.

4 (b) Let the length be d

Height be h

$d+h=3$

Volume $=\dfrac{\pi d^2 h}{4}$

$d=3-h$ or $h=3-d$

$V=\dfrac{\pi d^2(3-d)}{4}$

$=\dfrac{3\pi d^2}{4}-\dfrac{\pi d^3}{4}$

$\dfrac{dv}{d\partial}=\dfrac{3(2)\pi d}{4}-\dfrac{3\pi d^2}{4}$

At maximum volume $dv/d\partial=0$

$0=3/2\,\pi d-\dfrac{3\pi d^2}{4}$

$\dfrac{3\pi d^2}{4}=\dfrac{3\pi d}{2}$

$d=2m$

but radius $=d/2$

$=2/2$

$\therefore$ Radius $=1m$

Also

$d+h=3$

$h=3-2$

$h=1m$

$\therefore$ Height $=1m$

Hence proved!

Extract 4.3: A sample of correct response to part (c) of question 4

In Extract 4.3, the candidate used the formula that includes diameter and height to verify the explanations.

2.5 Question 5: Integration

The question consisted of parts (a) and (b). Part (a) intended to determine the competence of the candidates in applying the techniques of integration. The statement was: *evaluate* $\int_{\frac{1}{2}}^{1} x\sqrt{(1-x^2)}dx$ *correct to 4 decimal places*. Part (b) measured the competence of the candidates in applying the knowledge of integration. It stated that *find the area bounded by the curve* $y = 2\cos x$, *the lines* $x = 0$, $x = 2\pi$ *and the* $x-axis$.

A total of 34,547 candidates responded to this question. Of these, 5,868 (16.99%) candidates obtained marks ranging from 3.5 to 10. Therefore, the overall performance of the candidates in this question was weak. Figure 6 is a summary of the candidates' performance in this question.

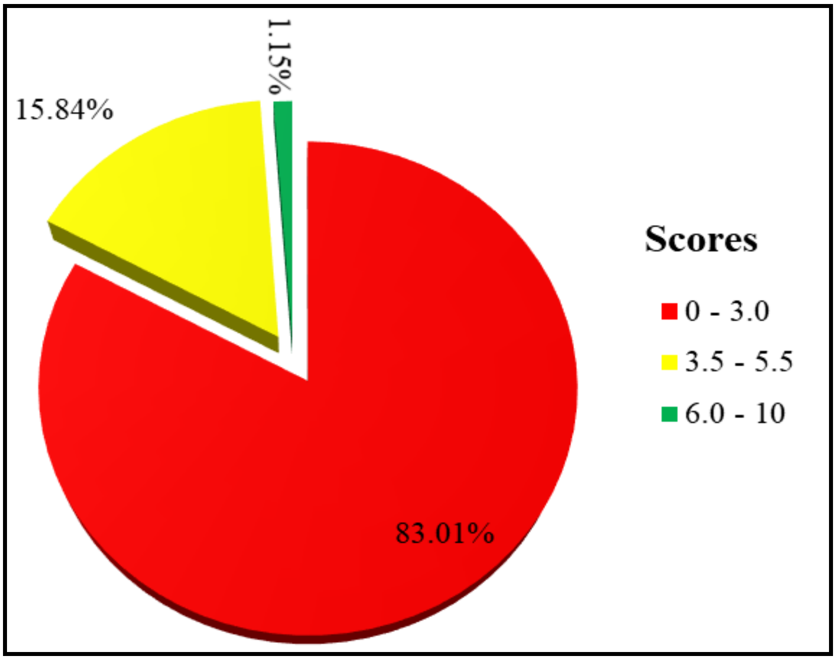


Figure 6: *The candidates' performance in question 5*

Figure 6 shows that 83.01 per cent of the candidates got low marks. In part (a), many candidates did not realize the presence of the expression and its derivative in the integrand. Most of these candidates applied incorrect rules of integrations. For instance, some candidates rewrote $\int_{\frac{1}{2}}^{1} x\sqrt{1-x^2}dx$ as $\int_{\frac{1}{2}}^{1} xdx.\int_{\frac{1}{2}}^{1}\sqrt{1-x^2}dx$ indicating the misconception

$\int_a^b f(x).g(x)dx = \int_a^b f(x)dx.\int_a^b g(x)dx$. Likewise, some candidates rewrote $\int_{\frac{1}{2}}^1 x\sqrt{1-x^2}dx$ as $\int_{\frac{1}{2}}^1 xdx + \int_{\frac{1}{2}}^1 \sqrt{1-x^2}dx$ indicating wrong assumption $\int_a^b f(x).g(x)dx = \int_a^b f(x)dx + \int_a^b g(x)dx$. The candidates were supposed to realize that the integrand is the product of an expression $\sqrt{1-x^2}$ and its derivative x. Therefore, they would evaluate the integral by applying the function and its derivative technique. Few candidates applied trigonometric substitution but failed to completely change the variable or identify the appropriate integration technique after changing the variable. For instance, most candidates who let $x = \sin t$ failed to get $\int \sin t \cos^2 t dt$. Instead, they worked out to the integrand containing both x and t.

In part (b), many candidates failed to identify the sub-regions included in the region bounded by the curves and their respective limits of integration. Thus, they calculated $\int_0^{2\pi} 2\cos x dx$ and obtained $Area = 0$ square unit (Extract 5.1). The candidates were supposed to draw the graph of $y = \cos 2\theta$ or solve the equation $\cos 2\theta = 0$ to determine the appropriate limits, that is from $x = 0$ to $x = \frac{\pi}{2}$, $x = \frac{\pi}{2}$ to $x = \frac{3\pi}{2}$ and from $x = \frac{3\pi}{2}$ to $x = 2\pi$. Further, some candidates did not consider the upper and lower function of the particular sub-region. These candidates evaluated $\int_0^{\frac{\pi}{2}} 2\cos x dx + \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} 2\cos x dx + \int_{\frac{3\pi}{2}}^{2\pi} 2\cos x dx$ instead of $\int_0^{\frac{\pi}{2}} 2\cos x dx - \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} 2\cos x dx + \int_{\frac{3\pi}{2}}^{2\pi} 2\cos x dx$. Furthermore, some candidates drew incorrect graphs because they replaced x in $y = 2\cos x$ with real numbers instead of angles. Moreover, some candidates applied inappropriate formulae including the formula for calculating the volume of solid of revolution along the $x-$axis, $Area = \pi\int_a^b y^2 dx$ whereby $a = 0$, $b = 2\pi$ and $y = 2\cos x$, ending up with $Area = 25.0322$ square unit.

5b)

$$y = 2\cos x$$

$$f(x) = 2\cos x$$

$$\text{Area} = \int_{a}^{b} f(x)\,dx$$

$$\text{Area} = \int_{0}^{2\pi} 2\cos x\,dx$$

$$\text{Area} = 2\int_{0}^{2\pi} \cos x\,dx$$

$$\text{Area} = 2\left[\sin x\right]_{0}^{2\pi}$$

$$\text{Area} = 2\left[\sin 2\pi - \sin 0\right)$$

$$\text{Area} = 0.$$

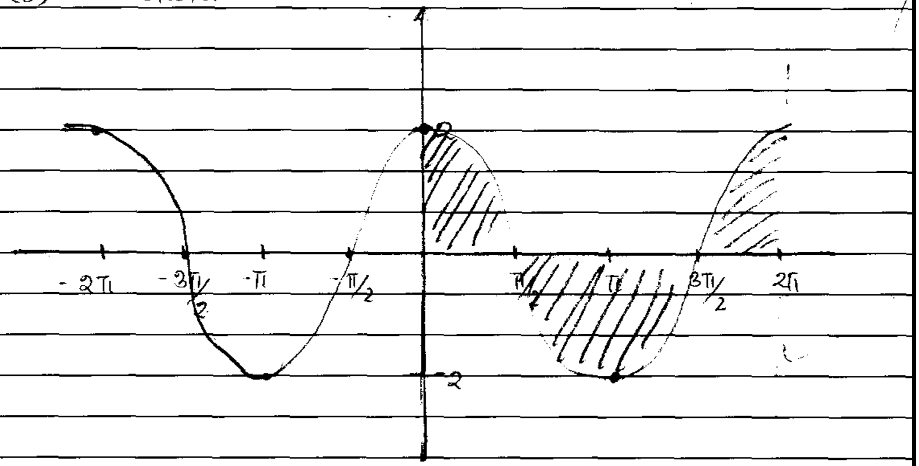
Extract 5.1: A sample of an incorrect response to part (b) of question 5

In Extract 5.1, the candidate evaluated $\int_{0}^{2\pi} 2\cos x dx$ instead of

$$\int_{0}^{\frac{\pi}{2}} 2\cos x dx - \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} 2\cos x dx + \int_{\frac{3\pi}{2}}^{2\pi} 2\cos x dx.$$

On the contrary, 59 (0.17%) candidates responded correctly to this question. In part (a), the candidates realized that the integrand includes both the expression $\left(1-x^2\right)$ and its derivative (x). These candidates let $u = 1-x^2$, from which they produced $xdx = -\frac{du}{2}$ and the limits $u = 0$ and $u = \frac{3}{4}$. Therefore, they changed $\int_{\frac{1}{2}}^{1} x\sqrt{\left(1-x^2\right)}dx$ into $-\frac{1}{2}\int_{\frac{3}{4}}^{0}\sqrt{u}du$ and consequently $\frac{1}{3}\left[u^{\frac{3}{4}}\right]_{\frac{3}{4}}^{0}$ which results in 0.2165. In part (b), the candidates correctly sketched the graph of $y = 2\cos x$ and applied the formula $A = \int_a^b y\,dx$ to evaluate the area of the intended region (Extract 5.2).

5 (b) Sketch



$A_T = A_1 + A_2 + A_3$

$A_1 = \int_0^{\pi/2} 2\cos x\, dx$

$= 2\left[\sin x\right]_0^{\pi/2}$

$= 2\left[\sin \pi/2 - \sin 0\right] = 2$ square units

$A_2 = \int_{\pi/2}^{3\pi/2} -2\cos x\, dx$

$= -2\left[\sin 3\pi/2 - \sin \pi/2\right] = 4$ square units

$A_3 = \int_{3\pi/2}^{2\pi} 2\cos x\, dx$

$= 2\left[\sin 2\pi - \sin 3\pi/2\right] = 2$ square unit

$A_T = (2 + 4 + 2)$ square units

$\therefore$ Total area $= 8$ square units

Extract 5.2: A sample of correct response to part (b) of question 5

Extract 5.3 indicates that the candidate was competent in evaluating trigonometric integrals.

2.6 Question 6: Statistics

The question measured the competence of candidates in using the formula for calculating mean, variance and median. This question consisted of the following parts:

(a) *The mean and variance of 7 observations are 8 and 16 respectively. Amongst, the five observations are 2, 4, 10, 12 and 14. Find the other two observations.*

(b) *The following table shows heights of 40 trees measured to the nearest meters:*

Heights	*4 – 8*	*9 – 13*	*14 – 18*	*19 – 23*	*24 – 28*	*29 - 33*
Number of trees	*2*	*4*	*7*	*14*	*8*	*5*

Find the median.

The question was attempted by 34,549 candidates, and out of these, 9,594 (27.77%) candidates scored 3.5 marks or above. Therefore, the overall performance of the candidates in this question was weak. Figure 7 shows the candidates' performance in this question.

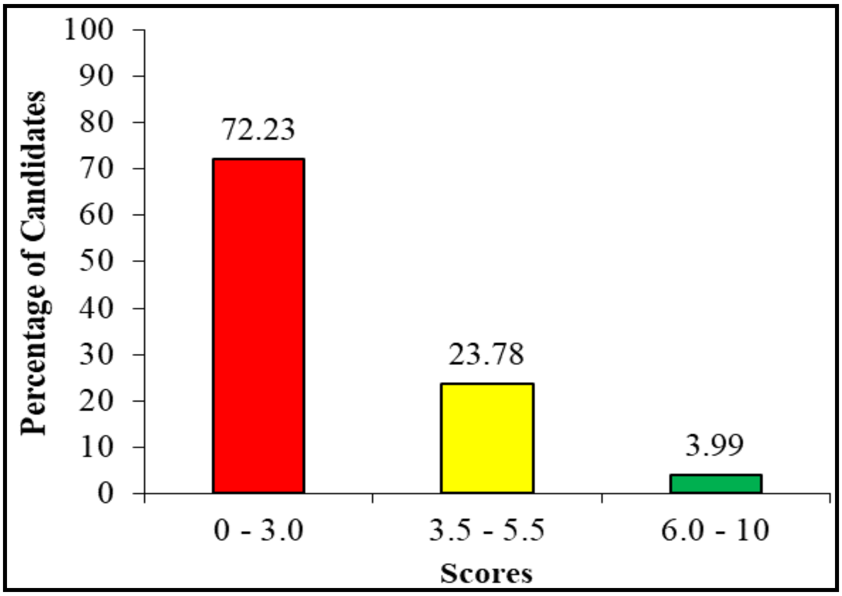


Figure 7: *The candidates' performance in question 6*

The performance indicates that many candidates faced various challenges. Many candidates answered part (a) based on the concept of average while ignoring the concept of variance (Extract 6.1). For instance, some candidates computed the average of the five observations given without considering the two missing observations. Also, other candidates wrongly assumed that x and $x+1$ were the two unknown observations. Therefore, they developed an incorrect equation $8=\frac{43+2x}{7}$ that lead to incorrect answers. Further, some candidates calculated the variance of the five observations given by applying the incorrect formula $Var(X)=\frac{\sum x-\bar{x}}{N}$ and thus ended up with incorrect answers including 21.16.

In part (b), some candidates applied an incorrect formula for calculating the median. Some of them wrote $Median = L+\left(\frac{\frac{N}{2}+\sum fb}{\sum fw}\right)i$ instead of $Median = L+\left(\frac{\frac{N}{2}-\sum fb}{\sum fw}\right)i$. Other candidates recalled the correct formula but failed to determine the values of some of its variables. For example, some candidates wrote $L=19.5$ indicating that these candidates added 0.5 to the lower limit of the median class instead of subtracting it. Also, some candidates got an incorrect value of the class size, $i=4$ instead of $i=5$. Other candidates failed to determine the median class whereas most of them preferred the class 14 – 18 to others, and therefore, they wrote $L=13.5$, $\sum fb=6$ and $fw=7$. Moreover, some candidates applied the inappropriate formulae (Extract 6.2).

6.a For Mean $8=\frac{y}{7}$

$y=56$,

~~For va~~ $\cancel{\sum_{n=1}^{n}}$ $\sum_{i=1}^{n} x = 56$.

A $y = 56-42$

$y = 14$.

$\therefore$ Other two observations are 14.

Extract 6.1: A sample of an incorrect response to part (a) of question 6

In Extract 6.1, the candidate calculated the sum of the missing observations instead of values of the observations.

06 (b) Given data

Height	No(x)	f	fx	cumulative frequency
4-8	2	6	12	6
9-13	4	11	44	17
14-18	7	16	112	33
19-23	14	21	294	54
24-28	8	26	208	80
29-33	5	31	155	111

Total =

from median

$$L + \left(\frac{t_1}{t_1 + t_2}\right) i$$

$L = 19 - 0.5 = 18.5$

$t_1 = 7$

$t_2 = 6$

$i = 1$

Then $18.5 + \left(\frac{7}{7+6}\right)1$

Median is $18.5 + (0.538)i$

$18.5 + 0.5 \doteq 19.0$

Median is ~~18~~ 19.0

Extract 6.2: A sample of an incorrect response to part (b) of question 6.

In Extract 6.2, the candidate used the formula for calculating mode instead of median.

The data further depict that 744 (2.15%) candidates correctly responded to this question as they scored all ten (10) marks. The competent candidates responded to part (a) by assuming that the missing observations are *x* and *y*. Then, they correctly used the formula for calculating average and variance to formulate and solve the equations to obtain the required observations 6 and 8 (Extract 6.3).

In part (b) the candidates realised that the total number of frequency (N) is 40 implying that $\frac{N}{2} = 20$ and therefore, the median class is 19 – 23. Thus, they correctly determined the class size, the lower boundary of median class, frequency of median class and total frequency of the classes with lesser values than that of median class which are $i = 5$, $L = 18.5$, is $f_w = 7$ and $f_b = 6$ respectively. Finally, they substituted the values in the formula $Median = L + \left(\frac{N/2 - f_b}{f_w}\right)i$ and computed to obtain $Median = 21$. Also, few candidates answered this part by representing the data using ogive and estimating the median.

6	(a) $\bar{x} = 8$
	$Var(x) = 16$
	$Var(x) = \frac{\sum x^2}{N} - \left(\frac{\sum x}{N}\right)^2$
	$16 = \frac{\sum x^2}{7} - (8)^2$
	$\sum x^2 = 560$
	Let the two observations be x and y
	$\frac{2+4+10+12+14+x+y}{7} = 8$
	$x + y = 14$ — (i)
	$460 + x^2 + y^2 = 560$
	$x^2 + y^2 = 100$ — (ii)
	from equation (i)
	$y = 14 - x$
	$x^2 + (14-x)^2 = 100$
	$x^2 + (14^2 - 28x + x^2) = 100$
	$x^2 + 196 - 28x + x^2 = 100$
	$2x^2 - 28x + 96 = 0$
	$x = 8$ or 6
	$y = 14 - x$
	$y = 14 - 8$
	$y = 6.$
	$\therefore$ The two observations are 8 and 6

Extract 6.3: A sample of correct response to part (a) of question 6

In Extract 6.1, the candidate correctly formulated and solved equations to obtain the missing observations.

2.7 Question 7: Probability

The question tested the candidates' competence in computing permutation, the probability of mutually exclusive events and combined events. It comprised the following parts:

(a) *Show that* ${}^{n+1}P_2 = n(n+1)$.

(b) *A and B are mutually exclusive events with* $P(A) = \frac{1}{2}$ *and* $P(B) = \frac{1}{4}$. *Find* $P(A \cap B')$.

(c) *Two balls are drawn randomly without replacement from a bag containing 3 black balls and 2 white balls.*

(i) *Use tree diagram to analyze the probability of each drawing.*

(ii) *Find the probability that both balls are white.*

A total of 34,545 candidates responded to this question, of which 8,095 (23.43%) scored between 3.5 and 10 marks. Therefore, the candidates had weak performance in this question. Figure 8 shows the percentage of candidates who obtained low, average and high scores.

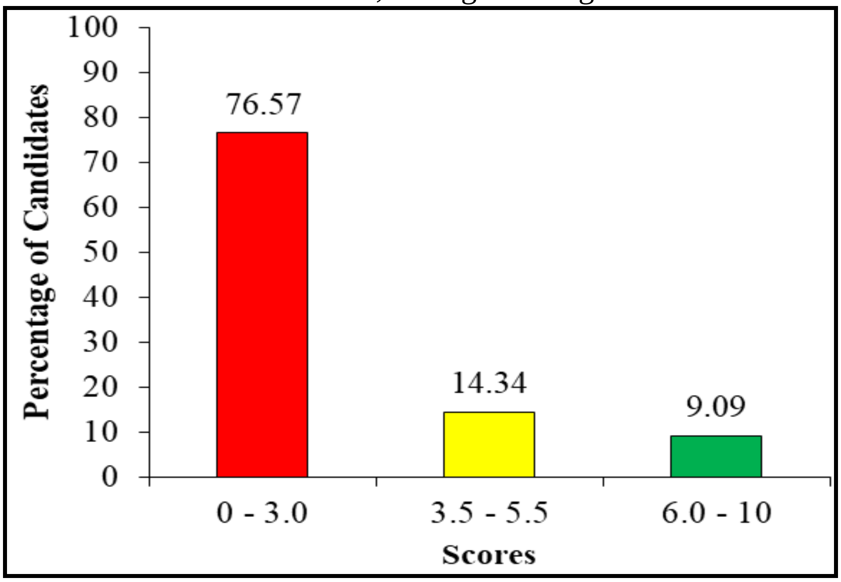


Figure 8: *The candidates' performance in question 7*

On the other hand, 76.57 per cent, equivalent to 23,603 candidates got low marks. In part (a) many candidates wrote the correct conclusion but their works contained some misconceptions. Most of these candidates applied the definition of combination, ${}^{n}C_{r} = \frac{n!}{(n-r)!r!}$ and therefore, they incorrectly defined ${}^{n+1}P_2$ as $\dfrac{(n+1)!}{((n+1)-2)!2!}$. Similarly, some candidates wrote ${}^{n+1}P_2 = \dfrac{n+1}{2}$ indicating that they defined ${}^{n}P_r$ as $\dfrac{n}{r}$. Moreover, some applied the definition of permutations correctly and got ${}^{(n+1)}P_2 = \dfrac{(n+1)!}{((n+1)-2)!}$ but committed errors in simplifying a factorial expression. For instance, some candidates wrote $\dfrac{(n+1)!}{((n-1)!}$ as $\dfrac{(n-1)!+(2!)}{((n-1)!}$.

In part (b), many candidates ignored the condition for mutually exclusive events and thus, their responses lacked coherence and correctness. Most of these candidates used the formula $P(A \cap B) = P(A) \times P(B)$ (Extract 7.1) instead of the fact "for any mutually exclusive events A and B, $P(A \cap B) = 0$". Other candidates wrote $P(A \cap B') = P(A) \times P(B')$ while taking $P(B') = 1 - P(B)$. This approach would be correct if A and B' were independent events, but the question is quite about it. Moreover, a significant number of candidates wrote $P(A \cap B') = P(A) - P(B)$ and thus, subtracted $\dfrac{1}{4}$ from $\dfrac{1}{2}$ to obtain $P(A \cap B') = \dfrac{1}{4}$.

In part (c), many candidates incorrectly interpreted $\{BB, BW, WB, WW\}$ and $\{WW\}$ as a sample space and event respectively. Therefore, they applied the formula $P(E) = \dfrac{n(E)}{n(S)}$ and got $P(WW) = \dfrac{1}{4}$. It should be known that a tree diagram describes the possible alternatives of selecting two balls, and therefore, each entry must be the probability of a particular event. Other candidates ignored the instruction "without replacement". These candidates wrote incorrect probabilities for some branches of the tree diagram. Furthermore, some candidates wrote $P(WW) = \frac{2}{5} + \frac{1}{4} = \frac{13}{20}$ implying that $P(W_1W_2) = P(W_1) + P(W_2)$ instead of $P(W_1W_2) = P(W_1) \times P(W_2)$.

(b) $p(A) = \frac{1}{2}$

$P(B) = \frac{1}{4}$

$p(A \cap B) = P(A) \times P(B)$

$p(A \cap{}' B) = \left(\frac{1}{2}\right) \times \left(\frac{1}{4}\right)$

$p(A \cap B) = \frac{1}{8}$

but $P(A \cap B) + P(A \cap B') = 1$

$p(A \cap B) + P(A \cap B') = 1$

$p(A \cap B)' = 1 - p(A \cap B)$

$p(A \cap B)' = 1 - \frac{1}{8}$

$P(A \cap B)' = \frac{7}{8}$

Extract 7.1: A sample of incorrect response to part (b) of question 7.

In Extract 7.1, the candidates applied inappropriate formulae $P(A \cap B) = P(A) \times P(B)$ and $P(A \cap B) + P(A \cap B') = 1$.

The analysis further showed that 399 (1.16%) candidates scored all 10 marks allotted to this question. In part (a), the candidates were knowledgeable about the definitions of permutation$\left({}^{n}\mathrm{P}_{r}\right)$ and factorial notation$(n!)$ Therefore, they rewrote the ${}^{n+1}P_2$ as $\dfrac{(n+1)!}{((n+1)-2)!}$ and followed the definition of $n!$ to change it into $\dfrac{(n+1)(n)(n-1)!}{(n-1)!}$ which simplifies to $n(n+1)$.

In part (b), the candidates were aware of the fact that $A \cap B = \phi$ for any mutually exclusive events A and B. Therefore, they developed various correct set relations which involve $A \cap B'$ and applied appropriate formulae to determine the probability of $A \cap B'$ (Extract 7.2). Other candidates realized that if events A and B are mutually exclusive, then $A \cap B'$ is equivalent to A and consequently $P(A \cap B') = P(A)$. Thus, they simply wrote $P(A \cap B') = \frac{1}{2}$.

Competent candidates answered part (c) by drawing a correct tree diagram (Extract 7.3). Then, they studied the tree diagram and obtained the probabilities of the events W_1 and W_2 as $\frac{2}{5}$ and $\frac{1}{4}$ respectively. Thus, they calculated the probability of drawing both white balls by computing the product of $\frac{2}{5}$ and $\frac{1}{4}$ hence, ending up with $P(W_1 W_2) = \frac{1}{10}$.

7 (b) For mutual exclusive

$P(A \cap B) = 0.$

But $P(A' \cap B') = 1 - P(A \cup B)$

$P(A' \cap B') = P(A \cup B)'$

$P(A' \cap B') = 1 - 3/4$

$P(A' \cap B') = 1/4$

Also:

$P(A' \cap B') + P(A \cap B') = P(B').$

$P(A \cap B') = P(B') - P(A' \cap B').$

$P(B) + P(B') = 1$

$P(B') = 1 - P(B)$

$P(B') = 1 - 1/4$

$P(B') = 3/4$

$P(A \cap B') = 3/4 - 1/4$

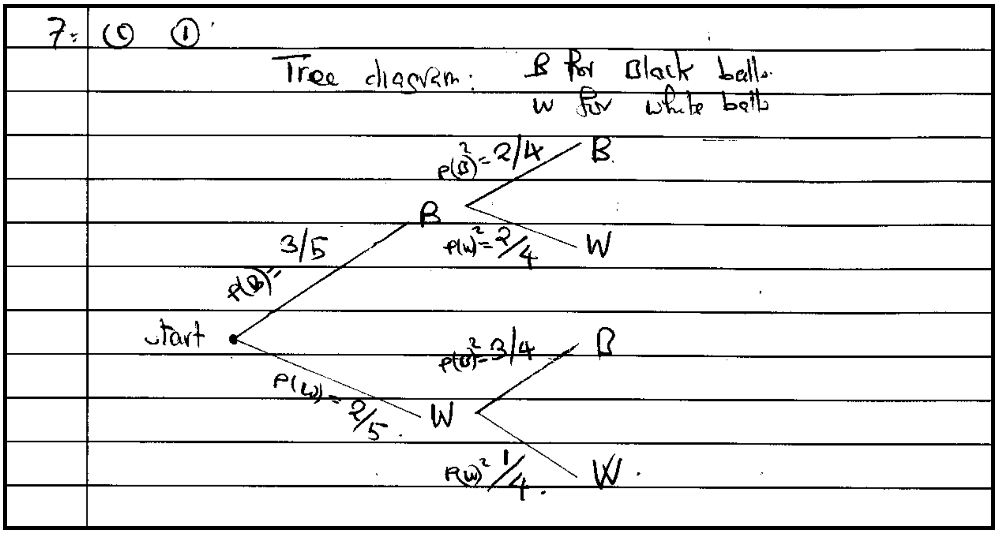
$P(A \cap B') = 2/4$

$P(A \cap B') = 1/2$

$\therefore P(A \cap B') = 1/2$

Extract 7.2: A sample of correct response to part (b) of question 7

In Extract 7.2, the candidate developed the correct formula $P(A' \cap B') + P(A \cap B') = P(B')$ and used it correctly to determine that $P(A \cap B') = \dfrac{1}{2}$.



Extract 7.3: A sample of correct response to part (c) of question 7

In Extract 7.3, the candidate indicated the correct probability of each event for all possible alternatives.

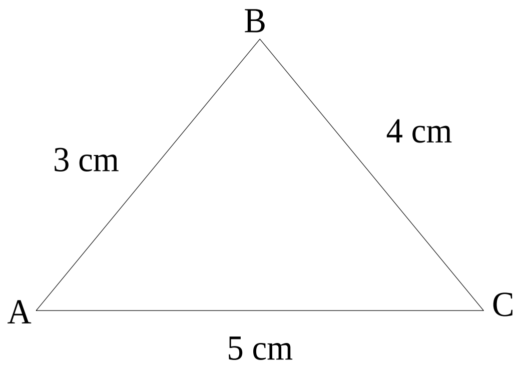
2.8 Question 8: Trigonometry

The question tested the candidates' competence in applying trigonometric identities. It also measured the ability of candidates to use the cosine rule or trigonometric ratios to calculate the angles of a triangle. The question stated as follows:

(a) *Write* $\cos 2B$ *in terms of* $\tan B$.

(b) *If* $\tan A = \dfrac{m}{m-1}$ *and* $\tan B = \dfrac{1}{2m-1}$, *show that* $A - B = \dfrac{\pi}{4}$.

(c) *Find the degree measure of* $A\hat{B}C$ *in the following figure:*



This question was attempted by 34,549 candidates, out of them 5,929 (17.16%) got 3.5 marks or more. Therefore, the overall performance of the candidates in this question was weak. Figure 9 shows the performance of the candidates in this question.

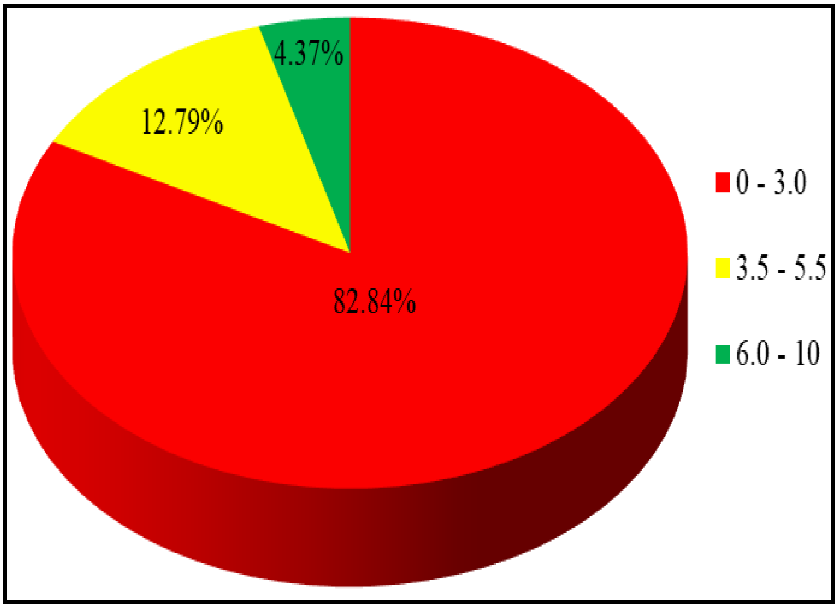
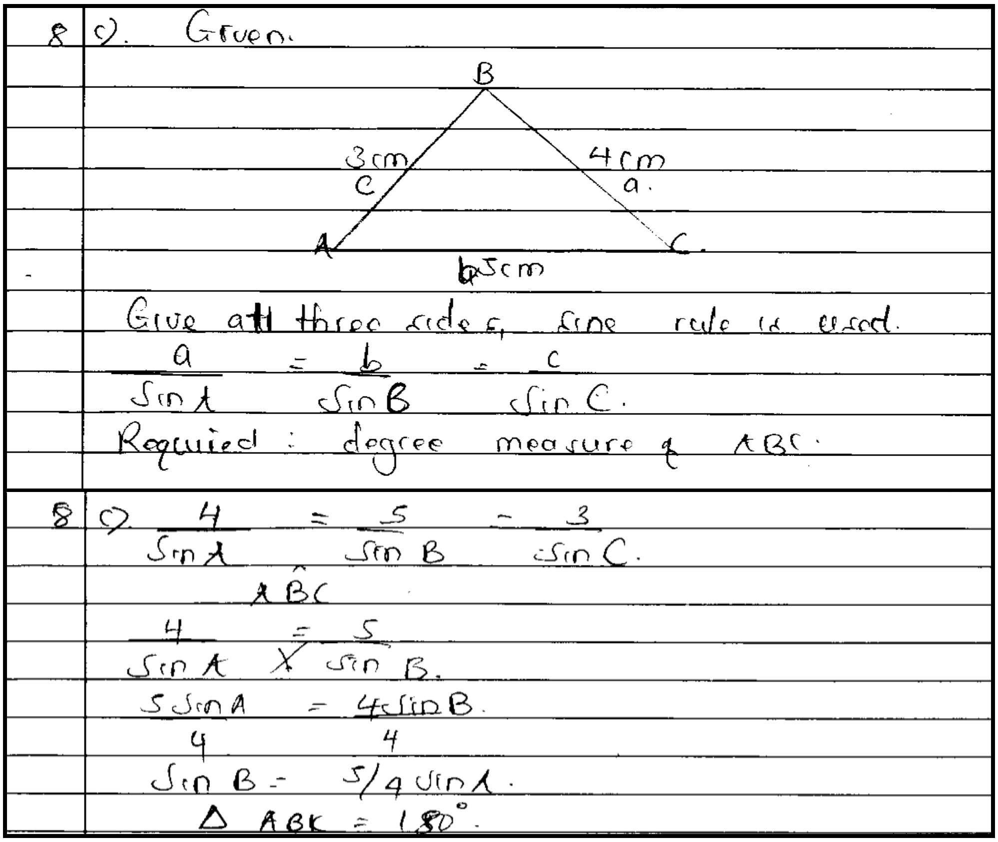


Figure 9: *The candidates' performance in question 7*

A total of 28,620 (82.84%) candidates scored 3.0 marks or less. In part (a), most of these candidates failed to recall the double angle formula for cosine and therefore, failed to write $\cos 2B$ in terms of $\cos B$ and $\sin B$. For example, some candidates rewrote $\cos 2B$ as $2\cos B \sin B$ which is an expansion of $\sin 2B$. Among these candidates, some divided $2\cos B \sin B$ by $2\cos^2 B$ resulting in $\tan B$ and concluded that $\cos 2B = \tan B$. Similarly, some candidates wrongly interpreted $\cos 2B$ as $\tan(B+B)$ and ended up with an incorrect conclusion, $\cos 2B = \frac{2\tan B}{1-\tan^2 B}$. Also, some candidates wrongly perceived that $\cos 2B = \frac{1+\cos^2 B}{2}$ instead of $\cos 2B = \frac{2\cos^2 B - 1}{2}$. The candidates were supposed to write $\cos 2B$ as $\cos^2 B - \sin^2 B$ and consequently $\frac{\cos^2 B - \sin^2 B}{\cos^2 B + \sin^2 B}$ after applying the identity $\cos^2 B + \sin^2 B = 1$. Then, they would divide each term of $\frac{\cos^2 B - \sin^2 B}{\cos^2 B + \sin^2 B}$ by $\cos^2 B$ and arrive at $\cos 2B = \frac{1-\tan^2 B}{1+\tan^2 B}$.

In part (b), most candidates gave inappropriate initial statements for the required verification. Some candidates wrote $\tan A = \tan B$. Thus, they replaced $\tan A$ and $\tan B$ with $\frac{m}{m-1}$ and $\frac{1}{2m-1}$ and consequently obtained $\frac{m}{m-1} = \frac{1}{2m-1}$ and solved for *m*. Therefore, they did not meet the requirements of the question. Further, some candidates directly evaluated $A - B$ by simplifying $\frac{m}{m-1} - \frac{1}{2m-1}$. Through this approach, the candidates got the expression containing the variable m instead of the angle $\frac{\pi}{4}$. Similarly, few candidates realized the need to start with $\tan(A-B)$, but they wrote incorrect formulae including $\tan(A-B) = \frac{\tan A \tan B}{1+\tan A \tan B}$ and therefore obtained expressions which do not verify the statement. In part (c), many candidates applied the sine rule which requires skills in solving three simultaneous equations (Extract 8.1).

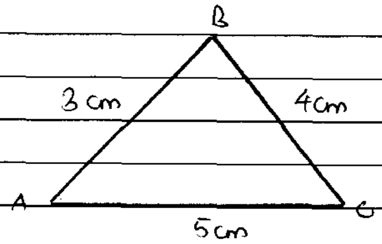


Extract 8.1: A sample of an incorrect response to part (c) of question 8

In Extract 8.1, the candidate failed to expose and solve the equations after applying the sine rule.

Despite the weak performance by majority of the candidates, 133 (0.38%) candidates scored all 10 marks in this question. In part (a), these candidates recalled the identities $\cos 2B = \cos^2 B - \sin^2 B$ and $\cos^2 B + \sin^2 B = 1$. Therefore, they wrote $\cos 2B$ as $\frac{\cos^2 B - \sin^2 B}{\cos^2 B + \sin^2 B}$ and divided both the numerator and denominator by $\cos^2 B$ to get $\cos 2B = \frac{1-\tan^2 B}{1+\tan^2 B}$. In part (b), the candidates applied the identity $\tan(A-B) = \frac{\tan A - \tan B}{1+\tan A \tan B}$ while replacing $\tan A$ and $\tan B$ with $\frac{m}{m-1}$ and $\frac{1}{2m-1}$ respectively. They further simplified the identity and obtained $\tan(A-B)=1$ implying $A-B=\tan^{-1}(1)$ and consequently $A-B=\frac{\pi}{4}$. Some candidates who correctly responded to part (c) were conversant with the cosine rule (Extract 8.2). Other candidates proved that the sides of the triangle obey Pythagoras theorem by showing that $3^2 + 4^2 = 5^2$. They further concluded that the degree measure of angle B is 90°. These candidates were also aware of the fact that the triangle whose sides obey Pythagoras theorem is a right - angled triangle and its right angle is the angle which is opposite to the longest side.

(c)



from cosine rule

$(\overline{AC})^2 = (\overline{AB})^2 + (\overline{BC})^2 - 2 \times \overline{AB} \times \overline{BC} \times \cos \hat{B}$

$(5)^2 = (3)^2 + (4)^2 - 2 \times 3 \times 4 \times \cos \hat{B}$

$25 = 25 - 2 \times 3 \times 4 \times \cos \hat{B}$

$24 \cos \hat{B} = 25 - 25$

$\cos \hat{B} = 0$

$\hat{B} = \cos^{-1}(0)$

$\hat{B} = 90^\circ$

$\therefore A\hat{B}C = 90^\circ$

Extract 8.2: A sample of correct response to part (c) of question 8

In Extract 8.3, the candidate correctly identified the angles and the corresponding opposite sides and then correctly applied the cosine rule.

2.9 Question 9: Exponential and Logarithmic Functions

The question intended to examine the candidates' competence in drawing the graphs of exponential and logarithmic functions and determining their domain and range. This question consisted of the following parts:

(a) *Draw the graph of* $f(x) = 2^{x-5}$ *and* $g(x) = \log_2(2x+3)$ *on the same xy plane.*

(b) *Use the graphs drawn in 9(a) to determine the domain and range of f(x) and g(x).*

A total of 5,866 (16.98%) candidates obtained from 3.5 to 10 marks. Generally the performance in this question was weak. Figure 10 shows the overall performance in percentage.

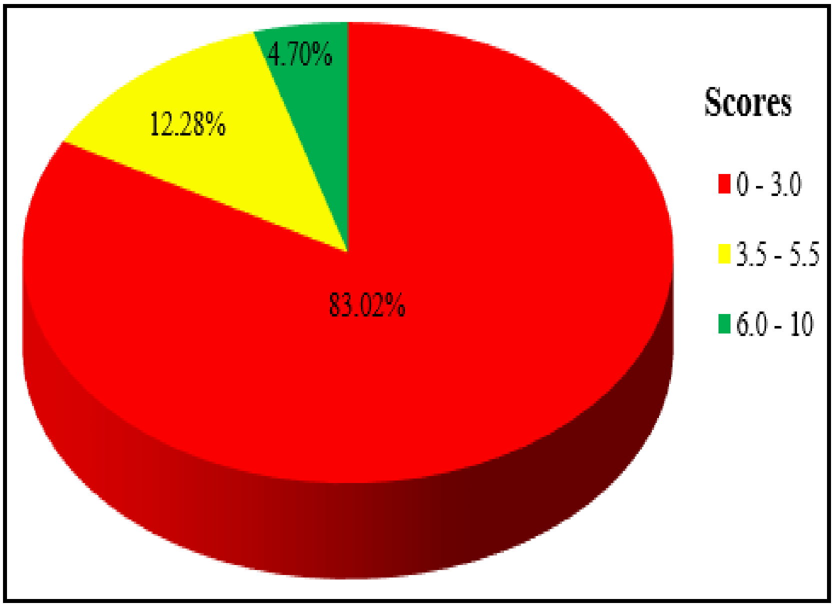
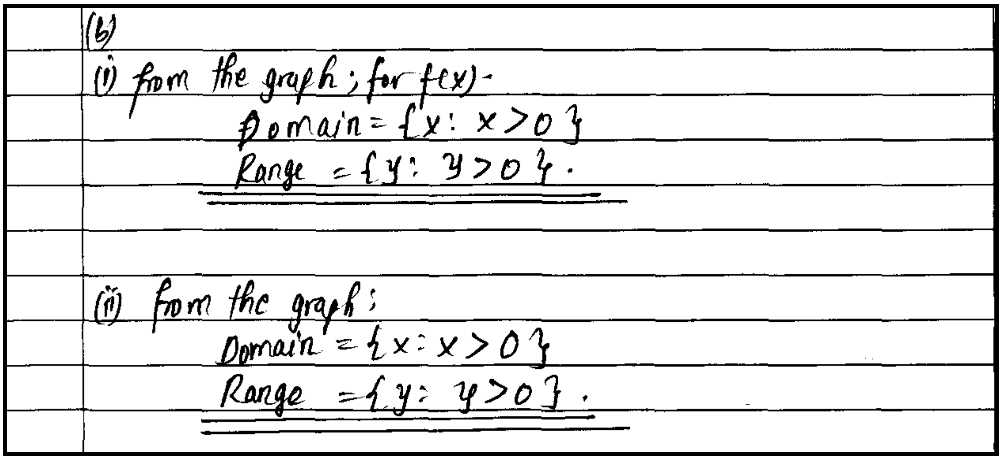


Figure 10: *The candidates' performance in question 9*

In part (a), the majority sketched the graphs which excluded some values along the x – axis. These sketches indicate that the candidates used few points without considering the features of the graphs of exponential functions. Some candidates also failed to determine the values of $f(x) = 2^{x-5}$ as x becomes a very large negative number. Therefore, the graphs did not approach the x - axis. Instead, some graphs crossed the x – axis and hence included negative values along y – axis. These graphs indicate that the candidates incorrectly evaluated the numbers with negative exponents, including 2^{-4}, 2^{-3}, 2^{-2} and 2^{-1} and ended up with negative instead of positive values.

Similar misconceptions were also observed in drawing the graphs of logarithmic functions. Further, most candidates did not consider the condition for the logarithmic function being defined. They did not write or solve the inequality $2x+3>0$ and therefore, failed to realise that $g(x)=\log_2(2x+3)$ is defined for all values of x greater than $-\frac{3}{2}$. As a result, some of these candidates computed the values of $g(x)$ for some values of x which are less than $-\frac{3}{2}$ and ended up with incorrect answers. Furthermore, some candidates wrongly assumed that $g(x) = \log_2(2x+3)$ is the inverse of $f(x) = 2^{x-5}$. Thus, they traced the curve of $g(x) = \log_2(2x+3)$ by studying the curve of $f(x) = 2^{x-5}$. The candidates were not aware that the inverse of the exponential function takes the logarithmic form, but the particular logarithmic function changes depending on the components of the given exponential function.

The incorrect answers in part (a) led to incorrect response in part (b). The incorrect answers observed in part (b) included domain $= \{x : -2 \le x < 2\}$ and range $= \{y : y \le 2\}$ for $f(x) = 2^{x-5}$ and domain $= \{x : x \ge -1\}$ and range $= \{y : y > 0\}$ for $g(x) = \log_2(2x+3)$. They were supposed to state that domain $= \{x : x \in \Re\}$ and range $= \{y : y \in \Re, y > 0\}$ for the given exponential function and domain $= \{x : x \in \Re, x > -\frac{3}{2}\}$ and range $= \{y : y \in \Re\}$ for the given logarithmic function.


(b)
(i) from the graph; for f(x)-
Domain = {x: x > 0}
Range = {y: y > 0}.

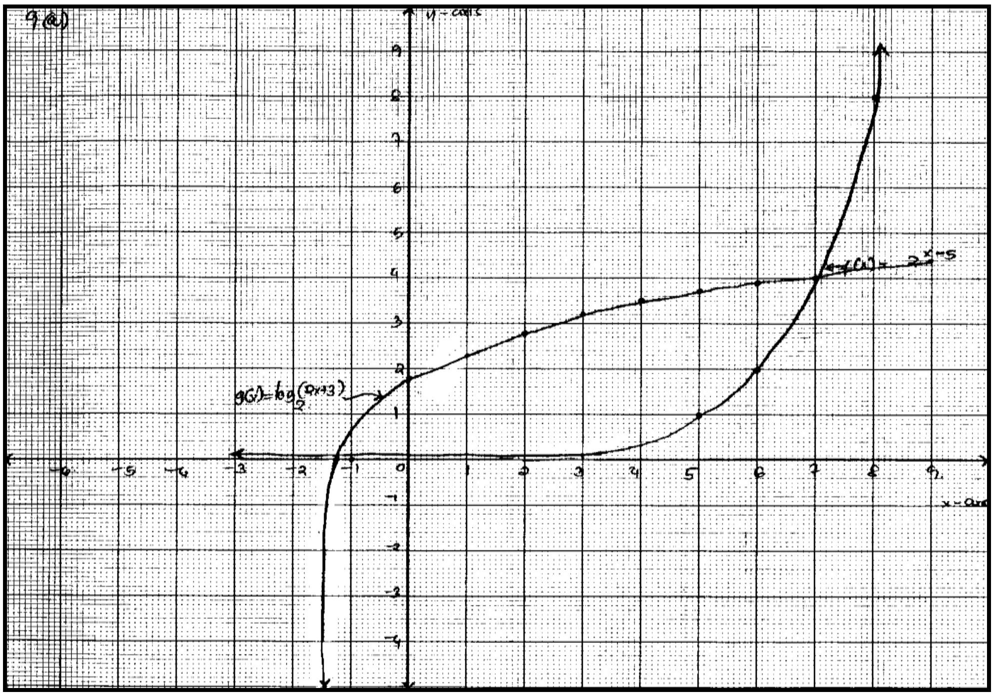
(ii) from the graph;
Domain = {x: x > 0}
Range = {y: y > 0}.

Extract 9.1: A sample of correct response to part (b) of question 9

As Extract 9.1 shows, the candidate wrote incorrect answers that domain and range for both $f(x)$ and $g(x)$ are numbers greater than zero.

Despite the weak performance, 75 (0.22%) candidates got full marks. In part (a), the candidates correctly computed the values of $f(x)$ for some values of x. Thus, they developed the points including the point at which the curve crosses y- axis, that is $\left(0, \frac{1}{32}\right)$. The candidates also realised that $f(x)$

increases as x increases and vice versa, however, it approaches to zero as x becomes a very large negative number. For the case of logarithmic function, these candidates correctly identified the interval in which $\log_2(2x+3)$ is defined by solving the inequality $2x+3>0$ resulting in $x>-\frac{3}{4}$. Therefore, they computed the points satisfying the function $g(x)=\log_2(2x+3)$ by substituting some values of x which are greater than $-\frac{3}{2}$. Further, they determined the y – intercept by computing $g(0)=\log_2 3 \approx 1.58$ and consequently $(0, 1.58)$. The candidates also knew that the curve crosses the x – axis at a point where $g(x)=0$. Therefore, they solved the equation $\log_2(2x+3)=0$ resulting to $x=-1$ implying the point $(-1, 0)$. With this information, the candidates drew the correct graph of $g(x)$ (Extract 9.2). Furthermore, the candidates studied the coverage of the graphs along x – axis to determine the domain and along the y – axis to determine the range. Therefore, they correctly wrote $\text{domain} = \{x : x \in \Re\}$ and $\text{range} = \{y : y \in \Re, y > 0\}$ for exponential function while for the given logarithmic function they wrote $\text{domain} = \{x : x \in \Re, x > -\frac{3}{2}\}$ and $\text{range} = \{y : y \in \Re\}$.



Extract 9.2: A sample of correct response to part (a) of question 9

In Extract 9.2, the candidates correctly sketched the graph of $f(x)$ and $g(x)$ by plotting the points and tracing them on the *xy* - plane by free hand.

2.10 Question 10: Matrices and Linear Programming

The question was set from both *Matrices* and *Linear Programming*. The question measured the candidates' competence in applying the properties of inverse matrices and representing the linear programming problem by the graph. The question included the following parts:

(a) *Given the matrices* $D = \begin{bmatrix} a & -4 & -6 \\ -8 & 5 & 7 \\ -5 & 3 & 4 \end{bmatrix}$ *and* $E = \begin{bmatrix} 1 & 2 & -2 \\ 3 & b & 1 \\ -1 & 1 & -3 \end{bmatrix}$. *If D is the inverse of E, determine the value(s) of* a *and* b.

(b) *A firm manufactures two products, A and B. The firm sells product A at a profit of 5 shillings per unit and product B at a profit of 3 shillings per unit. Each product is processed on two machines,* M_1 *and* M_2. *One unit of product A requires one minute of processing on* M_1 *and two minutes of processing on* M_2 *per day. One unit of product B requires two minutes of processing on* M_1 *and one minute of processing on* M_2 *per day. Machine* M_1 *works for 5 minutes per day while machine* M_2 *works for 6 minutes per day. Represent the information by using a graph and indicate the feasible region.*

This question was answered by 34,538 candidates, among them 23,397 (67.74) got 3.5 marks or more. Therefore, the overall performance of candidates in this question was good. Figure 11 shows the percentage of candidates who scored low, average and high marks.

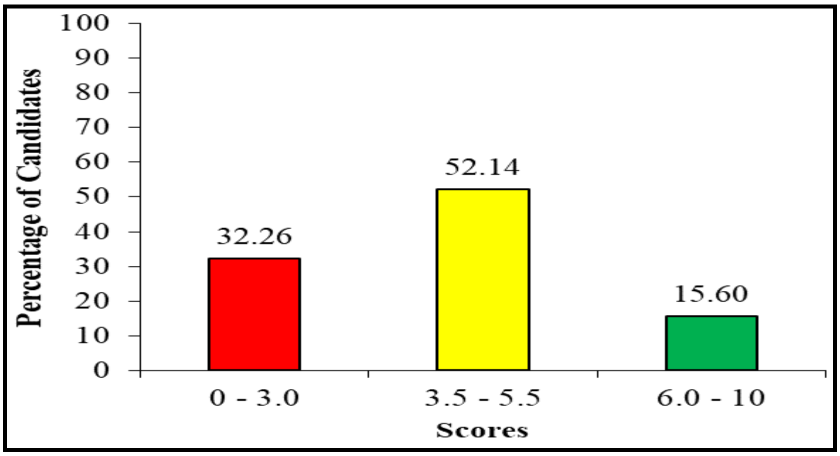


Figure 11: *The candidates' performance in question 10*

The majority answered part (a) by starting with the statement $DE = I$ and consequently $\begin{bmatrix} a & -4 & -6 \\ -8 & 5 & 7 \\ -5 & 7 & 4 \end{bmatrix}\begin{bmatrix} 1 & 2 & -2 \\ 3 & b & 1 \\ -1 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. This approach indicates that the candidates were conversant with the property of matrices “the product of n by n matrix (A) and its inverse (A^{-1}) results in an identity matrix (I), that is $AA^{-1} = I$”. The candidates also correctly performed matrix multiplication to obtain $\begin{bmatrix} a-6 & 2a-4b-6 & -2a+14 \\ 0 & -9+5b & 0 \\ 0 & -6+3b & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

Then, they equated some of the corresponding elements to formulate and solve the strategic equations, particularly $a-6=1$ and $-6+3b=0$ to get $a=7$ and $b=2$. Apart from this approach, a considerable number of candidates equated some elements of the matrix D to the corresponding elements of the inverse of matrix E and therefore, they got the incorrect answers (Extract 10.1).

In part (b), these candidates correctly realised that the asked quantities are the number of units of products A and that of product B produced per day and thus, assigned the variables x and y respectively. Then, they correctly formulated the constraints $x+2y \le 5$, $2x+y \le 6$, $x \ge 0$ and $y \ge 0$. Finally, the candidates drew the graphs of the constraints and indicated the feasible region (Extract 10.2).

10(a)

$$\begin{bmatrix} \frac{-8b-1}{-5b+9} & \frac{4}{-5b+9} & \frac{2+2b}{-5b+9} \\ \frac{8}{-5b+9} & \frac{5}{-5b+9} & \frac{-7}{-5b+9} \\ \frac{-3-b}{-5b+9} & \frac{-3}{-5b+9} & \frac{b-6}{-5b+9} \end{bmatrix} = \begin{bmatrix} a & -4 & -6 \\ -8 & 5 & 7 \\ -5 & 3 & 4 \end{bmatrix}.$$

$$\frac{8}{-5b+9} = -8$$

$$-8(-5b+9) = 8$$

$$40b - 72 = 8$$

$$\frac{40b}{40} = \frac{80}{40}$$

$$b = 2$$

$$\frac{-3b-1}{-5b+9} = a$$

$$a = \frac{-3\times2-1}{-5\times2+9}$$

$$a = \frac{-6-1}{-10+9}$$

$$a = \frac{-7}{-1}$$

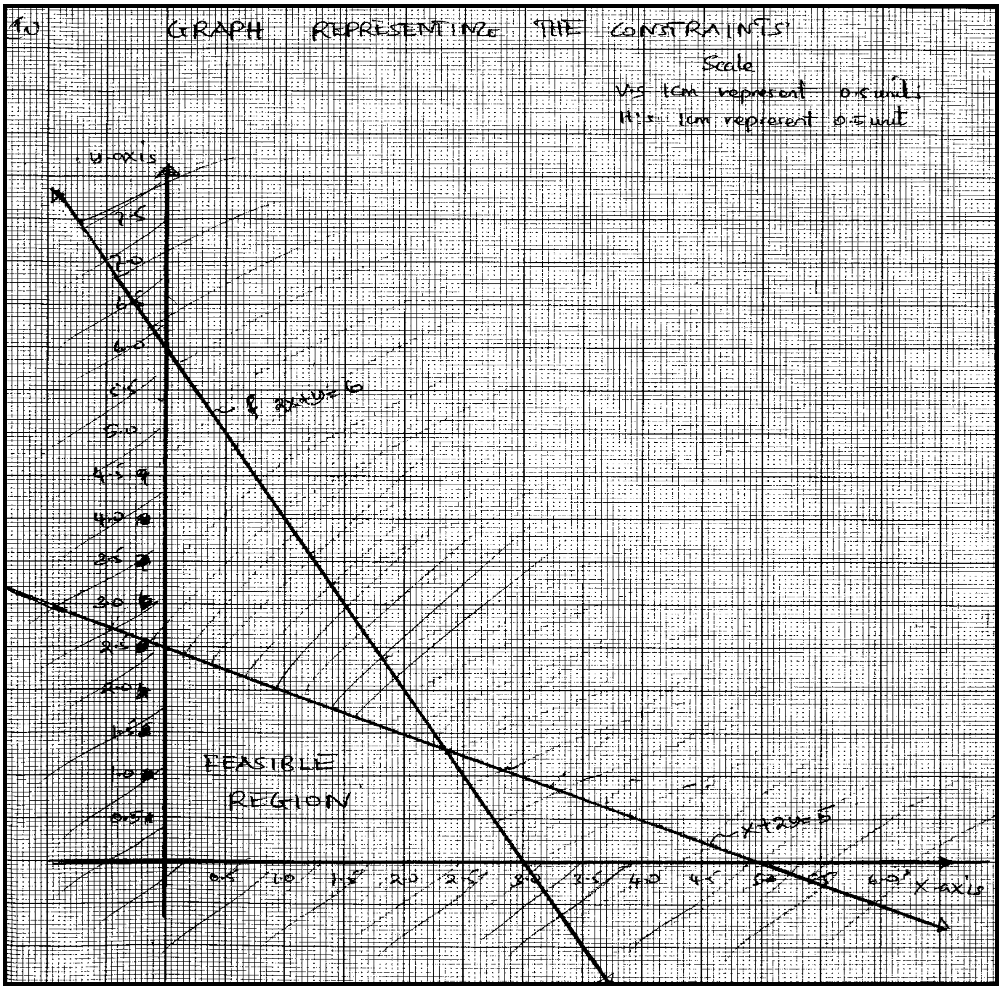
$$a = 7$$

$\therefore$ Value of $a = 7$

Value of $b = 2$

Extract 10.1: A sample of correct response to part (a) of question 10

In Extract 10.1, the candidate correctly evaluated the inverse of matrix *E* and then compared its elements with the corresponding elements of the matrix *D*.



Extract 10.2: A sample of correct response to part (b) of question 10

In Extract 10.2, the candidate correctly drew the graphs of the constraints and indicated the feasible region.

In spite of good performance, 11,141 (32.26%) candidates got low marks, of which, 3,808 (11.03%) got zero. Some candidates wrongly interpreted the two matrices as the singular matrices (Extract 10.2). Also, some candidates incorrectly assumed that the product of a matrix and its inverse result in a null matrix. They wrote $D \times E = 0$ and equated some elements of the product to zero. For example, the candidates who got $a = 6$, considered the elements of the first row in the first column. Further, there were candidates who equated the corresponding elements of the matrices D and E. These candidates got $a = 1$ and $b = 5$. In addition, the candidates who opted to equate the corresponding elements of D to those of the inverse of matrix E (or vice versa) faced difficulties in calculating either determinants or cofactors of the matrix. Generally, the candidates were not conversant with the fact that a minor found in i row and j column (M_{ij}) is multiplied by $(-1)^{i+j}$ to form a factor. Moreover, a significant number of the candidates performed matrix multiplication incorrectly. Most of these candidates multiplied the elements of a column by those of a row instead of multiplying the elements of the row by those of the column.

In part (b), many candidates interpreted the problem wrongly as they formulated incorrect constraints including $x + 2y \geq 5$ and $2x + y \geq 6$. They were supposed to be aware of the assumption that the total amount of the resource being utilized should be less than or equal to the amount of the available resource. For this case, the appropriate sign is $\leq$ (not $\geq$) and therefore, it results in $x + 2y \leq 5$ and $2x + y \leq 6$. Other candidates lacked skills in drawing the graph. For instance, a number of candidates interchanged x and y coordinates as they located $(2.5, 0)$ and $(3, 0)$ on the xy plane, implying $(0, 2.5)$ and $(0, 3)$ respectively. The errors led to an incorrect feasible region.

10 a) $|D| = a\begin{vmatrix} 5 & 7 \\ 3 & 4 \end{vmatrix} + 4\begin{vmatrix} ^-8 & 7 \\ ^-5 & 4 \end{vmatrix} - 6\begin{vmatrix} ^-8 & 5 \\ ^-5 & 3 \end{vmatrix}$

$|D| = a\,(20 - 21) + 4\,(^-32 - {^-35}) - 6\,(^-24 - {^-25})$

$|D| = {^-1a} + 4\,(3) - 6\,(1)$

$|D| = {^-a} + 12 - 6$

But $|D| = 0$

$0 = {^-a} + 6$

$\frac{^-6}{^-1} = \frac{^-a}{^-1}$

$\therefore\ a = 6$

For b

$E = \begin{pmatrix} 1 & 2 & ^-2 \\ 3 & b & 1 \\ ^-1 & 1 & ^-3 \end{pmatrix}$

$|E| = 1\begin{vmatrix} b & 1 \\ 1 & ^-3 \end{vmatrix} - 2\begin{vmatrix} 3 & 1 \\ ^-1 & ^-3 \end{vmatrix} - 2\begin{vmatrix} 3 & b \\ ^-1 & 1 \end{vmatrix}$

$|E| = 1\,(^-3b - 1) - 2\,(^-9 - {^-1}) - 2\,(3 + {^+1b})$

$|E| = (^-3b - 1) - 2\,(^-8) - 2\,(3 + {^+1b})$

$^-3b - 1 + 16 - 6 - 2b$

But $|E| = 0$

$0 = {^-3b} - 1 + 16 - 6 - 2b$

$0 = {^-3b} - 2b - 1 + 16 - 6$

$0 = {^-b} + 9$

$^-9 = {^-b}$

10 b) $\frac{^-9}{^-1} = \frac{^-b}{^-1}$

$b = 9$

$\therefore\ a = 6$

$b = 9$

Extract 10.3: A sample of incorrect response for part (a) of question 10.

In Extract 10.3, the candidate incorrectly responded to part (a) based on the statements $|D| = 0$ and $|E| = 0$.

3.0 ANALYSIS OF CANDIDATES' PERFORMANCE IN EACH TOPIC

The Basic Applied Mathematics examination of 2022 comprised ten (10) questions set from ten (10) topics. The data reveal that the candidates performed well in the topics of *Algebra* (75.68%) and *Linear Programming* (67.74%). This good performance indicates that the candidates were competent in applying the formula for determining the general term of an arithmetic progression, formulating equations/inequalities from word problems, solving the equations, and drawing graphs of linear inequalities. The candidates had an average performance in the topics of *Calculating Devices* (58.60%) and *Functions* (56.74%).

On the other hand, the overall performance of the candidates in other topics was weak. The topics included *Exponential and Logarithmic Functions*, *Integration*, *Trigonometry*, *Differentiation*, *Probability* and *Stati stics*. The weak performance is due to the failure of the candidates to draw graphs of exponential and logarithmic functions, apply rules of differentiation, calculate the area between two curves, and apply the trigonometric identities and rules. **Appendix I** shows the candidates' overall performance for all examined topics in 2022 paper, while **Appendix II** shows the performances in both 2021 and 2022 papers.

4.0 CONCLUSION AND RECOMMENDATIONS

4.1 Conclusion

The overall performance of the candidates in the Basic Applied Mathematics paper of 2022 was average. However, the performance decreased by 2.19 per cent when compared to the candidates' performance in 2021. The analysis also revealed that the overall performance in the topics of *Exponential and Logarithmic Functions* and *Integration* was persistently weak in 2020, 2021 and 2022. The decrease in performance was attributed to candidates' failure to apply computational skills and formulae to solve problems; and inability to interpret curve.

4.2 Recommendations

In order to facilitate the acquisition of the required competencies among students, it is recommended that teachers should:

(a) demonstrate and lead the students to describe features of exponential and logarithmic functions.

(b) guide students on applications of integration in solving various problems including the area between two curves.

(c) guide students to discuss how the double angle formulae are used to solve problems.

(d) guide students to apply differentiation to solve real–life problems.

(e) lead students to deduce the definition of permutation and combination.

(f) guide students to brainstorm the definition of mutually exclusive events and calculate the probability of two mutually exclusive events.

Appendix I

Analysis of Candidates' Performance per Topic in 141 Basic Applied Mathematics 2022

S/N	Topic	Question Number	Percentage of Candidates who Scored an Average of 3.5 Marks or Above	Remarks
1	Algebra	3	75.68	Good
2	Linear Programming	10	67.74	Good
3	Calculating Devices	1	58.60	Average
4	Functions	2	56.74	Average
5	Statistics	6	27.77	Weak
6	Probability	7	23.43	Weak
7	Differentiation	4	21.47	Weak
8	Trigonometry	8	17.16	Weak
9	Integration	5	16.99	Weak
10	Exponential and Logarithmic Functions	9	16.98	Weak

Appendix II

Candidates' Performance in each Topic for 2021 &2022