THE NATIONAL EXAMINATIONS COUNCIL OF TANZANIA

CANDIDATES' ITEM RESPONSE ANALYSIS REPORT FOR THE CERTIFICATE OF SECONDARY EDUCATION EXAMINATION (CSEE) 2015

042 ADDITIONAL MATHEMATICS
(For School Candidates)

THE NATIONAL EXAMINATIONS COUNCIL OF TANZANIA

CANDIDATES' ITEM RESPONSE ANALYSIS REPORT FOR THE CERTIFICATE OF SECONDARY EDUCATION EXAMINATION (CSEE) 2015

042 ADDITIONAL MATHEMATICS

(For School Candidates)

Published by
The National Examinations Council of Tanzania
P.O. Box 2624
Dar es Salaam, Tanzania

TABLE OF CONTENTS

FOREWORD

The candidates' item response analysis report for CSEE 2015 provides a feedback to the public in general about the performance of the candidates in attempting examination questions in Additional Mathematics.

The Certificate of Secondary Education Examination marks the end of the four years of ordinary level of secondary education. It is a summative evaluation which shows the effectiveness of the education system in general and the education delivery system in particular. Essentially, the candidates' responses to the examination questions indicate what the education system has been able or unable to offer to the candidates in their four years of Secondary Education.

The analysis presented in this report contributes towards understanding some of possible reasons behind the candidates' performance in Additional Mathematics. The report highlights the factors that made majority of the candidates perform well in the examination.

The feedback provided will enable the educational administrators, school managers, teachers and other education stakeholders to take appropriate measures in order to improve the candidates' performance in future examinations administered by the Council.

Finally, the Council would like to thank the Examination Officers, Examiners and all others who participated in preparing this report.

Dr. Charles E. Msonde
EXECUTIVE SECRETARY

1.0 INTRODUCTION

This report is based on the analysis of performance of candidates who sat for the Certificate of Secondary Education Examination (CSEE) Additional Mathematics paper in November 2015. The paper was set basing on the 2007 examination format which was composed from the 2010 Additional Mathematics Syllabus for Secondary Schools.

The paper consisted of two sections, namely A and B with a total of 16 questions. In section A each question weighed 6 marks while in section B 10 marks each. Candidates were required to answer all questions in section A and choose only 4 questions from section B.

In 2015, a total of 401 candidates sat for the Examination, of which 88.53 percent passed the Examination while in 2014, a total of 408 sat for Examination, of which 88.48 percent passed. This indicates that the percentage of the candidates who passed the Examination in 2015 has increased by 0.05 percent.

The subsequent section analyses the candidates' performance by indicating the task of each question which was expected to be done by the candidates. It also shows the strengths and weakness demonstrated by candidates in attempting each question. The samples of extracts for each question are inserted to support the cases presented.

In the analysis, the pass mark of the candidates is 30 percent of the allotted marks in each question. The performance in a topic is the average of performance of candidates of the questions set from that topic. The performance is categorized into three groups depending on the percentage of candidates who scored 30 percent or more. It is good from 45 to 100 percent, average from 30 to 44 percent and poor from 0 to 29 percent. Furthermore, green, yellow and red colours are used to denote good, average and poor performance respectively.

2.0 ANALYSIS OF CANDIDATES' PERFORMANCE PER QUESTION

SECTION A

This section consisted of 10 questions weighing 6 marks each. The candidates were required to answer all questions.

2.1 Question 1: Numbers

The question had two parts, namely (a) and (b). In part (a), the candidates were required to find the next three terms in the following sequences

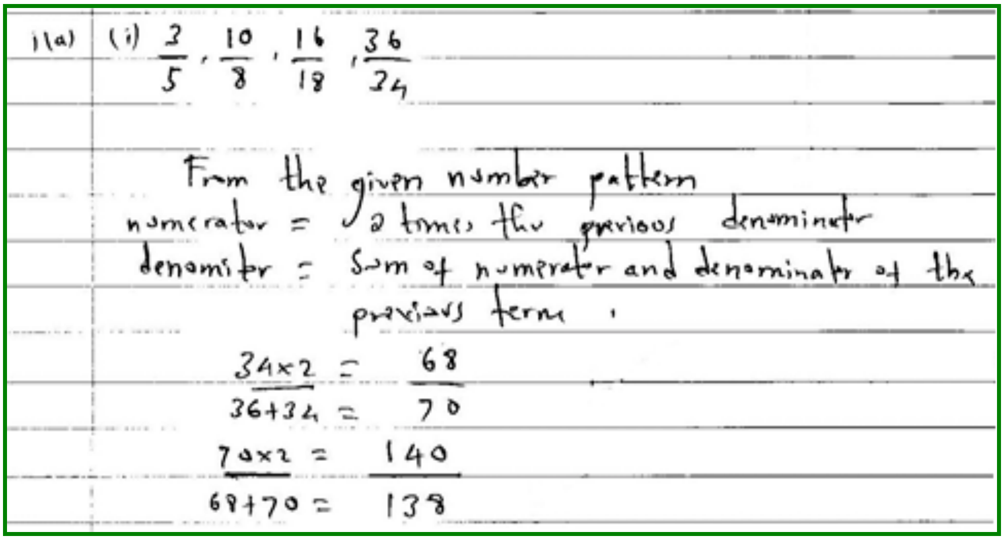
$\frac{3}{5}, \frac{10}{8}, \frac{16}{18}, \frac{36}{34},$ ___, ___, ___ and 1, 4, 9, 16, 25, ____, ___, ___.

In part (b), the candidates were required to find the approximate value of M in the equation $M = \frac{6.782 + 2.974}{7.332 - 2.422}$ by rounding each term to 2 significant figures.

This question was attempted by 397 candidates, of which 17.1% scored from 0 to 1.5 out of 6 marks with 3.3 percent scoring a zero mark. Majority of candidates (82.9%) scored from 2 to 6 marks, indicating that this question was well performed.

Many candidates (82.9%) had adequate knowledge on the topic of Numbers as they were able to recognize the relationship of numerators and denominators in finding the next terms. A sample of good response from the candidates is illustrated in Extract 1.1.

Extract 1.1


1(a) (i) $\frac{3}{5}, \frac{10}{8}, \frac{16}{18}, \frac{36}{34}$

From the given number pattern
numerator = 2 times the previous denominator
denomitor = Sum of numerator and denominator of the previous term.

$\frac{34\times2}{36+34} = \frac{68}{70}$

$\frac{70\times2}{68+70} = \frac{140}{138}$

$$\frac{138\times 2}{140+138} = \frac{276}{278}$$

$$\frac{3}{5}, \frac{10}{8}, \frac{16}{18}, \frac{36}{34}, \frac{68}{70}, \frac{140}{138}, \frac{276}{278}$$

(ii) 1, 4, 9, 16, 25

next term is square of the number

1, 4, 9, 16, 25, 6^2, 7^2, 8^2

1, 4, 9, 16, 25, 36, 49, 64

next three terms are 36, 49, 64

In Extract 1.1 the candidate managed to indicate clearly how the preceding terms were obtained.

In part (b), majority of the candidates found the approximate values for each term in the expression $\frac{6.782+2.974}{7.332-2.422}$ to 2 significant figures and made correct computations to obtain the value of M as illustrated in Extract 1.2 which shows a sample of candidates' good responses.

Extract 1.2

1(b) Solution

$$M = \frac{6.782 + 2.974}{7.332 - 2.422}$$

$6.782 \approx 6.8$ to 2 significant figures

$2.974 \approx 3.0$ to 2 significant figures

$7.332 \approx 7.3$ to 2 significant figures

$2.422 \approx 2.4$ to 2 significant figures

$$M = \frac{6.8 + 3.0}{7.3 - 2.4} = \frac{9.8}{4.9} = 2$$

In Extract 1.2, the candidate managed to approximate each term correct to 2 significant figures and determined the value of M correctly.

It was noted that, a few candidates (17.1%) lacked the knowledge on number patterns. Some of the candidates wrote answers that had no relationship to the given patterns. They were not able to obtain the appropriate trend for numerators and denominators that the next numerator was twice the denominator in the previous term, while the denominator of the next term was the sum of the numerator and denominator of the previous term. Also, they were not able to recognize that the sequence in part (ii) was obtained by squaring the natural numbers. A sample of candidates' poor answers is shown in extract 1.3.

Extract 1.3

1. a; $\frac{3}{5}, \frac{10}{8}, \frac{16}{18}, \frac{36}{34}, \frac{43}{37}, \frac{49}{48}, \frac{65}{64}$, .

ii, 1, 4, 9, 16, 25, 29, 34, 41.

b; $M = \frac{6.782 + 2.974}{7.332 - 2.422}$

$= \frac{9.756}{4910}$

$M = 1.973$

$\therefore$ Value of M is 1.97 (2 significant figure).

Extract 1.3 is a sample of a response from a candidate who listed the preceding numbers without using the required rules.

2.2 Question 2: Sets

The candidates were required to use the four sets $A=\{1, 2, 3, 4\}$, $B=\{2, 4, 6, 8\}$ $C=\{3, 4, 5, 6\}$ and $\mu=\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ to find A', $(A \cap C)'$ and $(B - C)'$.

This question was attempted by 100 percent of the candidates, of which 83.8 percent scored from 2 to 6 out of 6 marks allocated for this question and 16.2 percent scored from 0 to 1.5 marks. Thus the candidates' performance in this question was good.

Some of the candidates were able to answer some parts of the question as they had knowledge on few concepts of Sets. However, 38.2 percent of the candidates scored full marks as they were able to apply the Set properties to find the complement of the sets correctly. Extract 2.1 is an illustration of good responses from one of the candidates.

Extract 2.1

2 Data

$\mathcal{U} = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$A = \{1, 2, 3, 4\}$

$B = \{2, 4, 6, 8\}$

$C = \{3, 4, 5, 6\}$

a) $A' = \{5, 6, 7, 8, 9\}$

b) $(A \cap C)'$

$(A \cap C)' \equiv A' \cap \; A' \cup C'$

where

$A' = \{5, 6, 7, 8, 9\}$

$C' = \{1, 2, 7, 8, 9\}$

$\Rightarrow A' \cup C' = \{1, 2, 5, 6, 7, 8, 9\}$

$\therefore (A \cap C)' = \{1, 2, 5, 6, 7, 8, 9\}$

c) $(B - C)'$

$(B - C) \equiv B \cap C'$

$(B \cap C')' = (B' \cup C)$

$(B' \cup C) = \{1, 3, 4, 5, 6, 7, 9\}$

$\therefore (B - C)' = \{1, 3, 4, 5, 6, 7, 9\}$

In Extract 2.1, the candidate correctly determined the elements of the sets A', $(A \cap C)'$ and $(B - C)'$.

On the other hand, the analysis revealed that, 16.2 percent of the candidates who attempted this question performed poorly. Most of these candidates lacked the knowledge of a complement of a set. A sample of candidates' poor response is illustrated in extract 2.2.

Extract 2.2.

2 (a) A'

Given, $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9,\}$

$A = \{1, 2, 3, 4\}$

$A' = \{5, 6, 8\}$

$\therefore A' = \{5, 6, 8\}$

b) $(A \cap C)^1$

$A \cap C = \{1, 2, 3, 4, 5, 6\}$

$(A \cap C)^1 = \{8\}$

$\therefore (A \cap C)^1 = \{8\}$

(c) $(B - C)^1$

$B - C = \{2, 3, 4, 5, 6, 8\}$

$(B - C)^1 = \{1\}$

$\therefore (B - C)^1 = \{1\}$

In Extract 2.2, the candidate listed only a few elements of the complements of the given sets instead of listing all elements.

2.3 Question 3: Functions

The question had two parts; (a) and (b). In part (a), the candidates were required to find the value of $\alpha^2 + \beta^2$ given that α and β are the roots of quadratic equation $x^2 - 2x - 4 = 0$. In part (b), the candidates were required to use the remainder theorem to find the remainder when the polynomial $P(x) = x^3 + 2x^2 - 4x + 1$ is divided by $D(x) = x - 3$.

This question was attempted by 391 (97.5%) candidates, out of whom 86.2 percent scored from 2 to 6 out of 6 marks. Only 13.8 percent scored from 0 to

1.5. This implies that, the performance of candidates in this question was good.

Most of the candidates were able to correctly answer the question which shows that, they had adequate knowledge of functions. A sample of good answers from the candidates is illustrated in Extract 3.1.

Extract 3.1

3 a) $x^2 - 2x - 4 = 0$

$\Rightarrow \alpha + \beta = \frac{-b}{a} = \frac{+2}{1}$

$\alpha + \beta = 2$

$\Rightarrow \alpha\beta = \frac{c}{a} = \frac{-4}{1} = -4$

Then, $\alpha^2 + \beta^2$,

from $(\alpha+\beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta$

$\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= (2)^2 - (2x-4)$

$= 12$

$\therefore \alpha^2 + \beta^2 = 12$

b) $P(x) = x^3 + 2x^2 - 4x + 1$

dividend $D = x - 3$

When, $x - 3 = 0$ $\quad x = 3$

$P(3) = (3)^3 + 2(3)^2 - 4(3) + 1$ = remainder

$= 27 + 18 - 12 + 1$

$= 28 + 6 = 34$

$\therefore$ The remainder $= 34$

In Extract 3.1 the candidate related the roots and coefficients of quadratic equation in solving part (a) and also used the remainder theorem correctly to find the remainder in part (b).

The few candidates, who performed poorly in this question, could not relate the roots and the coefficients of the equation in part (a) as it was expected. It shows that, the idea of the remainder theorem was not known to some of the candidates, so they used long division method to find the remainder contrary to the instruction of the question. Extract 3.2 and 3.3, show samples of poor candidates' responses.

Extract 3.2

3a Solution

$x^2 \; 2x - 4 = 0$

$\alpha^2 + \beta^2 = (\alpha + \beta) \, 2 \, \alpha\beta$

$\alpha = {}^{-b}/a$ and $\beta = {}^{c}/a$

$({}^{-b}/a + {}^{c}/a] \; 2 \; {}^{-b}/a \cdot {}^{c}/a$

$a = 1, \; b = -2$ and $c = -4$

$= [{}^{2}/1 + {}^{-4}/1] \; 2 \; [{}^{2}/1 \cdot {}^{-4}/1]$

$= [2 - 4 \times 2 \times -8] = 32$

$\therefore \; \alpha^2 + \beta^2$ is 32

In Extract 3.2, the candidate wrongly factorized the equation in step 2.

Extract 3.3

3b Solution.

$P(x) = x^3 + 2x^2 - 4x + 1$

$D(x) = x - 3$

$x^2 - 4x + 8$

$x - 3 \,\big)\, x^3 + 2x^2 - 4x + 1$

$- \; x^3 + 6x^2$

$-4x^2 - 4x$

$+ \; 4x^2 + 12x$

$8x + 1$

$- \; 8x - 24$

25

$\therefore \; x^2 - 4x + 8$ remender of 25

In Extract 3.3, the candidate wrongly used long division method, hence failed to obtain the remainder.

2.4 Question 4: Algebra

This question had two parts; (a) and (b). In part (a), the candidates were required to make t subject of the formula in the equation $s = ut - \frac{1}{2}gt^2$. In part (b), the candidates were required to solve the pair of simultaneous equations $\begin{cases} xy = 10 \\ 3x + 2y = 16 \end{cases}$

The question was attempted by 99.8 percent of the candidates. Statistics show that, 77.2 percent scored from 2 to 6 marks and 22.8 percent scored from 0 to 1.5 marks which indicates a good candidates' performance.

Many candidates did well the question by clearly showing all the required procedures to obtain the answers. In part (a), they were able to re- arrange the given equation to form a quadratic equation in terms of *t*. Also, they used the general quadratic formula to make *t* subject. In part (b), majority of the candidates were able to make *x* or *y* subject from one of the given equations and then substituted in the other equation to form a quadratic equation which they correctly solved and obtained the values of *x* and *y*. A sample of candidates' good responses is illustrated in Extract 4.1.

Extract 4.1

4 (a) Given: $s = ut - \frac{1}{2}gt^2$

then: $2s = 2ut - gt^2$

$gt^2 - 2ut + 2s = 0$

from General quadratic formula;

$t = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

where $a = g$, $b = -2u$, $c = 2s$

$t = \dfrac{2u \pm \sqrt{4u^2 - 4(g)(2s)}}{2g}$

$t = \dfrac{2u \pm \sqrt{4u^2 - 8gs}}{2g}$

$\therefore t = \dfrac{u \pm \sqrt{u^2 - 2gs}}{2g}$

(b) Solution:

Take (ii), then $3x+2y=16$

$3x = 16-2y$

$x = \frac{16-2y}{3}$

Substituting onto (i)

$\Rightarrow \left(\frac{16-2y}{3}\right) y = 10$

$16y - 2y^2 = 30$

$2y^2 - 16y + 30 = 0$

$y^2 - 8y + 15 = 0$

$y^2 - 3y - 5y + 15 = 0$

$y(y-3) - 5(y-3) = 0$

$(y-5)(y-3) = 0$

either, $y = 5$ or $y = 3$

but $x = \frac{16-2y}{3}$

if $y = 5$

$x = \frac{16-10}{3}$

$x = \frac{6}{3}$

$x = 2$

if $y = 3$

$x = \frac{16-6}{3}$

$x = 10/3$

$x = 10/3$

Hence; when $x = 2$, $y = 5$ and

when $x = 10/3$, $y = 3$

Extract 4.1 shows that, the candidate performed well all steps involved in solving quadratic equations, hence got the correct solution.

On the other hand, some of the candidates (22.8%) were not able to re-arrange the terms of the given equation in part (a) in order to make *t* the subject. In part (b), the candidates were unable to solve the simultaneous equations in which one is quadratic and the other linear equation. This signifies that, these candidates lacked the knowledge of Algebra. Extract 4.2 and 4.3 are samples of candidates' poor responses.

Extract 4.2

4\. a)

$$S = ut - \frac{1}{2}gt^2$$

$$S = \frac{ut}{1} - \frac{1}{2}\frac{gt^2}{2}$$

$$S = \frac{2ut - 1gt^2}{2}$$

$$\frac{S}{1} = \times \frac{2ut - gt^2}{2}$$

$$\sqrt{2S} = \sqrt{2ut - gt^{\cancel{2}}}$$

$$2S = 2ut - gt$$

In Extract 4.2, the candidate failed to get rid of square of *t* and applied wrongly the general formula of solving quadratic equation.

Extract 4.2

4\. b)

$$\begin{cases} xy = 10 \\ 3x + 2y = 16 \end{cases}$$

$$\begin{matrix} 3 \\ 1 \end{matrix}\begin{cases} x + y = 10 \\ 3x + 2y = 16 \end{cases}$$

$$3x + 3y = 30$$
$$3x + 2y = 16$$
$$y = 14$$
$$y = 14$$
$$x + y = 10 \text{ but } y = 14$$
$$x + 14 = 10$$
$$x = -4$$
$$\therefore x = -4 \text{ and } y = 14$$

In Extract 4.3, the candidate wrongly used elimination method to find the values of *x* and *y* instead of substitution method.

2.5 Question 5: Symmetry and Geometrical Constructions

The question had two parts; (a) and (b). In part (a), the candidates were required to calculate the size of an exterior angle of a polygon with 12 sides. In part (b), they were required to find the number of sides of a polygon if the sum of its interior angles is 1520°.

This question was attempted by 96.3 percent of the candidates out of which 86.5 percent scored from 2 to 6 marks and 13.5 percent scored from 0 to 1.5 marks. These data imply good performance of the candidates in this question.

The candidates, who managed to score good marks, applied correct formula which is used to find the number of sides of a polygon, and got the required answer. Extract 5.1 is an example of good answer.

Extract 5.1

5 a Given exterior angle of a polygon with 12 sides

soln

From exterior angle $= \frac{360°}{n}$ where n is the number of sid

Exterior angle $= \frac{360°}{12}$

Exterior angle $= 30°$

Alternatively

Sum of interior angle $= (n-2)180°$

Interior angle $= \frac{(n-2)180°}{n}$

But interior angle + exterior angle $= 180°$

Exterior angle $= 180°$ − interior angle

$\therefore$ Exterior angle $= 180° - \frac{(n-2)180°}{n}$, where n is

The number of sides

Exterior angle $= 180° - \frac{(12-2)180°}{12}$

Exterior angle $= 180° - \frac{10 \times 180°}{12}$

Exterior angle $= 180° - 150°$

Exterior angle $= 30°$

Hint: The polygon above was considered to be regular

$\therefore$ Exterior angle $= 30°$

5b Given sum of interior angles $= 1520°$. Find the number of sides

soln

By taking the polygon to be regular

Sum of interior angles $= (n-2)\,180°$

$$\frac{(n-2)180°}{180°} = \frac{1520}{180°}$$

$$(n-2) = \frac{152}{18}$$

$$n = \frac{152}{18} + 2.$$

$$n = \frac{152+36}{18}$$

$$n = \frac{188}{18}$$

$$n \simeq 10 \text{ sides}$$

The polygon has 10 sides

In Extract 5.1, the candidate correctly applied the formulae for finding the exterior angle and the required number of sides.

A few candidates (13.5%) were not able to recall the formulae for finding interior and exterior angles that could be used to calculate the required values in parts 5(a) and 5(b). The candidates used various incorrect approaches to answer this question. For example, they used the formula for finding the area of a polygon and a circle. A sample of candidates' poor response is shown in extract 5.2.

Extract 5.2

05. (a). Solution

n° of sides = 12

$$\text{n° of sides} = \frac{\text{Exterior angle}}{\cancel{360}\ 180}$$

$$12 = \frac{\text{E.A}}{\cancel{360}} \qquad \frac{\text{E.A}}{180^\circ} = 12$$

$\therefore$ Exterior Angle = $\cancel{4320^\circ}$ 2160°

(b). Solution

$$\text{n° of sides} = \frac{\text{Interior angles}}{180}$$

$$\text{n° of sides} = \frac{1520^\circ}{180^\circ}$$

$\therefore$ n° of sides = 9

In Extract 5.2, the candidate applied incorrect formula to calculate the size of the exterior angle and the number of sides of the polygon.

2.6 Question 6: Variations

The question had two parts; (a) and (b). The candidates were given that, T varies jointly with the square root of x and inversely as the square of y. Furthermore, they were given that, when x is 9, y is 8 and T is 6. In part (a), they were required to find the equation of the variation by writing T as function of x and y. In part (b), they were required to find T when $x = \frac{1}{4}$ and $y = \frac{1}{6}$.

This question was attempted by 96.8 percent of the candidates, out of which 77.8 percent scored from 2 to 6 marks, showing a good performance in this question. The remaining 22.2 percent scored from 0 to 1.5 marks.

Majority of the candidates who performed well were able to apply the concepts of joint and inverse variation in solving the question. These candidates had good computational and algebraic skills that enabled them to obtain the required answer. They also had adequate knowledge on variations. Extract 6.1 provides a sample of good responses from the candidates.

Extract 6.1

6. a) $T \propto \frac{\sqrt{x}}{y^2}$

$T = k\frac{\sqrt{x}}{y^2}$ (Where k is constant)

$6 = \frac{k\sqrt{9}}{8^2}$

$6 = \frac{3k}{64}$

$K = \frac{64 \times 6}{3}$

$K = 128$

a) Equation is $T = \frac{128\sqrt{x}}{y^2}$.

b) $T = \frac{128\sqrt{x}}{y^2}$

$T = \frac{128\sqrt{1/4}}{(1/6)^2}$

$T = \frac{128 \times 1/2}{1/36}$

$T = \frac{64}{1/36}$

$= 64 \div \frac{1}{36}$

$= 64 \times 36$

$\therefore T = 2304$

In Extract 6.1, the candidate correctly applied the concept of joint and inverse variation to find the value of T.

On the other hand, there were candidates who failed to substitute the given values of *x* and *y* to get the value of T in part (b). Extract 6.2 shows a sample of candidates' good response.

Extract 6.2

6a Point $x = 9$

$y = 8$

$T = 6$

(x, y) $(0, 0)$

$(6, 9)$ $(6, 8)$

slope $= \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{8 - 9}{6 - 5}$

Slope $= -\dfrac{1}{1}$

Slope $= -1$

Point $(6, 9)$

$M = \dfrac{y - y_1}{x - x_1}$

$\dfrac{-1}{1} = \dfrac{y - 9}{x - 6}$

$-x + 6 = y - 9$

$-x + 6 - y + 9 = 0$

$-x - y + 6 + 9 = 0$

$-x - y + 15 = 0$

∴ The equation is $-x - y + 15 = 0$

b)

$x = 1/4$

$y = 1/6$

$(1/4, 0)$ and $(0, 1/6)$

$T = \dfrac{y_2 - y_1}{x_2 - x_1}$

$T = \dfrac{1/6 - 0}{0 - 1/4}$

$T = -\dfrac{2}{3}$

∴ The value of T is $-2/3$

In Extract 6.2, the candidate used the given values to find slope instead of period of the pendulum.

2.7 Question 7: Differentiation and Integration

In this question the candidates were required to differentiate $y = 2\pi x - 3x^2$ from the first principles.

This question was attempted by 360 (89.8%) candidates, out of whom 62.2 percent scored from 2 to 6 marks with 29.2 percent scoring full 6 marks. This indicates that, this question was well performed.

The candidates who performed well in this question were able to use the first principles to differentiate the given equation. This is illustrated in Extract 7.1 which is a sample of candidates' good response.

Extract 7.1

7 Soln:

Data

$y = f(x)$

$f(x) = 2\pi x - 3x^2$

$f(x+h) = 2\pi(x+h) - 3(x+h)^2$

→ from first principle formula

$$f(x)' = \lim_{h \to 0} \left\{ \frac{f(x+h) - f(x)}{h} \right\}$$

$$= \lim_{h \to 0} \left\{ \frac{2\pi(x+h) - 3(x+h)^2 - (2\pi x - 3x^2)}{h} \right\}$$

$$= \lim_{h \to 0} \left\{ \frac{2\pi x + 2\pi h - 3(x^2 + 2xh + h^2) - 2\pi x + 3x^2}{h} \right\}$$

$$= \lim_{h \to 0} \left\{ \frac{\cancel{2\pi x} + 2\pi h - \cancel{3x^2} - 6xh - 3h^2 - \cancel{2\pi x} + \cancel{3x^2}}{h} \right\}$$

$$= \lim_{h \to 0} \left\{ \frac{2\pi h - 6xh - 3h^2}{h} \right\}$$

$$= \lim_{h \to 0} \left\{ \frac{\cancel{h}(2\pi - 6x - 3h)}{\cancel{h}} \right\}$$

$$= \lim_{h \to 0} \{ 2\pi - 6x - 3h \}$$

but $h \to 0$

$$= 2\pi - 6x - 3(0)$$

$$= 2\pi - 6x$$

$$\therefore f'(x) = 2\pi - 6x$$

In Extract 7.1, the candidate correctly used the first principles rule to obtain the derivative of the given function.

The candidates who performed poorly this question used various incorrect approaches to solve it. Some of them used the knowledge of general formula for differentiation of polynomial to find the derivative. Other candidates wrote the first principles rule without putting the limits, that is, $y' = \left[\frac{f(x+h) - f(x)}{h}\right]$ instead of $y' = \lim_{h \to 0}\left[\frac{f(x+h) - f(x)}{h}\right]$. However, some candidates failed to recognize the demand of the question and therefore provided the answers which were out of the concept tested as illustrated in Extract 7.2.

Extract 7.2

7 $y = 2\pi x - 3x^2$

$$y = \frac{3x^{2+1}}{2+1} - 2\pi x$$

$$y = \frac{3x^3}{3} - 2\pi$$

$$y = \frac{3x^3}{\cancel{3}} \times \cancel{3} - 2\pi \times 3$$

$$y = 3x^3 - 9\pi$$

$y = 3x^3 - 9 \times \cancel{22}\ 9 \times 3.14$

$y = 3x^3 - 28.26$

$y = \frac{\cancel{3}x^3}{\cancel{3}} - \frac{28.26}{9}$

$\therefore y = x^3 - \frac{28.26}{9}$

In Extract 7.2, the candidate wrongly integrated the first term of the equation and differentiated the second term.

2.8 Question 8: Trigonometry

The question had two parts; (a) and (b). In part (a), the candidates were required to eliminate θ from the parametric equations $x = a\tan\theta$ and $y = b\cos\theta$. In part (b), they were required to:

(i) Define the term "Supplementary angle"

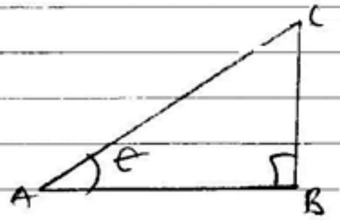
(ii) Find the value of x if $2x - 40^o$ and $80^o - 2x$ are supplementary angles.

This question was attempted by 374 (93.3%) candidates, out of whom 53.7 percent scored from 0 to 1.5 marks, 17.2 percent scored from 2 to 2.5 marks and 29.1 percent scored from 3 to 6 marks. This trend shows that, the performance of candidates in this question was average as 46.3 percent of the candidates scored 30 percent or above of the allotted marks in this question.

Some of the candidates, who scored from 0 to 1.5 marks, were not able to use the trigonometric identity $\tan^2\theta + 1 = \sec^2\theta$ to eliminate θ from the given equations in part (a). In part (b), some candidates wrongly gave the definition of supplementary angle as "angles whose sum is 90^0" instead of "two angles whose sum is 180^0". The sample of incorrect responses which were provided by candidates is shown in Extract 8.1 and 8.2.

Extract 8.1

8 a) Consider $\triangle ABC$



$x = a\tan\theta$ and $y = b\cos\theta$

by $\tan = \frac{BC}{AB}$

$\cos = \frac{AB}{AC}$

$\therefore\ x = a\left(\frac{AB}{AC}\right)$ and $y = b\left(\frac{AB}{AC}\right)$

In Extract 8.1, the candidate wrongly tried to eliminate θ by applying the definition of tangent and cosines instead of substitution method.

Extract 8.2

b)(i) Supplementary angle are equal angle of the polygon

ii) let $a = 2x - 40$, $b = 80 - 2x$ are

$\therefore\ a = b$

$2x - 40^\circ = 80 - 2x$

$2x + 2x = 80^\circ + 40^\circ$ (collect like terms)

$\frac{4x}{4} = \frac{\cancel{120}}{4}\ \frac{120^\circ}{4}$

$x = 30^\circ$

In Extract 8.2, the candidate wrongly defined supplementary angles as equal angles and applied it to find the value of *x* that led to end up with incorrect answer.

The candidates who were able to provide the required solutions used the required identity, $\tan^2\theta + 1 = \sec^2\theta$ to eliminate θ in part (a) and were able to define the term "supplementary angle"; hence they used it to find the value of x. A sample of good responses from the candidates is illustrated in Extract 8.3 and 8.4.

Extract 8.3

8 (a) Given; $\left.\begin{cases} x = a\tan\theta \\ y = b\cos\theta \end{cases}\right.$

then: recall; $1 + \tan^2\theta = \sec^2\theta$

$1 = \sec^2\theta - \tan^2\theta$

now; $x = a\tan\theta$

$\tan\theta = x/a$

$\tan^2\theta = x^2/a^2$ - - (i)

also; $\cancel{y}$ $y = b\cos\theta$

$\cos\theta = \frac{y}{b}$

$\sec\theta = b/y$

$\sec^2\theta = b^2/y^2$

Hence, $\sec^2\theta - \tan^2\theta = \frac{b^2}{y^2} - \frac{x^2}{a^2}$

$1 = \frac{b^2}{y^2} - \frac{x^2}{a^2}$

Hence; $\frac{b^2}{y^2} - \frac{x^2}{a^2} = 1.$

In Extract 8.3, the candidate was able to eliminate θ by introducing the identity $\tan^2\theta + 1 = \sec^2\theta$ in the parametric equation.

Extract 8.4

(b) i/ Supplementary angle is an angle whose sum with another angle gives 180°. i.e $\hat{A}+\hat{B}=180^\circ$.

ii/ ~~to~~ let $\hat{A} = 2x-40^\circ$

$\hat{B} = 80^\circ - 2x$

since $\hat{A}$ and $\hat{B}$ are supplementary

$\hat{A}+\hat{B} = 180^\circ$

then: $2x-40^\circ + 80^\circ - 2x = 180^\circ$

$0x + 40^\circ = 180^\circ$

$0x = 140^\circ$

$0 = 140^\circ$

x values have cancelled each other

Extract 8.4 shows that, the candidate was able to apply the definition of supplementary angle to answer part (b).

2.9 Question 9: Locus

The question had two parts; (a) and (b). In part (a), the candidates were required to define the term "Locus" as it is used in mathematics. In part (b), the candidates were required to find the equation of the locus of point $P(x, y)$ which is equidistant from point $A(0,1)$ and the line $x - y = 0$.

This question was attempted by 326 (81.3%) candidates whereby 85.6 percent scored from 0 to 1.5 marks with 40.8 percent scoring a 0 mark. A few candidates (14.4%) scored from 2 to 6 marks. This implies that, the candidates' performance in this question was poor because most candidates scored below the pass mark.

Most of the candidates who performed poorly were not able to define the term "locus", hence failed to use the concept of locus to find the required equation. Some candidates wrote the equation of a line through point A. A sample of poor candidates' response is illustrated in Extract 9.1.

Extract 9.1

1. a) Locus, this is a point which divide the line in a graph into two equal part.

b) Given:

$P(x,y)$, $A(0,1)$ line $x-y=0$

Required to find the equation of locus.

Solution:

From, Midpoint $= \left(\frac{(x_2+x_1)}{2}, \frac{(y_2+y_1)}{2}\right)$

$= \left(\frac{(x_1+0)}{2}, \frac{(y_2+1)}{2}\right)$

$= \left(\frac{x}{2}, \frac{y_2+1}{2}\right)$

Now let it $=0$

$= \left(\frac{x}{2}=0, \frac{y_2+1}{2}=0\right)$

$= (x=2\times 0, y_2+1=2\times 0)$

$= x=0 \quad y_2+1=0$

$= x=0 \quad y_2=-1$

Midpoints $= (0,-1)$

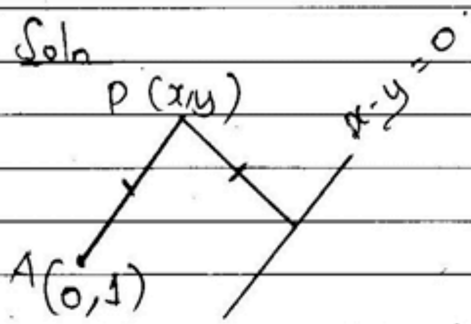
In Extract 9.1, the candidate gave wrong definition of locus and tried to find a midpoint, a concept which was not the requirement of the question.

A few candidates (14.4%) were able to define the term "locus" properly. They were able to apply the definition of locus to find the equation which was asked. They used the distance formula $d=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$ and the distance of a point from a line $d=\frac{|Ah+Bk+C|}{\sqrt{A^2+B^2}}$ correctly, and then equated the two distances to obtain the required equation of the locus. These candidates had adequate knowledge of Locus. This is illustrated by the answer of one of the candidates shown in Extract 9.2.

Extract 9.2

9\. (a) Locus - Is a point mostly indicated as $P(x,y)$ which obeys a certain condition given.

(b) Soln



$d_1 = d_2$.

distance from a line $d = \left| \frac{Ax + By + C}{\sqrt{A^2 + B^2}} \right|$

$x - y = 0$

$Ax + By + C = 0$

$A = 1$, $B = -1$, $C = 0$.

distance from point to point

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

$\sqrt{(x-0)^2 + (y-1)^2} = \left| \frac{1 \times x + -1 \times y + 0}{\sqrt{1^2 + (-1)^2}} \right|$

$\sqrt{x^2 + (y-1)^2} = \left| \frac{x - y}{\sqrt{2}} \right|$

Introduce square both sides.

$\left(\sqrt{x^2 + (y-1)^2}\right)^2 = \left(\frac{x-y}{\sqrt{2}}\right)^2$

$x^2 + (y-1)^2 = \frac{(x-y)^2}{2}$.

$$2x^2 + 2(y-1)^2 = (x-y)^2$$

$$2x^2 + 2(y^2-2y+1) = x^2-2xy+y^2$$

$$2x^2 + 2y^2 - 4y + 2 = x^2 - 2xy + y^2$$

$$2x^2 - x^2 + 2y^2 - y^2 - 4y + 2xy + 2 = 0$$

$$x^2 + y^2 - 4y + 2xy + 2 = 0$$

$$x^2 + y^2 + 2xy - 4y \cancel{=0} + 2 = 0$$

Equation of Locus is $x^2+y^2+2xy-4y+2=0$

In Extract 9.2, the candidate correctly used the distance formulae to find the required equation of the locus.

2.10 Question 10: Plan and Elevations

The question had two parts; (a) and (b). In part (a), the candidates were required to define the following terms “Front elevation” and “Plan view of an object”. In part (b), the candidates were required to draw the plan, front and side elevations of a cylinder which has a diameter of 1.5 cm and height of 2cm.

The question was attempted by 331 candidates, of whom 66.5 percent scored from 2 to 6 marks while 33.5 percent scored from 0 to 1.5 marks, which imply that the performance in this question was good.

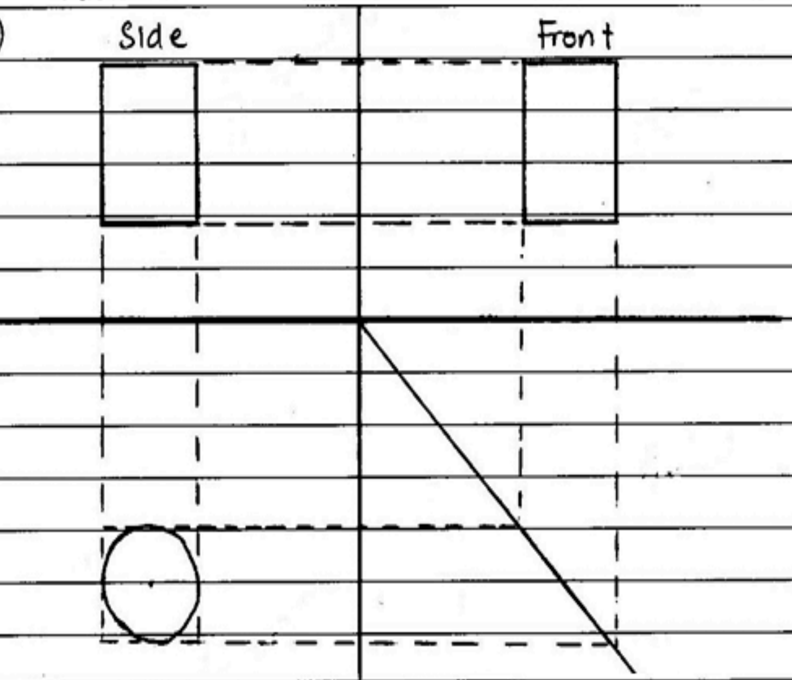
Majority of the candidates who performed well were able to define the terms front elevation and plan view of an object, and then correctly drew all elevations of a cylinder together with projection lines, showing that they had enough knowledge and skills in the topic of Plan and Elevations. Some candidates defined the terms but were not able to draw the plan and elevations of the cylinder. A sample of candidates’ good response is illustrated in Extract 10.1.

Extract 10.1

10. (a) (i) Front Elevation is the elevation of an object as seen from infront of an object to the observer

(ii) Plan view of an object is elevation of a object or view as seen from above of the object from the observer.

(b)



First Angle Projection: Plane And Elevation

In Extract 10.1, the candidate was able to define the terms Front elevation and plan view of an object, then drew the plan, front and side elevations correctly.

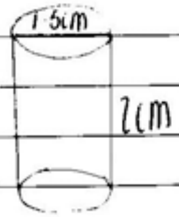
A few candidates (33.5%) were unable to define the terms correctly and also drew figures which were not related to the question. Some of the candidates failed to understand the requirement of the question and therefore just drew something like a certain container placed on top of a flat surface. Others were able to answer only some parts of this question. A sample of candidates' poor response is illustrated in Extract 10.2.

Extract 10.2

10 a)
i) Front elevation - Is the angle of elevation where an angle is made.

ii) Plan, view of an object - Is the view in which an object is being planned.

b)



In Extract 10.2, the candidate failed to define the front elevation and plan view of an object; also he/she could not draw any of the elevations required in this question.

SECTION B

This section consisted of 6 questions weighing 10 marks each. A candidate had to choose and answer any 4 questions.

2.11 Question 11: Coordinate Geometry

The question had three parts; (a), (b) and (c). In part (a), the candidates were required to find the coordinates of a point that divides the line segment joined by points $A(5,8)$ and $B(-8,5)$ in the ratio $3:2$. In part (b), they were required to find the tangents of the angle between the lines $4x+3y-12=0$ and $y-3x=0$. In part (c), they were required to find the centre and radius of the circle with equation $4x^2+4y^2+20x-16y+37=0$.

This question was attempted by 254 candidates of whom 85.4 percent scored 3 or above out of 10 marks that were allocated to this question indicating a good performance of the candidates.

Most of the observed candidates' scripts shows that, the candidates were able to answer correctly all the three parts. They managed to change the equation into factor form; hence they determined its centre and radius. The examples of good responses are illustrated in Extract 11.1, 11.2 and 11.3.

Extract 11.1

11. a) Internal division of a line segment

A (5, 8)
B (−8, 5)
m : n = 3 : 2.

$x_1 = 5 \quad y_1 = 8 \quad m = 3$
$x_2 = -8 \quad y_2 = 5 \quad n = 2$

$$(x, y) = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$

$$= \left(\frac{3(-8) + 2(5)}{3+2}, \frac{3(5) + 2(8)}{3+2}\right)$$

$$= \left(\frac{-24 + 10}{5}, \frac{15 + 16}{5}\right)$$

$$(x, y) = \left(\frac{-14}{5}, \frac{31}{5}\right)$$

$$(x, y) = \left(\frac{-14}{5}, \frac{31}{5}\right)$$

In Extract 11.1, the candidate correctly applied the ratio theorem to find the point which divides the line AB in ratio 3:2.

Extract 11.2

11. b) From $y = mx + c$ (m : gradient).

Line 1 :

$$4x + 3y - 12 = 0$$

$$y = \frac{-4}{3}x + 4.$$

$\Rightarrow$ Gradient m_1 of line 1 is $-\frac{4}{3}$

Line 2 : $y - 3x = 0$ $\quad y = 3x.$

Gradient m_2 of line 2 is 3.

$$\Rightarrow \tan\theta = \left|\frac{m_2 - m_1}{1 + m_2 m_1}\right|$$

$$= \left|\frac{3 - -4/3}{1 + 3 \times -4/3}\right| = \left|\frac{3 + 4/3}{1 - 4}\right|$$

$$\tan\theta = \left|\frac{13}{3} \div -3\right|$$

$$\tan\theta = \left|\frac{13}{3} \times \frac{1}{-3}\right|$$

$$\tan\theta = \left|\frac{13}{-9}\right|$$

$$\tan\theta = \frac{13}{9}$$

$$\tan\theta = 1.444.$$

$\therefore$ tangent of angle is 1.444.

In Extract 11.2, the candidate correctly showed all steps in finding the tangents of the angles between the lines $4x + 3y - 12 = 0$ and $y - 3x = 0$.

Extract 11.3

11. c) $\frac{4x^2}{4} + \frac{4y^2}{4} + \frac{20x}{4} - \frac{16y}{4} + \frac{37}{4} = \frac{0}{4}$

$x^2 + y^2 + 5x - 4y + \frac{37}{4} = 0$

$x^2 + y^2 + 2gx + 2fy + c = 0$

$\frac{2g}{2} = \frac{5}{2}$ $\qquad$ $\frac{-4}{2} = \frac{2f}{2}$

$g = \frac{5}{2}$ $\qquad$ $f = -2$

$\rightarrow$ $a = -g$ $\qquad$ $b = -f$

$a = \frac{-5}{2}$ $\qquad$ $b = --2$

$b = 2$

$c = \frac{37}{4}$

$c = a^2 + b^2 - r^2$

$\frac{37}{4} = \left(\frac{-5}{2}\right)^2 + (2)^2 - r^2$

$\left(\frac{37}{4} = \frac{25}{4} + 4 - r^2\right) \times 4$

$\rightarrow$ $a = -g$ $\qquad$ $b = -f$

$a = \frac{-5}{2}$ $\qquad$ $b = --2$

$b = 2$

$c = \frac{37}{4}$

$c = a^2 + b^2 - r^2$

$\frac{37}{4} = \left(\frac{-5}{2}\right)^2 + (2)^2 - r^2$

$\left(\frac{37}{4} = \frac{25}{4} + 4 - r^2\right) \times 4$

$37 = 25 + 16 - 4r^2$

$37 = 41 - 4r^2$

$\frac{4r^2}{4} = \frac{4}{4}$

$\sqrt{r^2} = \sqrt{1}$

$r = 1$.

$\therefore$ The centre of the circle is (a, b) is $\left(-\frac{5}{2}, 2\right)$ and the radius of the circle is 1 unit

In Extract 11.3, the candidate correctly factorized the equation of the circle and determined its centre and radius.

A few candidates (14.6%) who scored below 3 marks, in part (a) could not recall the formula for finding the coordinates of a point that divides a line in the 3: 2. Likewise, in part (b) they failed to recall the formula for finding angle between two lines that could help them to find tangent of the angle between the lines. Some of them were able to recall the formula but found angles between the lines instead of the tangents of those angles. In part (c), they could not apply the concept of perfect square so as to re-write the given equation of the circle in the factor form, $(x-h)^2 + (y-k)^2 = r^2$, from which the centre and radius could easily be identified. The sample answer of one of the candidates who performed poorly in this question is illustrated in Extract 11.4, 11.5 and 11.6.

Extract 11.4

11. (a) Soln

Given Point A, $(5, 8)$

B, $(3, 5)$

from $M_1 = 3$ $M_2 = 2$

$$X,y = \frac{M_1X_1 + M_2X_2}{M_1 \cdot M_2}, \frac{M_1y_1 + M_2y_2}{M_1 \cdot M_2}$$

$$(x,y) = \frac{3\times5 + 2\times8}{3\times2}, \frac{3\times8 + 2\times5}{3\times3}$$

$$(x,y) = \frac{15+-16}{6}, \frac{24+10}{9}$$

$$\therefore (X,y) = \left(-1/6, 34/9\right)$$

In Extract 11.4, the candidate wrongly wrote the product $m_1 \bullet m_2$ in the denominator of the formula instead of the sum $m_1 + m_2$.

Extract 11.5

11. (b) $13X - 12 = 0$

$$\frac{13X}{13} = 12/13$$

$$X = 12/13$$

$$y = 3X$$

$$y = 36/13$$

$$(X,y) = \left(12/13, 36/13\right)$$

from the tangents of the angle

$$y - y_1 = M(X - X_1)$$

Extract 11.5 shows that, the candidate wrongly found the equation of a line instead of the tangents of angles between those lines.

Extract 11.6

11 (c) $(2x+10)^2+100+(2y-4)^2-(4)^2+37=0$

$(2x+10)^2+(2y-4)^2+137-16=0$

$(2x+10)^2+(2y-4)^2+121=0$

$\sqrt{(2x+10)^2+(2y-4)^2}=\sqrt{-121}$

$2x+10+2y-4=121$

radius $=121$

Centre $=2x+10=0$

$\frac{2x}{2}=\frac{-10}{2}$

$x=-5$

$2y-4=0$

$\frac{2y}{2}=\frac{4}{2}$

$y=2$

radius $=121$

Centre $=(-5,2)$

In Extract 11.6, the candidate was unable to re-write the given equation into factor form which led into incorrect value of centre and radius.

2.12 Question 12: Statistics

The question had two main parts; (a) and (b). In part (a), the candidates were required to find the median and range from the following data of the amount of annual rainfall in centimeters for a period of 15 days;

25, 38, 27, 39, 42, 34, 27, 26, 24, 33, 32, 35, 44, 29 and 27. In part (b), the candidates were required to calculate the mean and standard deviation of ages of 50 adults which were shown in the table as follows:

Age	52-48	47-43	42-38	37-33	32-28	27-23	22-18
Number of people	4	6	7	11	9	8	5

This question was attempted by 348 (86.8%) candidates. The analysis shows that, 90.2 percent scored above 2.5 and 9.8 percent scored from 0 to 2.5 marks implying that, the general performance of candidates in this question was good.

Majority of the candidates who performed well were able to compute the median, range, mean and standard deviation as it was required. This is illustrated in Extract 12.1 and 12.2 which are examples of correct responses from the candidates.

Extract 12.1

12 (a) Soln.

Arrangement of numbea in ascending order.

24, 25, 26, 27, 27, 27, 29, 32, 33, 34, 35, 38, 39, 42, 44

$n = 15$ $\left(\frac{n-1}{2}\right)+1$

The 8^{th} number = 32

∴ Median = 32

Range = Highest number - Lowest number

Range = 44 - 24

Range = 20

∴ Median = 32, Range = 20

In Extract 12.1, the candidate arranged the given data in ascending order that enabled him/her to find the median and range.

Extract 12.2

12 (b) Soln.

Class interval	x	f	fx	$(x-\bar{x})$	$(x-\bar{x})^2$	$f(x-\bar{x})^2$
18 - 22	20	5	100	-14.1	198.81	994.05
23 - 27	25	8	200	-9.1	82.81	662.48
28 - 32	30	9	270	-4.1	16.81	142.29
33 - 37	35	11	385	0.9	0.81	8.91
38 - 42	40	7	280	5.9	34.81	243.67
43 - 47	45	6	270	10.9	118.81	712.86
48 - 52	50	4	200	15.9	252.81	1011.24
		$N=50$	$\Sigma f(x) = 1705$			$\Sigma f(x-\bar{x})^2 = 3775.5$

(b)(i) $\text{Mean} = \dfrac{\Sigma fx}{N}$

$= \dfrac{1705 \times 2}{50 \times 2} = \dfrac{3410}{100} = 34.1$

$\therefore \text{Mean} = 34.1$

(ii) $\text{Standard deviation } (\delta) = \sqrt{\dfrac{\Sigma f(x-\bar{x})^2}{n}}$

$= \sqrt{\dfrac{3775.5}{50}} = \sqrt{\dfrac{377.55}{5}} = \sqrt{75.51}$

$= 8.69$

$\therefore \text{Standard deviation} = 8.69$

In Extract 12.2, the candidate was able to prepare a frequency table, which was useful in computing the mean and standard deviation.

On the other hand, some of the candidates failed to answer the question due to inability to recall the formulae for calculating median, mean and standard deviation. Others were able to recall some of the formulae, but made wrong substitution of the values indicating that, they were not familiar with computation of measures of central tendency and measures of dispersion for both ungrouped and grouped data. The sample of responses from the candidates who performed poorly is illustrated in Extract 12.3 and 12.4.

Extract 12.3

12 a, The amount of annual rainfall (C°)

rainfall C°	F.	~~C.f~~. X	C.f
24 - 28	6	6	36
29 - 33	3	9.	27
34 - 38	3	12	36
39 - 43	2	14.	28
44 - 48	1	15	15.
	$\Sigma f = 15$	$\Sigma fx = 46$	

i, Solve

Median.

$$X = \frac{\Sigma f}{\Sigma fx} = \frac{15}{46} = 3.06$$

$X = 3.06C°$

Extract 12.3 shows that, the candidate wrongly computed the class mark, cumulative frequency and median.

Extract 12.4

b. The table of ages of 50 adults which Certai Village.

Age	Number of people	X	C.F.
18 - 22	5	5	25
23 - 27	8	13	104
28 - 32	9	22	198
33 - 37	11	33	393.
38 - 42	7	40	270
43 - 47	6	46	276
48 - 52	4	50	200
	$\Sigma f = 50$	$\Sigma fx = 176$	

12 b. i) Calculate Mean.

formular.

$$X = \frac{\Sigma f}{\Sigma f x}.$$

$$X = \frac{50}{176} = 3.52$$

$\therefore$ The Value of Mean is 3.52.

Extract 12.4 shows that, the candidate had a misconception on the formula for finding the mean as it is seen in the first step.

2.13 Question 13: Logic

The question had three parts (a), (b) and (c). In part (a), the candidates were required to write the truth value of the following mathematical statement: "If 2 is prime number, then 2 is not an even number". Part (b) of the question required the candidates to construct the truth table for the proposition $p \wedge (q \vee r)$. In part (c) the candidates were required to test the validity of the following argument:

> "Tanzania is making a new constitution. Either Tanzania is editing her constitution or Tanzania is making a new constitution. If Tanzania is making a new constitution then Tanzania has a constitution. Therefore, Tanzania has a constitution and Tanzania is making a new constitution."

The question was attempted by 270 candidates of whom 75.9 percent scored 3 or more out of 10 marks. Statistics show that, 24.5 percent scored below 3 marks with 4.1 percent scoring a zero mark. These data indicate that, the performance in this question was good.

Majority of the candidates who performed well were able to answer some of the three parts of the question while a few of them managed to provide correct answers to all parts. An example of a good response is illustrated in Extract 13.1, and 13.2.

Extract 13.1

13 sol:

let; 2 is a prime number be p
2 is not an even number be q

P is true (T)
q is False (F)

$p \rightarrow q$
$T \rightarrow F = F$

∴ the truth value of the mathematical statement "If 2 is a prime number, then 2 is not an even number" is FALSE (F)

b) sol.

p	q	r	$q \vee r$	$p \wedge (q \vee r)$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	F	F
F	T	T	T	F
F	T	F	T	F
F	F	T	T	F
F	F	F	F	F

Extract 13.1 shows that, the candidate was able to determine the truth value of the given mathematical statement and construct the truth table of the given proposition correctly.

Extract 13.2

let;
p be Tanzania is making a new constitution
q be Tanzania is editing her constitution
r be Tanzania has a constitution

$p, q \vee p, p \rightarrow r \therefore r \wedge p$

$[p \wedge (q \vee p) \wedge (p \rightarrow p)] \longrightarrow (r \wedge p)$

p	q	r	$q \vee p$	$p \rightarrow r$	$r \wedge p$	$[p \wedge (q \vee p) \wedge (p \rightarrow r)]$ (a)	$a \rightarrow (r \wedge p)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	T	T	T	T	T
T	F	F	T	F	F	F	T
F	T	T	T	T	F	F	T
F	T	F	T	~~F~~T	F	F	T
F	F	T	F	T	F	F	T
F	F	F	F	T	F	F	T

$\therefore$ since the argument is a tautology
$\therefore$ The argument is valid.

The candidate whose work is illustrated in Extract 13.2 was able to write the given argument in symbolic and test its validity correctly using a truth table.

The candidates who performed poorly were not able to write the truth value of the given mathematical statement in part (a). They also failed to construct the truth table of the given proposition in part (b) and were not able to write the argument in part (c) symbolically so as to test its validity either by using truth table or laws of algebra of propositions. Extract 13.3 shows an example of an incorrect response from the candidates.

Extract 13.3

13 b) Soln

The truth table

$2^n = 2^3$

$= 6$

$p \wedge (q \vee r)$

p	q	r	$q \vee r$
T	T	T	T
T	T	F	F
T	F	T	T
F	F	F	T
F	T	T	T
F	T	F	F

c) Tanzania is making new constitution

In Extract 13.3, the candidate wrongly constructed a truth table and copied one of the given statements in the argument as a solution to part (c).

2.14 Question 14: Probability, Permutation and Combination

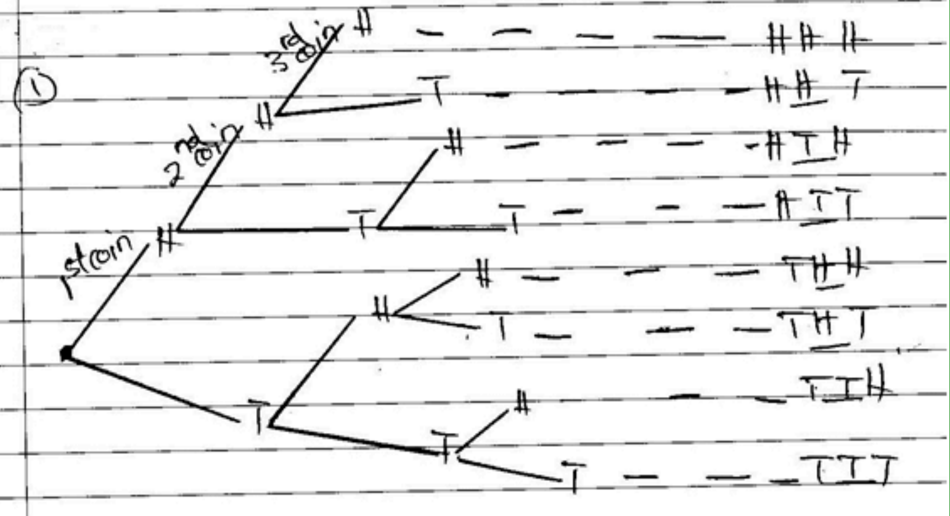
The question had two parts; (a) and (b). In part (a), the candidates were given that, three unbiased coins are tossed once, then they were required to draw the probability tree diagram to show the results of the experiment and find the probability of getting at most two heads. In part (b), the candidates were required to find the number of ways of selecting 9 balls from 6 red balls, 5 white balls and 5 blue balls if each selection consists of 3 balls of each colour.

This question was attempted by 211 candidates of whom 63.0 percent scored above 2.5 and 37 percent scored below 3 marks implying that, the performance in this question was good.

Majority of the candidates performed well in this question. The factors that contributed to the good performance include; the ability of candidates to apply the tree diagram in solving probability problems and good interpretation of selection and combination. Extract 14.2 and 14.3 show the samples of good responses from the candidates.

Extract 14.2

14 (i)



(ii) $P(E) = \frac{n(E)}{n(S)}$

$n(S) = 8$

$n(E) = 7$

(ii) $P(\text{at most 2 heads}) = \frac{7}{8}$

$\therefore$ The Probability is $7/8$

(b) from ${}^nC_r$

Combination $= {}^nC_r$

$= {}^6C_3 + {}^5C_3 + {}^5C_3$

${}^nC_r = \frac{n!}{(n-r)!\,r!}$

In Extract 14.2, the candidate was able to draw the tree diagram correctly and use it to identify the number of elements of sample space and events in order to calculate the probability of getting at most two heads.

Extract 14.3

(b) from ${}^{n}C_{r}$

combination $= {}^{n}C_{r}$

$= {}^{6}C_{3} \times {}^{5}C_{3} \times {}^{5}C_{3}$

${}^{n}C_{r} = \dfrac{n!}{(n-r)!\,r!}$

$= \dfrac{6!}{(6-3)!\,3!} \times \dfrac{5!}{(5-3)!\,3!} \times \dfrac{5!}{(5-3)!\,3!}$

$\dfrac{6\times5\times4\times3!}{3!\,3!} \times \dfrac{5\times4\times3!}{2!\times3!} \times \dfrac{5\times4\times3!}{2!\times3!}$

$\dfrac{6\times5\times4}{3\times2} \times \dfrac{5\times4}{2\times1} \times \dfrac{5\times4}{2\times1}$

$5\times4 \times 5\times2 \times 5\times2$

$20 \times 10 \times 10$

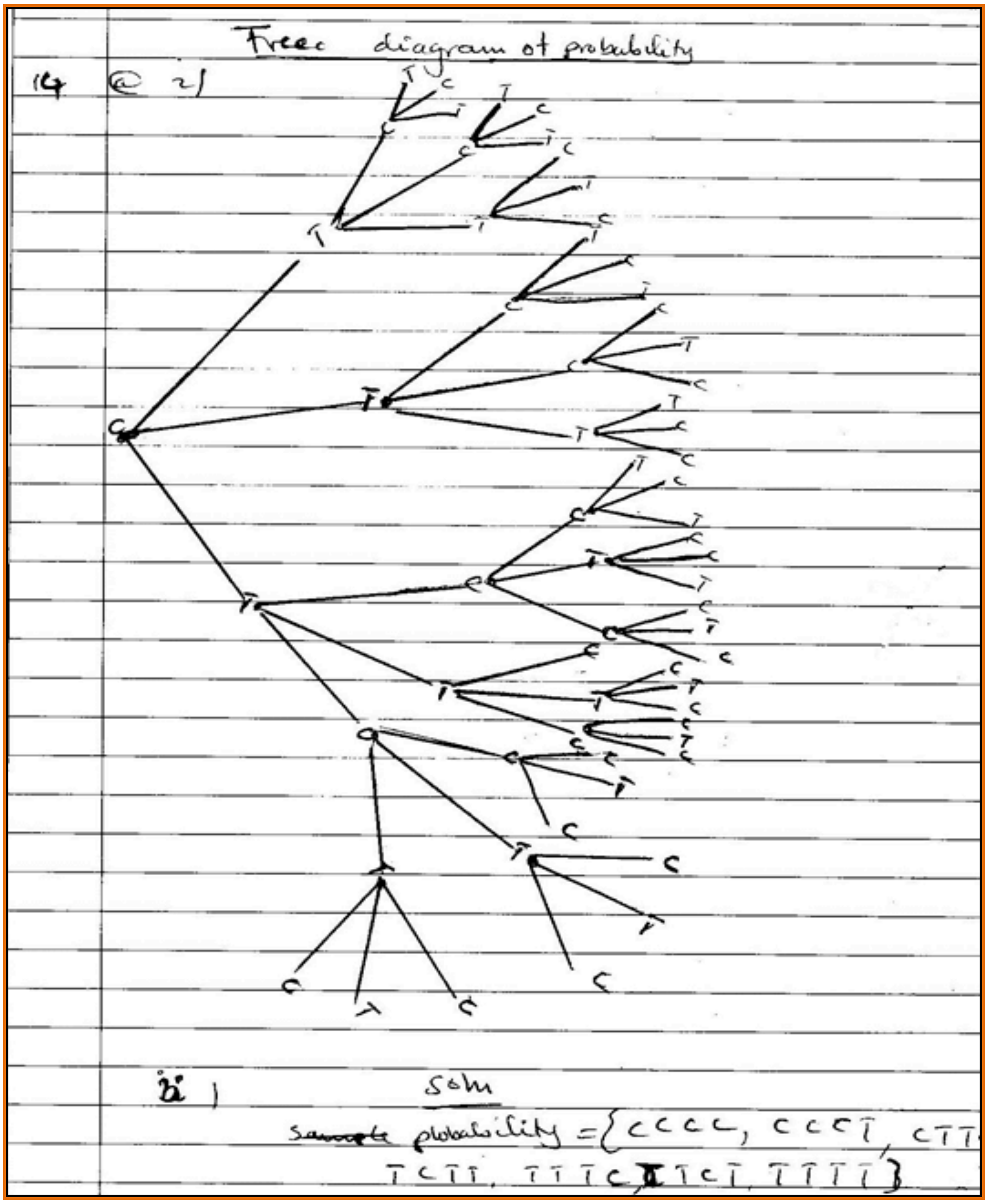
2000

$\therefore$ There are 2000 ways of selectio

In Extract 14.3, the candidate managed to use the formula of combination to find the required number of ways of selecting the 9 balls that consists of three balls of each colour.

Some of the candidates who failed to attempt part (a), of the question lacked knowledge and skills on tree diagram as applied in solving probability problems. Others confused the term “three unbiased coins’, so they drew tree diagram that involved the numbers 1, 2, 3 and H, T for Head and Tail respectively. They drew a tree diagram that starts with three instead of two branches. Likewise, in part (b) the majority of the candidates were unable to interpret the word “Combination” as used for selection of items in a given condition. Some of the candidates confused the concept of combination with permutation in which they wrote ${}^{n}c_{r} = \frac{n!}{(n-r)!}$ instead of ${}^{n}c_{r} = \frac{n!}{r!(n-r)!}$. The sample of candidates' poor response is illustrated in Extracts 14.4 and 14.5.

Extract 14.4



In Extract 14.4, the candidate wrongly drew a tree diagram starting with three branches in three and four steps instead of two branches in three steps.

Extract 14.5

(14 b) Solution

$9+6+5+5+3 = 28$ balls

$P = \frac{S(n)}{S(E)}$

$= \frac{28}{5}$

$= 5.6$

$\therefore$ probability $= 5.6$

In Extract 14.5, the candidate wrongly found the sum of the balls and wrote a funny formula ending up with wrong answers.

2.15 Question 15: Vectors, Matrices and Linear Transformations

The question had parts (a), (b) and (c). In part (a), the candidates were required to find $\underline{a}\times(\underline{b}\times\underline{c})$ given that $\underline{a} = \underline{i} + \underline{j} + \underline{k}$, $\underline{b} = \underline{i} - \underline{j} - \underline{k}$ and $\underline{c} = \underline{i} - 2\underline{j} + 3\underline{k}$. In part (b), they were required to solve the following system of simultaneous equations by substitution method;

$3y + 2x = z + 1$

$3x + 2z = 8 - 5y$

$3z - 1 = x - 2y$

In part (c), the candidates were required to find the image of point R (1,1) under the transformation matrix equation $(x', y') = \begin{pmatrix} 2 & 0 \\ 0 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$.

This question was attempted by 338 candidates whereby 56.8 percent scored above 2.5 out of 10 marks with 45 (13.3%) candidates scoring full marks.

The candidates who scored good marks were able to answer correctly two or all the three parts of the question which is evidence that, they had adequate knowledge and skills in the topic of Vectors, Matrices and Linear Transformation. The samples of candidate' good responses are illustrated in Extracts 15.1, 15.2 and 15.3.

Extract 15.1

15 a) $\underline{a}\times(\underline{b}\times\underline{c})$

$$\underline{b}\times\underline{c} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 1 & -1 & -1 \\ 1 & -2 & 3 \end{vmatrix}$$

$$= \underline{i}\begin{vmatrix} -1 & -1 \\ -2 & 3 \end{vmatrix} - \underline{j}\begin{vmatrix} 1 & -1 \\ 1 & 3 \end{vmatrix} + \underline{k}\begin{vmatrix} 1 & -1 \\ 1 & -2 \end{vmatrix}$$

$$= \underline{i}(-3 \ -2) - \underline{j}(3+1) + \underline{k}(-2+1)$$

$$= -5\underline{i} - 4\underline{j} + (-1)\underline{k}$$

$$= -5\underline{i} - 4\underline{j} - \underline{k}$$

$$\underline{a}\times(\underline{b}\times\underline{c}) = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 1 & 1 & 1 \\ -5 & -4 & -1 \end{vmatrix}$$

$$= \underline{i}\begin{vmatrix} 1 & 1 \\ -4 & -1 \end{vmatrix} - \underline{j}\begin{vmatrix} 1 & 1 \\ -5 & -1 \end{vmatrix} + \underline{k}\begin{vmatrix} 1 & 1 \\ -5 & -4 \end{vmatrix}$$

$$= \underline{i}(-1+4) - \underline{j}(-1+5) + \underline{k}(-4+5)$$

$$\therefore \underline{a}\times(\underline{b}\times\underline{c}) = 3\underline{i} - 4\underline{j} + \underline{k}$$

In Extract 15.1, the candidate demonstrated clearly the procedure to be followed on solving cross product problems.

Extract 15.2

b) $2x+3y-z=1 \quad \cdots (i)$

$3x+5y+2z=8 \quad \cdots (ii)$

$-x+2y+3z=1 \quad \cdots (iii)$

From (iii) eqn

$-x+2y+3z=1$

$x=2y+3z-1$

Sub. $x=2y+3z-1$ in (ii) eqn

$3x+5y+2z=8$

$3(2y+3z-1)+5y+2z=8$

15	b)	$y+z=1 \ldots (iv)$
		$y=1-z$
		Sub $y=1-z$ in (i) eq.
		$2x+3(1-z)-z=1$
		$2x+3-3z-z=1$
		$2x-4z=1-3$
		$2x-4z=-2$
		$x=-1+2z \quad \ldots (v)$
		Sub $x=-1+2z$ in (iii) eq.
		$-x+2y+3z=1$
		$-(-1+2z)+2y+3z=1$
		$1-2z+2y+3z=1$
		$1+2y+z=1$
		$z=-2y \quad . (vi)$
		Sub $z=-2y$ in (iv) eq.
		$y+z=1$
		$y-2y=1$
		$\therefore y=-1$
		Sub $y=-1$ in (vi) eq.
		$2y+z=0$
		$2(-1)+z=0$
		$\therefore z=2$
		Sub $z=2$ in (v) eq.
		$x-2z=-1$
		$x-2(2)=-1$
		$x-4=-1$
		$\therefore x=3$

In Extract 15.2, the candidate managed to use substitution method to find the value of x, y, and z in the system of simultaneous equations.

Extract 15.3

15 c) $\begin{pmatrix}x\\y\end{pmatrix}' = \begin{pmatrix}2 & 0\\0 & -2\end{pmatrix}\begin{pmatrix}1\\1\end{pmatrix}$

$= \begin{pmatrix}2+0\\0-2\end{pmatrix}$

$\begin{pmatrix}x\\y\end{pmatrix}' = \begin{pmatrix}2\\-2\end{pmatrix}$

$\therefore$ The image of point P(1, 1) under transformation is (2, −2)

In Extract 15.3, the candidate used the formula provided to find the image of the point (1, 1) as it was required.

However, the candidates who performed poorly in this question were unable to find the cross product in part (a). Some of them failed to add the product of the elements in the leading diagonal to the negative of the product of the minor diagonal. Some of them found the dot product showing that, they lacked knowledge of cross product. In part (b), most of the candidates performed poorly due to lack of the knowledge of algebra. They were unable to multiply, open the brackets and add, especially the terms with negative signs. In part (c) also, they made errors in multiplying 2×1 and 2×2 matrices. A sample of a poor answer to this question is illustrated in Extract 15.4.

Extract 15.4

15a) To given that $a = i + j + k$, $b = -1 - j - k$ and $c = i - 2j + 3k$

to find $a \times (b \times c)$

To Calculate.

$= a \times (b \times c)$

$= a \times b \times b \times c$

$= a \times 2b \times c$

$= a \times 2bc$

$= 2abc$

To the given that a = i + 1 + k, b = 1 - j - k and
c = i, 2j + 3k a × (b × c) =
a × (b × c) = 2abc
b. To solve the following system of ~~ef~~ simultaneous
equation by substitution methode.
To Calculate.
3y + 2x = z + 1
= 3yx = z + 1
= 3yx = z1.
3x + 2z = 8 - 5y
= 5x + z = 8 - 5y
= 5x + 8 - 5y
= 14x - 5y.
3x - 1 = x - 2y
= 2x = x - 2y
= 2x = x - y
= 2x = y

In Extract 15.4, the candidate wrongly applied the distributive property on $\underline{a}\times(\underline{b}\times\underline{c})$ which does not apply in finding the cross product of vectors.

2.16 Question 16: Differentiation and Integration

The question had three parts; (a), (b) and (c). In Part (a) the candidates were required to find y in terms of x in the equation $\frac{dy}{dx} = 3x^3 - 4x^2 + 5x + 1 + \frac{1}{x^2}$. In part (b), they were asked to find the value of the definite integral $\int_0^{\frac{\pi}{2}} (\cos x + 2\cos 2x)\,dx$. In part (c), they were instructed to use product rule to find the derivative of the expression $(2 - x^2)(3x + x^2)$.

The question was attempted by 166 (41.4%) candidates whereby 80.1 percent scored 3 or more marks with a few candidates scoring full 10 marks. On the other hand, 19.9 percent scored below 3 marks with 9.0 percent scoring a 0 mark. Generally, the performance of candidates in this question was good as most of them scored above the pass mark.

Many candidates were able to answer correctly two or all the three parts of the question indicating that, they had adequate knowledge on calculus. The Extracts 16.1 and 16.2 show examples of good responses.

Extract 16.1

16a.

$$\frac{dy}{dx} = 3x^3 - 4x^2 + 5x + 1 + \frac{1}{x^2}$$

$$\int dy = \int \left(3x^3 - 4x^2 + 5x + 1x^0 + x^{-2}\right) dx$$

$$y = \frac{3x^{3+1}}{3+1} - \frac{4x^{2+1}}{2+1} + \frac{5x^{1+1}}{1+1} + \frac{x^{0+1}}{0+1} + \frac{x^{-2+1}}{-2+1} + c$$

$$= \frac{3}{4}x^4 - \frac{4}{3}x^3 + \frac{5}{2}x^2 + x + \frac{x^{-1}}{-1} + c$$

$$= \frac{3}{4}x^4 - \frac{4}{3}x^3 + \frac{5}{2}x^2 + x - \frac{1}{x} + c$$

$$\therefore y = \frac{3}{4}x^4 - \frac{4}{3}x^3 + \frac{5}{2}x^2 + x - \frac{1}{x} + c$$

In Extract 16.1, the candidate re-arranged the given equation and then integrated and made y the subject of the equation as it was required.

Extract 16.2

16b.

$$= \int_0^{\pi/2} (\cos x + 2\cos 2x)\, dx$$

$$= \int_0^{\pi/2} \cos x + 2\int_0^{\pi/2} \cos 2x\, dx$$

$$= \cancel{\int_0^{\pi/2}} \left|\sin x\right|_0^{\pi/2} + 2\int_0^{\pi/2} \cos 2x\, dx$$

$$\text{let } t = 2x$$
$$dt/dx = 2$$
$$dt/2 = dx$$
$$= \left|\sin x\right|_0^{\pi/2} + 2\left|\cos t \, dt/2\right.$$
$$= \left|\sin x\right|_0^{\pi/2} + \left|\sin 2x\right|_0^{\pi/2}$$
$$= (\sin \pi/2 - \sin 0) + (\sin 2(\pi/2) - \sin 0)$$
$$(1 - 0) + (0 - 0)$$
$$= 1$$
$$\therefore \int_0^{\pi/2} \cos x + 2\cos 2x \, dx = 1.$$

In Extract 16.2, the candidate integrated clearly and then substituted the given limits correctly.

Extract 16.3

16 c

$$y = (2 - x^2)(3x + x^2)$$
$$\text{let } u = \cancel{2x} \; 2 - x^2 \qquad du/dx = -2x$$
$$\text{let } v = 3x + x^2 \qquad dv/dx = 3 + 2x$$
$$dy/dx = v\frac{du}{dx} + u\frac{dv}{dx}$$
$$= (3x + x^2)(-2x) + (2 - x^2)(3 + 2x)$$
$$= -6x^2 - 2x^3 + [6 + 4x - 3x^2 - 2x^3)$$
$$= -6x^2 - 2x^3 + 6 + 4x - 3x^2 - 2x^3$$
$$dy/dx = -4x^3 - 9x^2 + 4x + 6$$

In Extract 16.3, the candidate correctly applied the product rule to differentiate the expression $(2 - x^2)(3x + x^2)$.

The few candidates who scored low marks were unable to find y in terms of x in part (a) of the question. This implies that, they failed to re-arrange and integrate the given equation. Also, they failed to relate the knowledge and skills of differentiation and that of integration. In part (b), some of them were able to integrate the given expression but were unable to substitute the limits. This shows that, the candidates lacked the knowledge and skills of limits when dealing with integration. In part (c), they were unable to recall and apply the product rule as required for differentiation. Some candidates (9.0%) failed to answer all parts of the question as illustrated by the sample work from one of them in Extract 16.4.

Extract 16.4

16 (a) Soln

$$dy/dx = 3x^3 - 4x^2 + 5x + 1 + \frac{1}{x^2}$$

let $dy/dx = y$ 0 y

$$\cancel{y = 3x^3 - 4x^2 + 5x + 1 + 1/x^2 = 0}$$

$$\cancel{3y^3 - 4y^2 + 5y + 1 + 1/x^2 = 0}$$

$$y = 3x^3 - 4x^2 + 5x + 1 + 1/x^2$$

$$y = 9x^2 - 8x + 5 + \frac{1}{2x}$$

$$dy/dx = 9x^2 - 8x + 5 + \frac{1}{2x}$$

In Extract 16.4, the candidate expanded the terms of the given expression instead of rearranging and integrating the given equation.

Extract 16.5

16 (b) $\int_0^{\frac{\pi}{2}} (\cos x + 2\cos 2x)\,dx$

$\int_0^{90} \cos x\,dx + \int_0^{90} 2\cos 2x\,dx$

$\int_0^{90} \cos\frac{x^2}{2} + 2\int_0^{90} \cos \cancel{2}\frac{x^2}{2}\,dx$

$\cos\frac{x^2}{2} + 2\cos x^2 \Big|_0^{90}$

as $x = 90^\circ$

$\frac{\cos(90)^2}{2} + 2\cos(90)^2$

$\cos 90 = 0$

as $x = 0$

$\cos\frac{0}{2} + 2\cos 0$

$1/2 + 2\times 1$

$1/2 + \frac{2}{1} = \frac{1+4}{2} = 5/2$

$0 - 5/2 = -5/2$

$\therefore \int_0^{\frac{\pi}{2}} (\cos x + 2\cos 2x)\,dx = -5/2$

In extract 16.5, the candidate integrated the variable *x* which is a function of the trigonometric expression instead of integrating the trigonometric expressions.

3.0 ANALYSIS OF CANDIDATES' PERFORMANCE IN DIFFERENT TOPICS

In this subject, each topic was tested in one question, except the topic of Differentiation which was examined in question 7 and 16. The performance of the candidates in a topic is the average performance in the questions from that topic. The performance was good in the topics of Statistics, Symmetry, Geometrical Constructions, Equations, Remainder Theorem, Functions; Coordinate Geometry, Sets, Numbers, Variation, Algebra, Logic, Differentiation, Integration, Plan and Elevations, Permutations, Combinations, Probability, Vectors, Matrices, Linear Transformation and Trigonometry. Locus was the only topic which had poor performance while there was no topic which had average performance (see **Appendix A**).

The comparison of performance of candidates in 2014 and 2015 shows that, the performance has increased in the topics of Sets, Statistics, Symmetry and Geometrical Constructions, Coordinate Geometry, Logic, Plan and Elevations, Vectors and Locus. On the other hand, the performance has decreased in the topics of Numbers, Variation, Equations, Remainder Theorem, Functions, Algebra, Permutations and Combinations, Probability, Differentiation, Integration and Trigonometry. Generally the performance in 2015 is slightly lower than that of 2014 (see **Appendix B** and **C**).

The slight decrease in performance of candidates in the stated topics was caused by lack of knowledge and skills in those topics as well as failure to recall and apply mathematical rules, theorems and formulae in answering the questions to some candidates. Also, the performance was slightly low due to candidates' inability to understand and use English Language to identify the requirements of the question. Therefore, the teaching and learning activities should focus on these topics in order to increase candidates' performance in future.

4.0 CONCLUSION AND RECOMMENDATIONS

Conclusion

The analysis of the performance of candidates showed that, most of the candidates were able to recall and apply mathematical rules, theorems and formulae in answering the questions; hence they scored high marks in most questions.

On the other hand, the candidates who scored low marks lacked the knowledge and skills on the topics which were examined as they were not able to recall and apply mathematical rules, theorems and formulae in answering the questions. For instance, in question 9, the candidate misconceived the rule of finding the locus with the formula for finding midpoint, as a result he/she found midpoint instead of locus.

The candidates' performance was good in 15 questions and weak in 1 question (see **Appendix A**). It was highest in question 12, while it was lowest in question 9. Generally, the candidates' performance in Additional Mathematics was good.

Recommendations

In order the candidates to perform more better in future Additional Mathematics examinations, the following are the recommendations;

(a) Students should study all the topics in the syllabus and make sure that, they understand the underlying concepts, theorems, rules and formulae. Also, they should possess the ability to apply them in solving problems.

(b) Teachers should make sure that, they provide many exercises to candidates and guide them in solving the questions. They should also assist the candidates to make good preparation for Examinations by encouraging them to read books and do many exercises. Teachers should employ formative assessments before, during and after instruction in order to monitor the whole learning process and identify candidates who may have learning difficulties so as to provide a special attention to help them.

Appendix A

A Summary of Candidates Performance Topic Wise (2015)

S/N	Topic	Number of Question(s)	Average Percentage of performance of candidates who Scored 30% or more	Remarks
1	Statistics	1	90.2	Good
2	Symmetry/ Geometrical Constructions	1	86.5	Good
3	Equations & Remainder Theorem/ functions	1	86.2	Good
4	Coordinate Geometry	1	85.4	Good
5	Sets	1	83.8	Good
6	Numbers	1	82.9	Good
7	Variations	1	77.8	Good
8	Algebra	1	77.2	Good
9	Logic	1	75.9	Good
10	Differentiation/ Integration	2	71.15	Good
11	Plan and Elevations	1	66.5	Good
12	Permutations and Combinations/ Probability	1	63.0	Good
13	Vectors/ Matrices and Linear transformation	1	56.8	Good
14	Trigonometry	1	46.3	Good
15	Locus	1	14.4	Poor
Overall Average Performance			**70.94**	**Good**

Appendix B

A Summary Showing the Comparison of the Candidates' Performance in 2014 and 2015

S/N	Topic	Number of Question (s)	Average performance of candidates who Scored 30% or more	
			2014	2015
1	Sets	1	83.7	83.8
2	Numbers	1	83.7	82.9
3	Variations	1	91.8	77.8
4	Equations & Remainder Theorem/ functions	1	86.5	86.2
5	Statistics	1	83.4	90.2
6	Algebra	1	79.4	77.2
7	Permutations and Combinations/ Probability	1	74.3	63.0
8	Differentiation/ Integration	2	73.6	71.15
9	Trigonometry	1	67.9	46.3
10	Symmetry/ Geometrical Constructions	1	65.0	86.5
11	Coordinate Geometry	1	59.7	85.4
12	Logic	1	55.9	75.9
13	Plan and Elevations	1	45.2	66.5
14	Vectors/ Matrices and Linear transformation	1	43.2	56.8
15	Locus	1	1.8	14.4

Appendix C

A Histogram Showing the Comparison of the Candidates' Performance in 2014 and 2015