THE NATIONAL EXAMINATIONS COUNCIL OF TANZANIA

CANDIDATES' ITEM RESPONSE ANALYSIS REPORT FOR THE CERTIFICATE OF SECONDARY EDUCATION EXAMINATION (CSEE) 2018

041 BASIC MATHEMATICS

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FOREWORD

The National Examinations Council of Tanzania is delighted to issue this report on performance of candidates in Basic Mathematics examination for the Certificate of Secondary Education Examination (CSEE) 2018. The report was prepared in order to give feedback to students, teachers, policy makers and the public in general about the performance of the candidates.

The analysis of data revealed that, the candidates had good performance in the question that was set from the topic of *Accounts* and had an average performance in the questions that were set from the topics of: *Statistics, Linear Programming*, *Numbers*, *Fractions* and *Decimals*. Further analysis showed that, the candidates had weak performance in the questions that were set from the topics of: *Rates and Variations*; *Matrices and Transformations*; *Ratios, Profit and Loss; Sets*; *Algebra*; *Pythagoras Theorem*; *Trigonometry*; *Sequences and Series*; *Quadratic Equations*; *Exponents*; *Logarithms*; *Functions*; *Probability*; *Circles*; *Three Dimensional Figures*; *Vectors*; *Coordinate Geometry* and *Perimeters and Areas*.

The weak performance was contributed by several factors including the candidates' inability to: use laws, formulae, theorems and other mathematical concepts in answering the questions; identify the requirements of the questions; formulate expressions, equations and inequalities from the word problems; sketch the correct diagrams and graphs which were useful in answering the questions; and perform mathematical operations correctly.

The National Examinations Council of Tanzania expects that this report will be useful in improving the candidates' performance in future Basic Mathematics examinations.

Lastly, the National Examinations Council would like to thank the examiners, examination officers and all other personnel who participated in preparing this report. The Council will highly be grateful to receive constructive comments from education stakeholders to improve future reports.

Dr. Charles Msonde
EXECUTIVE SECRETARY

1.0 INTRODUCTION

This report is based on the analysis of the candidates' items responses in the 041 Basic Mathematics examination for CSEE 2018. The analysis mainly addresses the areas in which the candidates faced difficulties including the areas in which they performed well when answering the examination items.

The examination paper consisted of sections A and B, with a total of 16 questions. Section A had 10 questions each carrying 6 marks, whereas section B had 6 questions each carrying 10 marks. The candidates were required to answer all questions in section A and 4 questions from section B. For a candidate who answered more than 4 questions in section B, only the first 4 questions according to the order of his or her responses in the answer booklet were marked. The extra questions that the candidate answered were out of rubric, and were ignored forthwith.

In 2018, a total of 360,225 candidates sat for the 041 Basic Mathematics examination out of which, 71,703 (20.02%) candidates passed. In 2017, a total of 317,444 candidates sat for the 041 Basic Mathematics examination out of which, 60,621 (19.19%) candidates passed. This indicates that the performance in 2018 increased by 0.83 percent.

Section 2 of this report summarizes the analysis of the candidates' performance in each question. The analysis includes; descriptions of the requirements of the items, summary on how the candidates answered the items of each question, sample extracts of correct and incorrect responses and the reasons for good, average and weak performance in each question.

The candidates' performance was categorized by using the percentage of candidates who scored at least 30 percent of the marks allocated to a particular question. The performance was classified into three groups, that is "65 – 100" for *good* performance, "30 – 64" for *average* performance and "0 – 29" for *weak* performance as shown in the Figures and the Appendix using *green*, *yellow* and *red* colours respectively.

2.0 ANALYSIS OF CANDIDATES' PERFORMANCE IN EACH QUESTION

2.1 Question 1: Numbers, Fractions and Decimals

This question consisted of parts (a) and (b). In part (a), the candidates were required to find the value of $\frac{n}{m}$ in its simplest form given that $m = 0.\dot{2}\dot{7}$ and $n = 0.\dot{1}\dot{5}$. In part (b), they were required to find the GCF of 210, 357 and 252.

The performance of candidates in this question is summarized and presented in Figure 1. The figure shows that, 119,838 (33.3%) candidates scored from 2 to 6 marks. Therefore, this question had average performance.

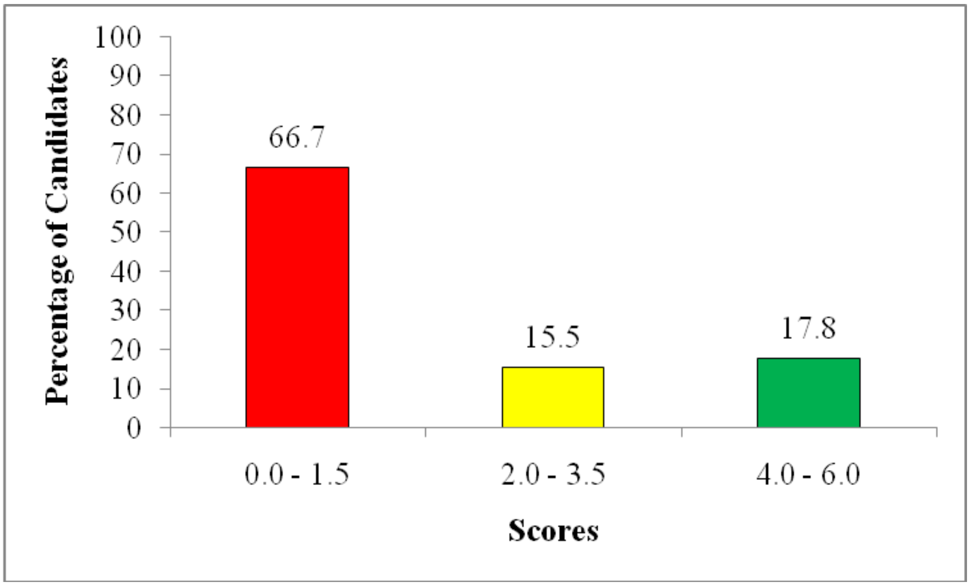


***Figure 1**: Candidates' Performance in Question 1.*

Further analysis shows that, 19,057 (5.3%) candidates answered this question correctly. The candidates were able to convert repeating decimals into fractions, divide fractions, simplify fractions and find the GCF by using either the listing method or prime factorization method. A sample answer from one of the candidates is shown in Extract 1.1.

1.a. Given $m = 0.\dot{2}\dot{7}$ $\frac{n}{m}$.

$n = 0.\dot{1}\dot{5}$

Let $m = 0.\dot{2}\dot{7}$ - (i)

Multiply by 100 both sides.

$100m = 27.\dot{2}\dot{7}$ - - (ii)

Subtract equation (i) from (ii)

$100m - m = 27.\dot{2}\dot{7} - 0.\dot{2}\dot{7}$

$\frac{99m}{99} = \frac{27}{99}$

$m = \frac{3}{11}$

Let $n = 0.\dot{1}\dot{5}$ - - (i)

Multiply by 100 both sides.

$100n = 15.\dot{1}\dot{5}$ - - (ii)

Subtract equation (i) from (ii)

$100n - n = 15.\dot{1}\dot{5} - 0.\dot{1}\dot{5}$

$\frac{99n}{99} = \frac{15}{99}$

$n = \frac{5}{33}$

$\frac{n}{m} = \frac{5/33}{3/11}$

$= \frac{5}{33} \div \frac{3}{11}$

$= \frac{5}{\cancel{33}\,3} \times \frac{\cancel{11}\,1}{3}$

$= \frac{5}{9}$.

$\therefore \frac{n}{m} = \frac{5}{9}$

(b.) GCF of 210, 357 and 252.

2	210	357	252
2	105	357	126
3	105	357	63
3	35	119	21
7	35	119	7
5	5	17	1
17	1	17	1
	1	1	1

GCF $= 3 \times 7$

$= 21$.

$\therefore$ GCF of 210, 357 and 252 $= 21$.

Extract 1.1: A sample response from a candidate who answered this question correctly.

On the other hand, 240,385 (66.7%) candidates who attempted this question scored low marks ranging from 0 to 1.5, out of which 184,239 (51.1%) candidates scored 0 mark.

In part (a), the candidates who scored zero could not express $m = 0.\dot{2}\dot{7}$ and $n = 0.\dot{1}\dot{5}$ as fractions. These candidates lacked knowledge of repeating decimals. For example, some of them wrote $m = 0.\dot{2}\dot{7} = \frac{27}{100}$ and $n = 0.\dot{1}\dot{5} = \frac{15}{100}$ while others wrote $\frac{n}{m} = \frac{0.\dot{1}\dot{5}}{0.\dot{2}\dot{7}}$. Others multiplied the given equations by 10^3 instead of 10^2, which was an important step in changing the repeating decimals into fractions.

Further analysis shows that, some candidates correctly got $m = \frac{3}{11}$ and $n = \frac{5}{33}$ but they wrongly evaluated $\frac{n}{m}$. For example, instead of writing $\frac{n}{m} = \frac{5}{33} \div \frac{3}{11} = \frac{5}{33} \times \frac{11}{3} = \frac{5}{9}$, most of them wrote $\frac{n}{m} = \frac{3}{11} \div \frac{5}{33} = \frac{3}{11} \times \frac{33}{5} = \frac{9}{5}$ or $\frac{n}{m} = \frac{5}{33} \times \frac{3}{11} = \frac{5}{121}$ indicating lack of knowledge and skills in dividing fractions.

In part (b), many candidates could not express the numbers as a product of prime factors, and ended up with incorrect answers. For example, several candidates faced difficulties in dividing the given numbers especially 357 and 252 by 3 or 7. Other candidates confused between GCF and LCM. They wrote $GCF = 2 \times 2 \times 3 \times 3 \times 5 \times 7 \times 17$ =21,420 instead of $GCF = 3 \times 7 = 21$ as required. Extract 1.2 shows a sample response of a candidate who failed to answer this question correctly.

1\. (a) $M = 0.\dot{2}\dot{7}$ and $n = 0.\dot{1}\dot{5}$

soln.

$M = 0.\dot{2}\dot{7} \times 27$

$= 0.\dot{2}\dot{7}.27$

$100x - x = 99x - 27.27$

$\frac{99x}{27} = \frac{\cancel{27}}{\cancel{27}}$

$x = 4.44$ answer

$n = 0.\dot{1}\dot{5}$

soln

$n = 0.\dot{1}\dot{5}$

$0.15 - 15.15$

$100x - x = 15.15 - 15$

$\frac{99x}{15} = \frac{15}{15}$

$x = 6.6$ answer.

$M = 4.44$, $n = 6.6$. answer

Fraction $= 6.6/4.44$.

1 b)

1	210	357	252
2	210	357	252.
2	105	357	146
3	105	357	73
5	35	119	73
7	7	119	73
73	1	119	73
199	1	119	1
	1	1	1

G.C.F = 1.

Extract 1.2: A sample response of a candidate who could not express m and n as fractions and was also unable to correctly divide 252 by 2, hence ended up with incorrect GCF.

2.2 Question 2: Exponents and Logarithms

This question had parts (a) and (b). In part (a), the candidates were required to evaluate $\log_{10} 40500$ given that $\log_{10} 2 = 0.3010$, $\log_{10} 3 = 0.4771$ and

$\log_{10} 5 = 0.6990$. In part (b), they were required to solve for x and y from the equation $\frac{3^{x+2}}{5^{2y-8}} = 2025$.

The summary of the candidates' performance in this question is presented in Figure 2. The figure shows that, 36,718 (10.2%) candidates scored from 2 to 6 marks. This reveals that, the candidates had weak performance. Also, 323,510 (89.8%) candidates scored marks from 0 to 1.5, out of which 303,837 (84.3%) candidates scored 0 mark.

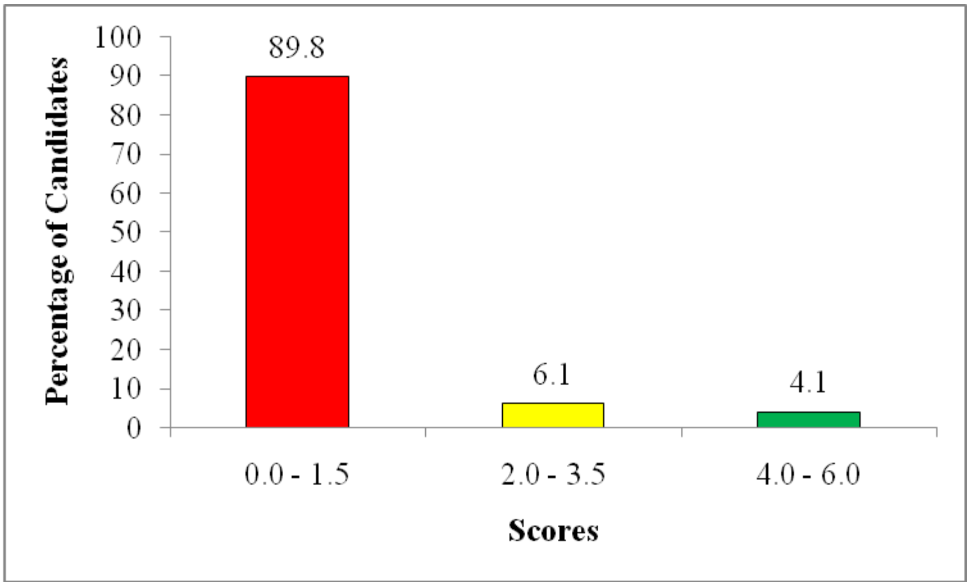


***Figure 2**: Candidates' Performance in Question 2.*

In part (a), most of the candidates were unable to express 40500 as a product of prime factors in exponential form, that is, $40500 = 2^2 \times 3^4 \times 5^3$ which was an essential step to arrive at the required answer. There were some candidates who were able to write 40500 as $2^2 \times 3^4 \times 5^3$ but failed to correctly apply the product rule and the power rule in logarithms. For example, some of the mistakes noted included expressing $\log_{10} 40500$ as: $2^2 \times 3^4 \times 5^3$; $0.3010 + 0.4771 + 0.6690$; $\log_{10} 405 \times \log_{10} 100$ and $\log_{10} 2^2 \times \log_{10} 3^4 \times \log_{10} 5^3$ instead of $\log_{10} 2^2 + \log_{10} 3^4 + \log_{10} 5^3$. Moreover, there were candidates who managed to express $\log_{10} 40500$ as $2\log_{10} 2 + 4\log_{10} 3 + 3\log_{10} 5$ but failed to correctly substitute $\log_{10} 2 = 0.3010$, $\log_{10} 3 = 0.4771$ and $\log_{10} 5 = 0.6990$ to get the required answer. Others were able to make correct substitution but unable to perform mathematical operations of the resulting expression, that is $2\times 0.3010+4\times 0.4771+3\times 0.6990$.

In part (b), the majority of candidates were able to rewrite $\frac{3^{x+2}}{5^{2y-8}}=2025$ as $\frac{3^{x+2}}{5^{2y-8}}=3^4\times 5^2$ but failed to apply the reciprocal law of exponents, that is $\frac{1}{a^n}=a^{-n}$. They wrote $\frac{3^{x+2}}{5^{2y-8}}=3^{x+2}\times 5^{2y-8}=3^4\times 5^2$ instead of $\frac{3^{x+2}}{5^{2y-8}}=3^{x+2}\times 5^{-(2y-8)}=3^4\times 5^2$. Also, it was noted that some candidates applied the reciprocal law but wrongly opened the brackets in the equation $5^{-(2y-8)}=5^2$ as they wrote $-2y-8=2$ instead of $-2y+8=2$ and hence ended up with incorrect value of y.

Further analysis reveals that, there were candidates who could not express 2025 as a product of exponents with bases 3 and 5. For example, some wrote $2025=3^2\times 5^2$ instead of $3^4\times 5^2$. Other misconceptions included: solving $3^{x+2}=2025$ and $5^{2y-8}=2025$ separately; equating the exponents of $\frac{3^{x+2}}{5^{2y-8}}$ to 2025, that is $x+2=2025$ and $2y-8=2025$; and considering the exponents as if they are of the same base because they wrote $\frac{3^{x+2}}{5^{2y-8}}=3^{(x+2)-(2y-8)}$ and $3^4\times 5^2=3^{4+2}$ suggesting that they lacked enough knowledge and skills in solving exponential equations. Extract 2.1 illustrates this case.

2(a) $\log_{10} 40500$

$\log_{10}(4 \times 10{,}000 + 5 \times 100)$

$\log_{10}(4 \times 100^2 + 5 \times 10^2$

$2^2 + 10^4 + 5 + 10^2$

$2\log 2 + 4\log_{10} + \log 3+2 + 2\log 10$

$2(0{\cdot}3010) + 4 \times 1 + \log 0{\cdot}4771 + 0{\cdot}3010 + 2\log_{10} 10$

$0{\cdot}6020 + 4 + 0{\cdot}7781 + 2$

$4{\cdot}6020 + 2{\cdot}7781$

$7{\cdot}3801$

Therefore $\log_{10} 40{,}500 = 7{\cdot}3801$ Answer

(b) $\frac{3^{x+2}}{5^{2y-8}} = 2025$

$3(x+2) = 2025$

$3x + ~~6~~ = 2025$

$3x + 6 = 2025$

$3x = 2025 - 6$

$\frac{\cancel{3}x}{\cancel{3}} = \frac{2019}{3}$

$x = 673$

The value of $x = 673$

2. (b) The value of y

$5(2y-8) = 2025$

$10y - 40 = 2025$

$10y = 2025 + 40$

$\frac{\cancel{10}y}{\cancel{10}} = \frac{2065}{10}$

The value of $y = 26{\cdot}5$

∴ The value of $y = 26{\cdot}5$

Extract 2.1: A sample response of a candidate who failed to apply the laws of exponents and logarithms in answering this question.

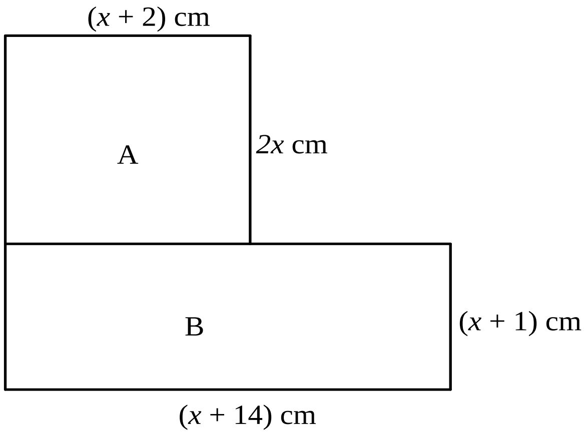
Despite the weak performance, 4,372 (1.2%) candidates answered this question correctly. These candidates had adequate knowledge and skills on the laws of exponents and logarithms as they correctly evaluated $\log_{10} 40500$ and solved the equation $\frac{3^{x+2}}{5^{2y-8}} = 3^4 \times 5^2$ for x and y by comparing the exponents of the same base. A sample response from one of the candidates is shown in Extract 2.2.

2 a)	Soln:
	Given $\log_{10} 40500$
	$\log_{10} 40500 = \log_{10}(2\times2\times3\times3\times3\times3\times5\times5\times5)$
	$= \log_{10}(2^2 \cdot 3^4 \cdot 5^3)$
	but, $\log_{10}(a\times b) = \log a + \log b$
	$\log_{10}(2^2 \cdot 3^4 \cdot 5^3) = \log_{10}2^2 + \log_{10}3^4 + \log_{10}5^3$
	$= 2\log_{10}2 + 4\log_{10}3 + 3\log_{10}5$
	given $\log 2 = 0.3010$, $\log 3 = 0.4771$ and $\log 5 = 0.6990$
	$= 2(0.3010) + 4(0.4771) + 3(0.6990)$
	$= 0.6020 + 1.9084 + 2.097$
	$= 4.6074$
	$\therefore \log_{10} 40,500 = 4.6074$
2b)	Soln
	$\frac{3^{x+2}}{5^{2y-8}} = 2025$
	$3^{x+2} \times (1/5)^{2y-8} = 2025$
	$3^{x+2} \times ((5)^{-1})^{2y-8} = 3^4 \times 5^2$
2b)	$3^{x+2} \times 5^{-2y+8} = 3^4 \times 5^2$
	$3^{x+2} = 3^4$
	$x + 2 = 4$
	$x = 4 - 2$
	$x = 2$
	$5^{-2y+8} = 5^2$
	$-2y + 8 = 2$
	$-2y = 2 - 8$
	$\frac{-2y}{-2} = \frac{-6}{-2}$
	$y = 3$
	$\therefore$ The value of x is 2 and y is 3

Extract 2.2: A sample response of a candidate who answered question 2 correctly.

2.3 Question 3: Sets and Algebra

The question comprised parts (a) and (b). In part (a), it was given that, in a school of 60 teachers, 46 drink Fanta, 18 drink Coca-Cola and 14 drink both Coca-Cola and Fanta. The candidates were required to find the number of teachers who drink neither Fanta nor Coca-Cola by using a Venn diagram. In part (b), the candidates were required to use the following figure to (i) write the expression for the area of rectangles A and B (ii) find the value of *x* if the total area of rectangles A and B is 98 square centimeters.



The candidates' performance in this question is presented in Figure 3. The figure shows that 62,183 (17.3%) candidates who attempted this question scored marks ranging from 2 to 6, whereby 20,290 (5.6%) candidates scored from 4 to 6 marks. Meanwhile, 298,045 (82.7%) candidates got marks ranging from 0 to 1.5 marks, out of which 211,404 (58.7%) candidates scored 0 mark. Generally, candidates had weak performance in this question.

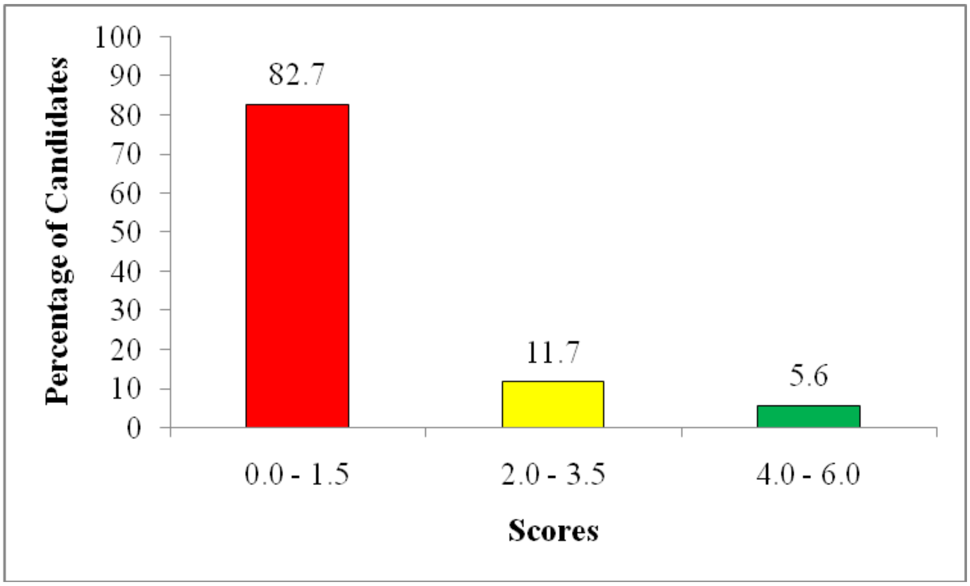
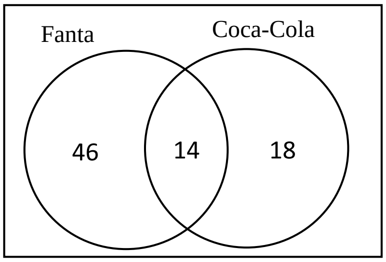
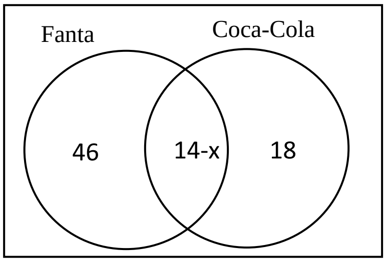
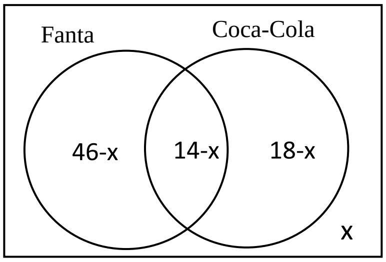


Figure 3: *Candidates' Performance in Question 3.*

The majority of candidates who scored low marks in part (a) could not correctly represent the given information in Venn diagram. The following are incorrect diagrams drawn by some of the candidates:







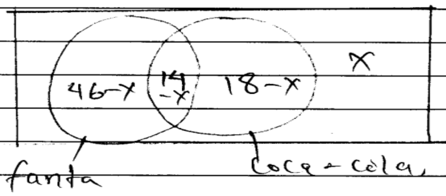
With such misconceptions, these candidates ended up with incorrect number of teachers who drink neither Fanta nor Coca-Cola. Likewise, other candidates did not use Venn diagram as instructed, instead they applied the set formula.

In part (b), many candidates were unable to formulate the correct equation for the total area of the given figure. For example, there were candidates who ignored one of the areas of rectangles A and B as they incorrectly wrote $(x+2)(2x)=98$ or $(x+1)(x+14)=98$. Other candidates could not correctly expand the expressions for the areas of rectangles A and B. Also, there were candidates who multiplied the areas of rectangles A and B instead of adding them. Others calculated the perimeter of the given figure. It was also noted that, some candidates added the expressions in the given figure, that is, $\left(x+2\right)+2x+\left(x+1\right)+\left(x+14\right)=98$ instead of finding the area as instructed.

Moreover, few candidates were able to get the equation for the total area of the given figure, that is, $3x^2+19x-84=0$ but failed to correctly solve it. For example, some of candidates who opted to use the quadratic formula were not able to perform the basic mathematical operations in $x=\dfrac{-19\pm\sqrt{361-4\times3\times(-84)}}{2\times3}$ especially in evaluating the square root.

Extract 3.1 is a sample response of a candidate who failed to answer this question correctly.

3(a)



$46+14+18+x = 60$

$46-x+14-x+18-x+x = 60$

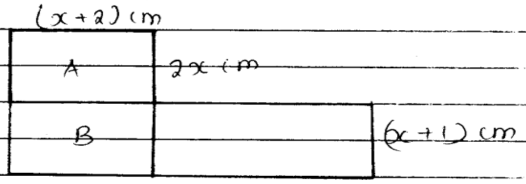
$78-2x = 60$

$\frac{-2x}{-2} = \frac{-12}{-2}$

$x = 9.$

$\therefore$ Therefore 9 donot drink fanta or cocacolla

(b) Given that;



(i) solution

$(x+2)$cm $+ (2x)$cm

$2x+2x$cm $+ x+1$cm

$= 4x\,cm^2 + x$

$= 5x\,cm^3$

(ii) $x+2+2x+x+1 = 98$

$x+2x+x+2+1 = 98$

$4x+3 = 98$

$4x = 98-3$

$\frac{4x}{4} = \frac{95}{4}$

$x = 23.75$

$\therefore x = 23.75\,cm^2$

Extract 3.1: Incorrect response of a candidate who lacked knowledge and skills on the use of Venn diagram and area of rectangles.

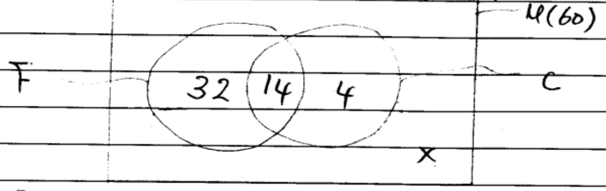
On the other hand, 2311 (0.6%) candidates who answered this question got it correct. These candidates were able to correctly draw the Venn diagram representing the given information and got the required number of teachers who drink neither Fanta nor Coca-Cola. These candidates had adequate knowledge and skills in solving set problems using Venn diagrams as illustrated in Extract 3.2. Also, these candidates demonstrated good understanding on the concept of the area of a rectangle which enabled them to get the equation for the total area, that is, $x^2+19x-84=0$ and solved it as required.

3. a) Solution

Let: F stand for drinkers of fanta

C Stand for drinkers of coca-cola

X stand for those who do not drink



$32+14+4+x=60$

$50+x=60$

$x=60-50$

$x=10$

$\therefore$ 10 teachers drink neither Fanta nor Coca-Cola

b) i) Solution

For rectangle A:

Area $= L\times W = (x+2)\,2x \text{ cm}^2$

$= 2x^2+4x \text{ cm}^2$

For rectangle B:

Area $= L\times W = (x+14)(x+1) \text{ cm}^2$

$= x^2+14x+x+14 \text{ cm}^2$

$= x^2+15x+14 \text{ cm}^2$

Total area $= (2x^2+4x+x^2+15x+14) \text{ cm}^2$

$= (3x^2+19x+14) \text{ cm}^2$

$\therefore$ Total area is given by $3x^2+19x+14 \text{ cm}^2$

ii) Solution

Given: $3x^2+19x+14=98$

$3x^2+19x+14-98=0$

$3x^2+19x-84=0$

3	From : $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
	$= \frac{-19 \pm \sqrt{361 - (4 \times 3 \times -84)}}{6}$
	$= \frac{-19 \pm \sqrt{361 + 1008}}{6}$
	$= \frac{-19 \pm \sqrt{1369}}{6}$
	$= \frac{-19 \pm 37}{6}$
	$= \frac{-19+37}{6}$ or $\frac{-19-37}{6}$
	$= 3$ or $-56/6$
	Then : $x = 3$ since we do not have negative length

Extract 3.2: A sample response of a candidate who answered this question correctly.

2.4 Question 4: Vectors and Coordinate Geometry

This question had parts (a) and (b). In part (a), the candidates were given three vectors $\underline{a} = 2x\underline{i} + 3\underline{j}$, $\underline{b} = (x^2 + y)\underline{i} + 4y\underline{j}$ and $\underline{v} = \frac{8}{3}\underline{i} + \frac{25}{12}\underline{j}$, and were required to find x and y if $\underline{v} = \frac{1}{4}\underline{a} + \frac{1}{3}\underline{b}$. In part (b), they were required to find the point of intersection of the lines given by the equations $x - 2y = -5$ and $2x + 7y - 34 = 0$.

Figure 4 shows a summary of the candidates' performance in this question. It is evident from the figure that, 19,107 (5.3%) candidates scored marks from 2 to 6, out of which 17,163 (4.8%) candidates obtained marks ranging from 2 to 3.5. The figure also shows that, 341,119 (94.7%) candidates scored from 0 to 1.5 marks, out of which 319,140 (88.6%) candidates scored 0 mark. Generally, candidates had weak performance in this question.

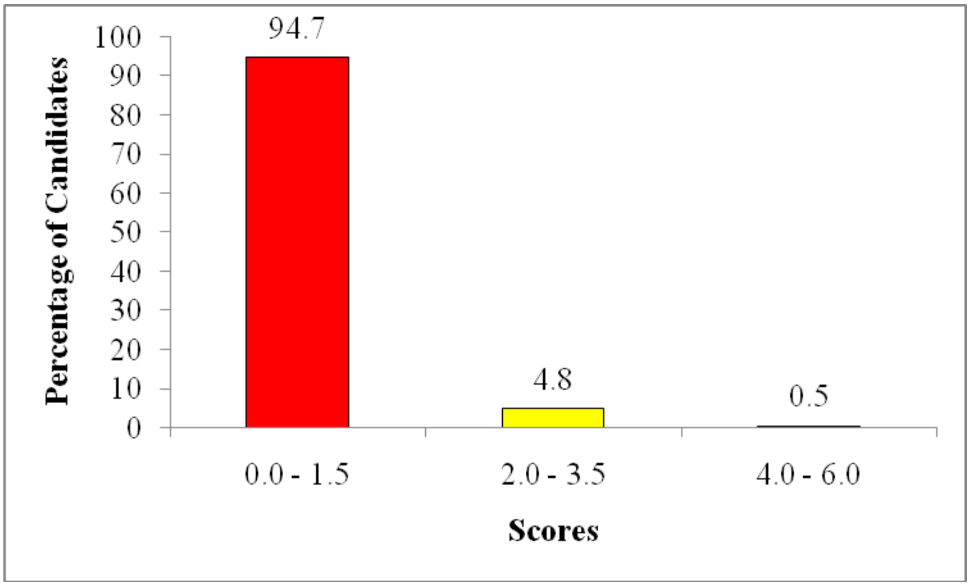


Figure 4: *Candidates' Performance in Question 4.*

In part (a), the majority of candidates who scored zero were not able to correctly substitute $\underset{\sim}{a} = 2x\underset{\sim}{i} + 3\underset{\sim}{j}$, $\underset{\sim}{b} = \left(x^2 + y\right)\underset{\sim}{i} + 4y\underset{\sim}{j}$ and $\underset{\sim}{v} = \frac{8}{3}\underset{\sim}{i} + \frac{25}{12}\underset{\sim}{j}$ into $\underset{\sim}{v} = \frac{1}{4}\underset{\sim}{a} + \frac{1}{3}\underset{\sim}{b}$. For example, the candidates incorrectly wrote $\underset{\sim}{v} = \frac{1}{4}\left(2x\underset{\sim}{i} + 3\underset{\sim}{j}\right) + \frac{1}{3}\left(x^2 + y\right)\underset{\sim}{i} + 4y\underset{\sim}{j}$ or $\underset{\sim}{v} = \frac{1}{4} \times 2x\underset{\sim}{i} + 3\underset{\sim}{j} + \frac{1}{3} \times \left(x^2 + y\right)\underset{\sim}{i} + 4y\underset{\sim}{j}$ instead of $\underset{\sim}{v} = \frac{1}{4}\left(2x\underset{\sim}{i} + 3\underset{\sim}{j}\right) + \frac{1}{3}\left(\left(x^2 + y\right)\underset{\sim}{i} + 4y\underset{\sim}{j}\right)$. These candidates failed to comply with proper use of brackets as a result they got wrong answers. Others were able to make correct substitution but could not perform the required operations correctly because they failed to multiply vectors by scalars. Also, some candidates faced difficulties as they could not compare the corresponding vector components in $\underset{\sim}{i}$ and $\underset{\sim}{j}$ when formulating the equations. The analysis of performance' scores and responses revealed that, candidates lacked adequate knowledge and skills in performing vector operations.

Some of the candidates were able to formulate the equations $\frac{x}{2} + \frac{x^2}{3} + \frac{y}{3} = \frac{8}{3}$ and $\frac{3}{4} + \frac{4}{3}y = \frac{25}{12}$ but failed to solve them, suggesting that they lacked knowledge and skills in solving simultaneous equations.

In part (b), the analysis shows that, some of the candidates who opted to use elimination method in solving the equations $x-2y=-5$ and $2x+7y-34=0$ failed to eliminate either the term involving *x* or the term involving *y*. For example, when eliminating *x* from the equations, some of them were able to multiply $x-2y=-5$ by 2 on both sides but failed to subtract $2x-4y=-10$ from $2x+7y=34$ as they wrote $(7y-{}^{-}4y)=(34-{}^{-}10)$ as $3y=24$ instead of $11y=44$ indicating lack of adequate knowledge and skills on subtraction of integers.

Also, some of the candidates who used substitution method could not correctly substitute one equation into the other. For example, there were candidates who incorrectly wrote $x=-2y-5$ instead of $x=2y-5$ before substituting it into $2x+7y=34$. These candidates lacked the basic algebraic skills.

Moreover, the candidates who used the graphical method were unable to draw the correct graph of each equation and ended up with wrong location of the point of intersection between the lines. Others did not understand that the point of intersection between the given lines is the common solution to both equations. As a result they ended up calculating the slopes of the lines and the midpoint using the intercepts which was contrary to the requirements of the question. A sample response of a candidate who could not answer this question correctly is shown in Extract 4.1.

4\. (a) Solution

$\underline{a} = 2xi + 3\underline{j}$,

$\underline{b} = (x^2 + y)i + 4y\underline{j}$, and

$\underline{v} = \frac{8}{3}i + \frac{25}{12}\underline{j}$

but $\underline{v} = \frac{1}{4}a + \frac{1}{3}\underline{b}$.

$v = \frac{1}{4}\left(2xi + 3\underline{j}\right) + \frac{1}{3}\left(x^2+y\right)i + 4y\,\underline{j}$

$\frac{1}{4}\left(\frac{2xi}{4} + \frac{3}{4}j\right) + \left(\frac{x^2}{3} + \frac{y}{3}\right) i + 4yj$

$\frac{2xi}{4} + \frac{3}{4}j + \frac{x^2 i}{3} + \frac{yi}{3} + 4yj$

$\frac{2xi}{4} + \frac{x^2 i}{3} + \frac{3\underline{j}}{4} + 4yj + \frac{yi}{3}$

$\frac{1}{2}xi + \frac{xi}{3} + \frac{3j}{4} + \frac{4yj}{1} + \frac{yi}{3}$

$x = \frac{1}{2}i + \frac{i}{3} + \frac{3j}{4} + y = 4j + \frac{i}{3}$.

$x = \frac{3i + 2i}{6} = \frac{5i}{6} + \frac{3j}{4} + 4j + i$

$x = \frac{5i + i}{6} = y = \frac{4j + 3j}{1} = \frac{7}{4}$

$\therefore$ the value of $x = i$ and $y = j$

4 (b) $x - 2y = -5$

$2x + 7y - 34 = 0$

$\begin{matrix} 2 \\ 1 \end{matrix}\begin{cases} x - 2y = -5 \\ 2x + 7y = +34 \end{cases}$

$-\begin{cases} 2x - 4y = -10 \\ 2x + 7y = 34 \end{cases}$

$\frac{3y}{3} = \frac{24}{3}$

$y = 8$

$x - 16 = -5$

$x = -5 + 16$

$x = 9$

$\therefore$ The point of Intersection are $(8, 9)$

Extract 4.1: A sample response of a candidate who lacked adequate knowledge and skills in multiplying vectors by scalars, adding vectors and solving simultaneous equations.

However, 925 (0.3%) candidates answered this question correctly. These candidates were able to correctly substitute $\underline{a} = 2x\underline{i} + 3\underline{j}$, $\underline{b} = (x^2 + y)\underline{i} + 4y\underline{j}$ and $\underline{v} = \frac{8}{3}\underline{i} + \frac{25}{12}\underline{j}$ into $\underline{v} = \frac{1}{4}\underline{a} + \frac{1}{3}\underline{b}$ and compared the corresponding vector components leading to equations $\frac{x}{2} + \frac{x^2}{3} + \frac{y}{3} = \frac{8}{3}$ and $\frac{3}{4} + \frac{4}{3}y = \frac{25}{12}$ that gave $y = 1$ and $x = 2$ or $x = -\frac{7}{2}$. The candidates showed good understanding on vector operations and the procedures to solve simultaneous equations. Also, the candidates were able to correctly find the point of intersection of the given equations of lines. This indicates that, the candidates had adequate knowledge and skills in solving simultaneous equations. Extract 4.2 shows a sample response of a candidate who answered this question correctly.

4. a)

$$\frac{8}{3}i + \frac{25}{12}j = \frac{1}{4}(2xi + 3j) + \frac{1}{3}[(x^2+y)i + 4yj]$$

multiplying ~~b~~ by 12 throughout

$$12 \times \left(\frac{8}{3}i + \frac{25}{12}j\right) = \left\{\frac{1}{4}(2xi + 3j) + \frac{1}{3}[(x^2+y)i + 4yj]\right\} \times 12$$

$$32i + 25j = 6xi + 9j + 4x^2i + 4yi + 16yj$$

$$\underline{32i + 25j = (6xi + 4x^2i + 4yi) + (9j + 16yj)}$$

$$32i = 6xi + 4x^2i + 4yi$$

$$32 = 6x + 4x^2 + 4y$$

$$4x^2 + 6x + 4y - 32 = 0 \text{ --- (i)}$$

also $25j = 9j + 16yj$ --- (ii)

$$25 = 9 + 16y$$

$$25 - 9 = 16y$$

$$\frac{16}{16} = \frac{16y}{16}$$

$$y = 1$$

substituting into equation (i)

$$4x^2 + 6x + 4(1) - 32 = 0$$

$$\frac{4x^2}{2} + \frac{6x}{2} + \frac{-28}{2} = \frac{0}{2}$$

$$2x^2 + 3x - 14 = 0$$

sum = 3

4 a) product $= -28$

factors $= 7, \quad -4$

$$2x^2 + 7x - 4x - 14 = 0$$

$$x(2x+7) - 2(2x+7) = 0$$

$$(x-2)(2x+7) = 0$$

$x_1 = 2$

x_2 : $2x + 7 = 0$

$$\frac{2x}{2} = \frac{-7}{2}$$

$$x = -3\tfrac{1}{2}$$

$x_1 = -3\tfrac{1}{2} \quad \cancel{x_2} = 2$

and $\underline{y = 1}$

b) $\begin{cases} x - 2y = -5 & (\times 2) \\ 2x + 7y = 34 & (\times 1) \end{cases}$

$-\begin{cases} 2x - 4y = -10 \\ 2x + 7y = 34 \end{cases}$

$$-4y - 7y = -10 - 34$$

$$\frac{-11y}{-11} = \frac{-44}{-11}$$

$$y = 4$$

$$x - 2(4) = -5$$

$$x - 8 = -5$$

$$x = -5 + 8$$

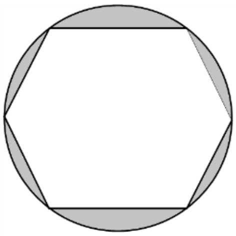
$$x = 3$$

$\therefore$ The point of intersection $= (3, 4)$

Extract 4.2: A sample response of a candidate who had adequate knowledge and skills on vector operations and solving simultaneous equations.

2.5 Question 5: Perimeters and Areas

In this question, the candidates were given the following regular hexagon inscribed in a circle whose perimeter is 42 cm, and were required to find; (a) the radius of the circle, (b) the area of the circle and the regular polygon and (c) the area of the shaded region.



A summary of the candidates' performance in this question is presented in Figure 5. The figure shows that, 347,844 (96.6%) candidates scored marks from 0 to 1.5. It also shows that, 12,384 (3.4%) candidates got scores ranging from 2 to 6. This indicates that the performance was weak.

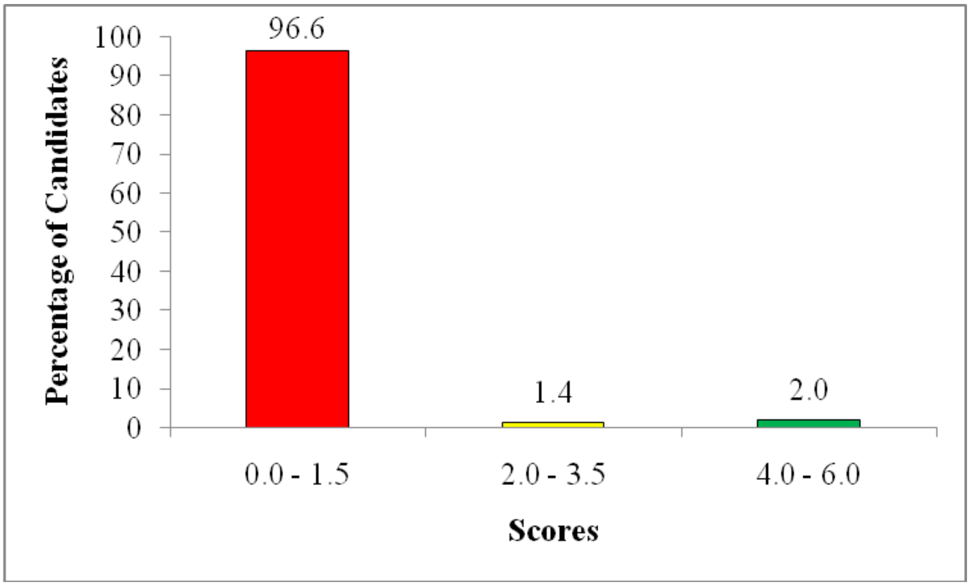


Figure 5: *Candidates' Performance in Question 5.*

Further analysis reveals that, 338,728 (94.0%) candidates scored 0 mark. The weak performance in this question was contributed by several factors as highlighted in the subsequent paragraphs.

In part (a), the candidates who scored zero were unable to find the radius r of the circle by using the perimeter $(p = 42 \text{ cm})$ of the given hexagon. Some of them regarded 42 cm as the radius of a circle. These candidates wrongly calculated the area of the circle as $\frac{22}{7} \times 42 \times 42$ and the area of the regular polygon as $\frac{1}{2}\times 6\times 42\times 42\times \sin\left(\frac{360°}{6}\right)$. Other candidates applied wrong formulae when finding r such as: $p=2nr\left(\frac{180°}{n}\right)$, $p=2nr\left(\frac{\sin 180°}{n}\right)$ or $p=2nr\sin\left(\frac{360°}{n}\right)$. Instead, they were supposed to use the formula $p=2nr\sin\left(\frac{180°}{n}\right)$ implying that $42=2\times 6r\sin\left(\frac{180°}{6}\right)$ giving $r=7\,\text{cm}$.

In part (b), it was also noted that, when finding the area of the hexagon inscribed in the circle, many candidates used wrong formulae like: $A=\frac{1}{2}nr\sin\left(\frac{360°}{n}\right)$ or $A=2r\sin\left(\frac{180°}{n}\right)$ instead of $A=\frac{1}{2}nr^2\sin\left(\frac{360°}{n}\right)$. Also, when finding the area of the circle, there were candidates who wrote incorrect formulae such as $A=2\pi r$ or $A=2\pi r^2$ instead of $A=\pi r^2$. These candidates clearly lacked adequate knowledge about the concepts of perimeter and area of a regular polygon inscribed in a circle.

In part (c), the analysis shows that, the candidates' inability to find the correct areas of circle and regular hexagon resulted into incorrect area of the shaded region. Moreover, few candidates did not understand that the area of the shaded region could be obtained by subtracting the area of the hexagon from the area of the circle. Extract 5.1 is a sample response of a candidate who failed to answer this question.

5a Soln

Perimeter of a hexagon is 42cm

$P = 2nr^2 \sin\left(\frac{180}{n}\right)$

$42 = 2 \times 6 \times r^2 \sin\left(\frac{\cancel{180}^{30}}{\cancel{6}}\right)$

$42 = 12r^2 \sin 30$

$42 = 12r^2 \times 0.5000$

$\frac{42}{6} = \frac{6r^2}{6}$

$\sqrt{r^2} = \sqrt{7}$

$r = \sqrt{7}$

$\therefore r = \sqrt{7}\ cm$

5(b) Area of the circle $= 2\pi r^2$

$= 2 \times \cancel{3.14}\ \frac{22}{7} \times (7 \times 7)$

$= (44 \times 7) cm^2$

Area of the circle is $308 cm^2$

Area of a regular polygon

number of triangles = [number of sides − 2]

$n\Delta = 6 - 2 \rightarrow$ number of triangles is 4

From Area of a triangle $\frac{1}{2} b \times h$

$= (\frac{1}{2} \times \cancel{6}\ 7 \times 7)\ 4$

$= \frac{49}{2} \times 4 = 49 \times 2$

Area of a regular polygon is $98 cm^2$

c $A = \pi r^2 - \frac{1}{2} n r^2 \sin\left(\frac{360}{n}\right)$

$A = \frac{22}{7} \times \sqrt{7} \times \sqrt{7} - 9.093$

$A = \frac{22}{7} \times \sqrt{49} - 9.093$

$A = \frac{22}{\cancel{7}} \times \cancel{7} - 9.093$

$A = 22 - 9.093$

$\therefore A = 12.927 cm^2$

Extract 5.1: A sample response of a candidate who failed to apply the correct formulae for perimeter of a regular polygon and the area of circle and ended up with incorrect answers.

Although the general performance was weak, 3,019 (0.8%) candidates answered this question correctly as illustrated in Extract 5.2.

5.	a) $P = 42\,cm$ $n = 6$
	Perimeter $= 2nr \sin\left(\frac{180°}{n}\right)$
	$42 = 2 \times 6 r \sin\left(\frac{180°}{6}\right)$
	$42 = 12r \sin 30°$
	$42 = 12r \times 0.5$
	$\frac{42}{6} = \cancel{16}\ \frac{6r}{6}$
	$r = 7cm$
	$\therefore$ $\underline{\text{Radius} = 7\,cm}$
	b) Area of the circle $= \pi r^2$
	$= 3.14 \times 7cm \times 7cm$
	$= 154\,cm^2$
	$\therefore$ Area of the circle $= \underline{154\,cm^2}$
	Area of the polygon $= \frac{1}{2} nr^2 \sin\left(\frac{360°}{n}\right)$
	$= \frac{1}{2} \times 6 \times 7^2 \sin\left(\frac{360°}{6}\right)$
	$= 3 \times 49 \sin 60°$
	$= 147 \times 0.8660$
	$\therefore$ Area $= 127.282\,cm^2$
	$\therefore$ Area of the circle $= \underline{154\,cm^2}$ and area of the polygon $= \underline{127.282\,cm^2}$
	c) Area of shaded region $=$ Area of the circle $-$ Area of the polygon
	$= 154\,cm^2 - 127.282\,cm^2 = 26.718\,cm^2$
	$\therefore$ The area of shaded region $= \underline{26.718\,cm^2}$

Extract 5.2: A sample response of a candidate who calculated the area of the shaded region correctly.

2.6 Question 6: Rates and Variations

This question comprised parts (a) and (b). In part (a), the question was: "Mukasa received Ushs 1,000,000 from his sister in Uganda. How much amount is this when converted into Tshs at a bank? (Ushs 1 = Tshs 0.65)". In part (b), it was given that the energy (*E*) stored in an elastic band varies as the square of the extension (*x*). When the elastic band is extended by 4 cm, the energy stored is 240 Joules. The candidates were required to find: the energy stored when the extension is 6 cm; and the extension when the stored energy is 60 joules.

The performance of candidates in this question is presented in Figure 6. It shows that, 83,291 (23.1%) candidates scored marks from 2 to 6. Also, 276,936 (76.9%) candidates scored marks from 0 to 1.5, out of which 253,157 (70.3%) candidates scored 0 mark. Therefore, candidates had weak performance in this question.

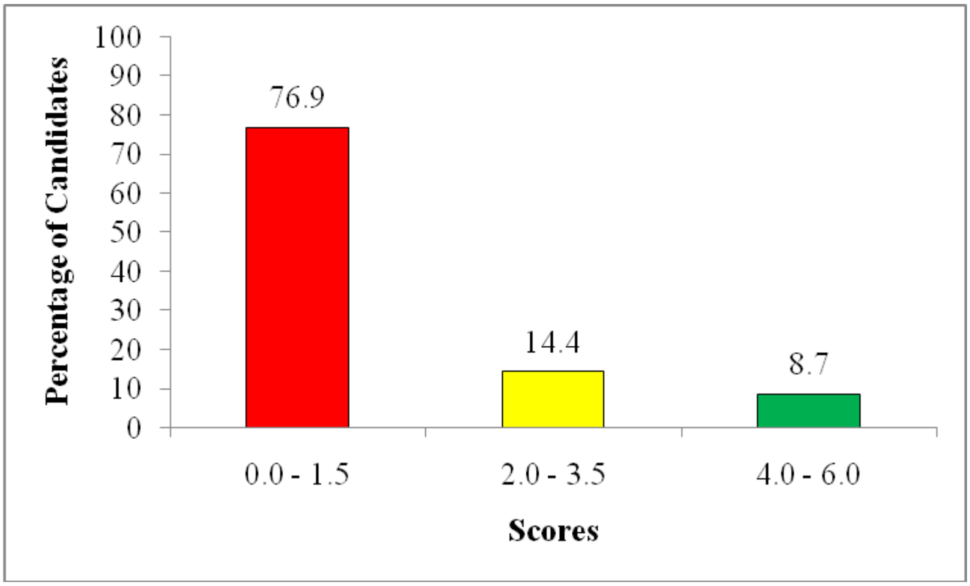


Figure 6: *Candidates' Performance in Question 6.*

In part (a), the candidates who scored zero failed to interpret the requirements of the question leading to incorrect conversion of the amount given in Ugandan currency into Tanzanian currency. For example, they performed wrong operations like $\frac{1{,}000{,}000}{0.65}$ or $\frac{0.65}{1{,}000{,}000}$ instead of $1{,}000{,}000 \times 0.65$ as required, indicating lack of knowledge and skills in solving problems related to currency conversion. Also, some of them could not correctly evaluate $0.65 \times 1{,}000{,}000$ because they came up with answers like $1{,}000{,}000 \times 0.65 = 65{,}000$ or $1{,}000{,}000 \times 0.65 = 65{,}000{,}000$ justifying that the candidates lacked the skills of multiplying a decimal by a whole number.

In part (b), the majority of candidates failed to write the correct mathematical statement representing the given word problem. For example, they wrote wrong statements like $E \alpha \sqrt{x}$ or $E \alpha x$, instead of $E \alpha x^2$. Other candidates wrote $E \alpha \frac{1}{x^2}$ which is an inverse variation.

Some candidates were able to formulate the equation $E = kx^2$ but failed to find the proportionality constant (k), which was an essential step in arriving at the required answers. For example, they interchanged the given extension x and energy E writing $4 = k \times 240^2$, instead of $240 = k \times 4^2$, hence ended up with incorrect value of k. This suggests that these candidates lacked adequate knowledge and skills on the concepts of direct variation. Extract 6.1 is a sample response from one of the candidates who failed to answer this question.

6. (a) 1,000000. = T ?

0.65

$\frac{1000000 \times 100}{0.65 \times 100}$

~~2000000~~

~~100000000~~

~~65006~~

13

Mukasa will get 923.76 shillings of Tanz

6. b) $E \propto \sqrt{x}$

$E = k\sqrt{x}$

$\frac{840}{\sqrt{4}} = \frac{k\sqrt{4}}{\sqrt{4}}$

$k = \frac{240}{2}$

$k = 120$

$E = 120\sqrt{x}$

$E = 120\sqrt{6}$

$E = 120 \times 2.4$

$E = 12 \times 24$

$E = 268$ Joules

$E = 120\sqrt{x}$

$\frac{60}{120} = \frac{120\sqrt{x}}{120}$

$\sqrt{x} = \frac{61}{122}$

$(\sqrt{x})^2 = (0.5)^2$

$x = 2.5$

$x = 0.25$ cm.

Extract 6.1: A sample response of a candidate who lacked enough knowledge and skills in solving problems related to exchange rates and direct variation.

Despite the weak performance, the analysis of data further revealed that, 16,256 (4.5%) candidates were able to answer this question correctly. Extract 6.2 shows a sample of a good response from one of the candidates.

6 a) Ushs - 1,000,000

solution

1 Ushs = 0·65 Tshs

1,000,000 Ushs = x ?

$x = \frac{1,000,000 \text{ Ushs} \times 0{\cdot}65 \text{ Tsh}}{1 \text{ Ushs} \times ?}$

= 650,000 Tanzanian shilling

∴ Mukasa received 650,000/= in Tshs

b) solution

$\mathcal{E} \propto x^2$

$\mathcal{E} = Kx^2$

when x = 4cm and $\mathcal{E}$ = 240 joules

$\Longrightarrow \mathcal{E} = Kx^2$

$240 = k(4)^2$

but; $(4)^2 = 16$

$\frac{240}{16} = \frac{k16}{16}$

∴ $K = 15$

But also,

when (x) - extension is 6cm

find $\mathcal{E}$ - ?

from

$\Longrightarrow \mathcal{E} = Kx^2$

$\mathcal{E} = 15 \times 6^2$

$= 15 \times 36$

$\mathcal{E} = 540$

∴ Energy is 540 joules

Thus

when given energy is 60 joules

to find extension = ? (x)

$K = 15$

from $\mathcal{E} = Kx^2$

60 joules = $15 \times x^2$

$\frac{60 \text{ jouls}}{15} = \frac{15 \times x^2}{15}$

$\sqrt{x^2} = \sqrt{4}$

x = 2cm

∴ The extension is 2cm

Extract 6.2: A sample response from a candidate who correctly applied the concepts of currency conversion rates and direct variation in answering question 6.

2.7 Question 7: Ratios, Profit and Loss

This question had parts (a) and (b). In part (a), the candidates were required to find the amount of money received by the first relative if Tshs 140,000/= was shared among three relatives such that the first relative got twice as much as the second, and the second got twice as much as the third. In part (b), the candidates were required to find (i) the loss made and (ii) the percentage loss when a desktop computer bought for Tshs 900,000 was sold for Tshs 720,000.

The candidates' performance in this question is summarized in Figure 7. The figure reveals that, 65,245 (18.1%) candidates scored marks from 2 to 6, out of which 12,300 (3.4%) candidates scored from 4 to 6 marks. However, 294,985 (81.9%) candidates scored marks from 0 to 1.5, out of which 203,050 (56.4%) candidates scored 0 mark. In general, the performance in this question was weak.

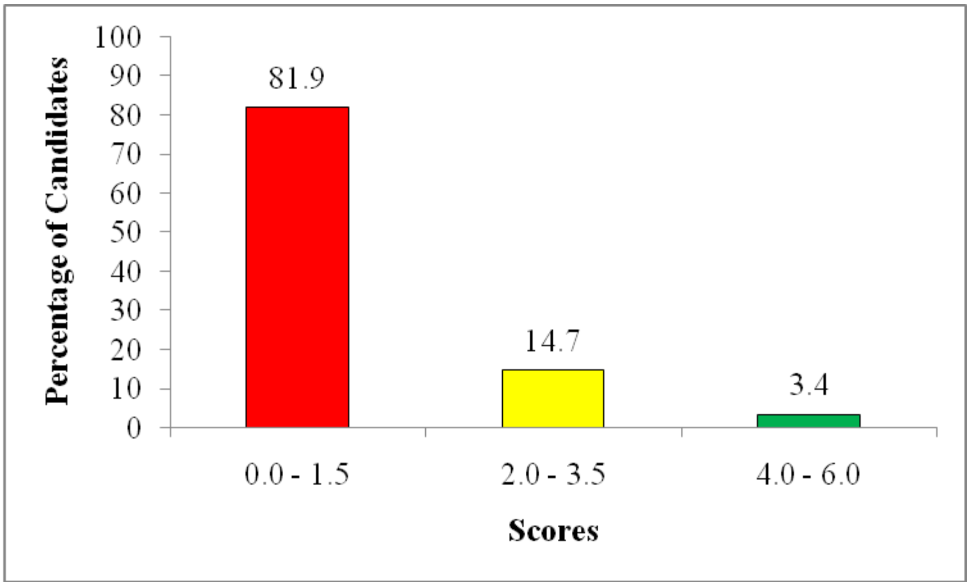


Figure 7: *Candidates' Performance in Question 7.*

In part (a), the candidates who scored zero could not apply the concept of ratios to solve the given word problem. The candidates were unable to divide the given amount into proportional parts. Some candidates wrote incorrect ratios like 1:3 or 1:2:3 for the first, second and third relative respectively, instead of 4:2:1. Some of the examples on how the candidates calculated the amount of money the first relative should receive include: $\left(\frac{140{,}000}{2}\right)$, $(140{,}000+140{,}000)$, $\left(\frac{1}{6}\times 140{,}000\right)$ or $\left(\frac{1}{3}\times 140{,}000\right)$ instead of

$\left(\frac{4}{7}\times 140{,}000\right)$. This shows that the candidates lacked adequate knowledge and skills on dividing a quantity into proportional parts.

In part (b) (i), the majority of candidates failed to find the loss made because they applied wrong formulae including; $loss = selling\,price - buying\,price$ which is the formula for profit made, $I = \frac{PRT}{100}$ which is the formula for finding the simple interest or $loss = selling\,price + buying\,price$ instead of $loss = buying\,price - selling\,price.$ This confirms that these candidates lacked adequate knowledge and skills on calculating the loss. Few candidates were able to write $loss = 900{,}000 - 720{,}000$ but could not perform subtraction operation correctly.

In part (b) (ii), most candidates failed to find the percent loss as they used wrong formulae including; $Percentage\ loss = \frac{Loss\ made}{Selling\ price}\times 100\%$ or $Percentage\ loss = \frac{Selling\ price}{Buying\ price}\times 100\%$. These candidates lacked knowledge and skills on calculating the percentage loss. Others were able to recall the correct formula for the percentage loss, that is, $Percentage\ loss = \frac{Loss\ made}{Buying\ price}\times 100\%$ but could not evaluate $\frac{180000}{900000}\times 100\%$ correctly. Extract 7.1 is an example of an incorrect response from one of the candidates.

7.	a) 140,000 to three relatives
	$\frac{1}{3}\times 140{,}000$
	$\frac{140{,}000}{3} = 46000$ each got
	but the first got
	46000×2
	$=92000$
	$\therefore$ The first got 92000 Tsh.

7 b)	i) loss made = Selling price − Cost price
	720,000 900,000
	loss made = $^{-}180,000$
	ii) The percentage loss = $\frac{\text{Selling price}}{\text{Cost price}} \times 100\%$
	$\frac{720,\cancel{000}}{900,\cancel{000}} \times 100\%$
	$\frac{72\cancel{0}}{90\cancel{0}} \times 100\%$
	$\frac{72}{9\cancel{0}} \times 10\cancel{0}\%$
	$= \frac{720}{9}$
	The percentage = 80%

Extract 7.1: A sample response of a candidate who could not apply the concept of ratios and the formulae for loss and percentage loss in solving the given real life problems.

In spite of weak performance, 9,175 (2.5%) candidates attempted the question correctly. These candidates showed good understanding of the concepts of ratios and percentage loss in answering this question as shown in Extract 7.2.

7	a) Let the three relatives be A, B and C
	Given A = 2B
	B = 2C
	A : B = 2 : 1
	B : C = 2 : 1
	A : B = (2 : 1) 2
	B : C = (2 : 1) 1
	A : B = 4 : 2
	B : C = 2 : 1
	A : B : C = 4 : 2 : 1
	Thus A + B + C in ratio = 4 + 2 + 1
	Total ratio = 7
	Required share of A.
	A = $\frac{4}{7} \times 140,000$ Tsh
	= 4 × 20,000 Tsh
	= 80,000 Tsh
	∴ The first got Tsh 80,000.

b) i) Loss made = Buying prize - Selling prize

$= 900\,000 - 720,000$

$= 180,000$

$\therefore$ The loss made is 180,000 Tsh

ii) Percentage loss

$\%\text{loss} = \frac{\text{Loss made}}{\text{Buying prize}} \times 100$

$= \frac{180000}{900,000} \times 100$

$= \frac{1}{5} \times 100 = 20\%$

$\therefore$ The percentage loss is 20%.

Extract 7.2: A sample response of a candidate who answered question 7 correctly.

2.8 Question 8: Sequences and Series

This question consisted of parts (a) and (b). In part (a), the candidates were required to (i) write down the second, third, fourth and fifth terms of an arithmetic progression having A_1 as the first term and d as the common difference and (ii) establish the formula for the sum of the first five terms of the arithmetic progression by using the results in part (a) (i). In part (b), they were required to (i) find the third, fourth and fifth terms of a geometric progression whose first and second terms are 3 and 9 respectively and (ii) verify that the sum of the first n terms is given by $S_n = G_1 \frac{r^n - 1}{r - 1}$ using the results in part (b) (i).

Figure 8 shows that, 44,332 (12.3%) candidates scored marks ranging from 2 to 6. It is also evident from the figure that, 315,891 (87.7%) candidates scored marks from 0 to 1.5, out of which 307,322 (85.3%) candidates scored 0 mark. Generally, candidates had weak performance in this question.

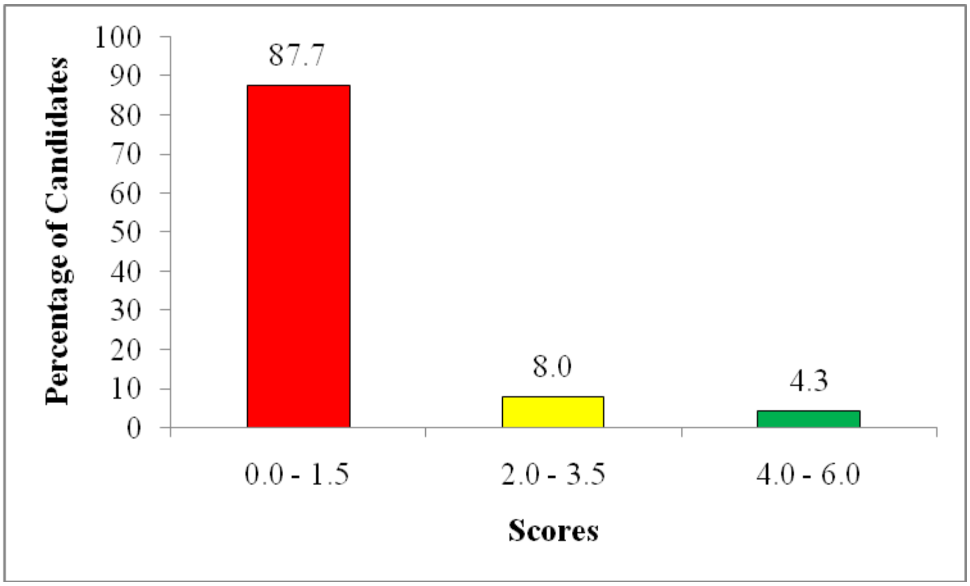


Figure 8: *Candidates' Performance in Question 8.*

In part (a), the majority of candidates failed to get the correct answers as they were not able to establish the required terms. Most of them applied wrong formulae for the *n*th term of an arithmetic progression like $A_n = A_1(n-1)d$ or $A_n = A_1 + (n+1)d$ instead of $A_n = A_1 + (n-1)d$. Others wrote $A_1 = 1$ ending up with incorrect expressions $1+d,\ 1+2d,\ 1+3d$ and $1+4d$ for the second, third, fourth and fifth terms respectively, instead of $A_1 + d,\ A_1 + 2d,$ $A_1 + 3d$ and $A_1 + 4d$ as required.

In some cases, there were candidates who got the required terms but could not add them to get the expression $5A_1 + 10d$ for the sum of the first five terms of the arithmetic progression. For example, some of them ignored or forgot the first term as they wrote $S_5 = A_2 + A_3 + A_4 + A_5$ instead of $S_5 = A_1 + A_2 + A_3 + A_4 + A_5$. Other candidates used the formula for the sum of the first *n* terms, that is, $S_n = \frac{n}{2}\left(2A_1 + (n-1)d\right)$ which was contrary to the given instructions.

In part (b), the majority were not able to get the required answers because they incorrectly calculated the common ratio r. For example, some of the candidates considered the values 3 and 9 as the terms of an arithmetic progression, hence they wrote $r = 9 - 3 = 6$ while others wrote $r = \frac{3}{9} = \frac{1}{3}$ instead of $r = \frac{9}{3} = 3$.

Also, other candidates failed to correctly apply the formula $G_n = G_1 r^{n-1}$ to get G_3, G_4 and G_5. Likewise, others confused between the terms of the arithmetic progression and the terms of the geometric progression.

However, few candidates correctly got G_3, G_4 and G_5 but failed to use the formulae $S_n = G_1 \frac{r^n - 1}{r - 1}$ and $S_5 = G_1 + G_2 + G_3 + G_4 + G_5$ to complete the verification. Extract 8.1 illustrates some of the mistakes that the candidates made in answering this question.

Generally, the candidates lacked adequate knowledge and skills on the *n*th term and the sum of the first *n* terms of an arithmetic progression and geometric progression.

8. a) $A_1 = 1$

$d = 1$

i) second term

$A_n = A_1 + (n-1)d$

$A_2 = 1 + (2-1)1$

$A_2 = 1 + 1$

$A_2 = 2$ ∴ Second term $= 1+d$

Third term

$A_3 = 1 + (3-1)d$

$A_3 = 1 + 2d$

~~Fo~~ Third term $= 1 + 2d$

Fourth term

$A_4 = A_1 + (n-1)d$

$A_4 = 1 + (4-1)d$

$A_4 = 1 + 3d$

Fourth term $= 1 + 3d$

Fifth term

$A_5 = A_1 + (n-1)d$

$A_5 = 1 + 4d$

∴ Fifth term $= 1 + 4d$.

ii) $S_n = \frac{n}{2}\left(2A_1 + (n-1)d\right)$

Sum of first 5 terms

$S_5 = \frac{n}{2}\left(2A_1 + (n-1)d\right)$

8(b)(i) $3, 9, 15$

$G_1 = 3 \quad r = 6$

(i) $G_3 = G_1 r^2 = 3 \times 6^2$

$= 3 \times 36$

$G_3 = G_1 r^{n-1}$

$= 3r^{3-1}$

$= 3 \times 6^2$

$= 3 \times 36$

$= 108$

$G_4 = G_1 r^4$

$= 3 \times 6^4$

$=$

(ii) $S_n = 3\left(\frac{6^5 - 1}{6-1}\right)$

$= 3\left(\frac{7576 - 1}{5}\right)$

$= 3\left(\frac{7575}{5}\right)$

$= 3 \times 1515$

$= 4545$

$S_n = 4545$

Extract 8.1: A sample response of a candidate who lacked enough knowledge and skills on arithmetic and geometric progressions.

Despite the fact that the general performance was weak, 811 (0.2%) candidates answered this question correctly as shown in Extract 8.2.

8 (a) solution.

(i) Given: A_1 and d.

Then; from:

$$A_n = A_1 + (n-1)d$$
$$A_2 = A_1 + (2-1)d$$
$$\therefore A_2 = A_1 + d$$

Then:

$$A_n = A_1 + (n-1)d$$
$$A_3 = A_1 + (3-1)d$$
$$\therefore A_3 = A_1 + 2d$$

Then;

$$A_n = A_1 + (n-1)d$$
$$A_4 = A_1 + (4-1)d$$
$$\therefore A_4 = A_1 + 3d$$

Then;

$$A_n = A_1 + (n-1)d$$
$$A_5 = A_1 + (5-1)d$$
$$\therefore A_5 = A_1 + 4d$$

(ii) solution:

$$S_5 = A_1 + A_2 + A_3 + A_4 + A_5$$
$$S_5 = A_1 + A_1 + d + A_1 + 2d + A_1 + 3d + A_1 + 4d$$
$$S_5 = 5A_1 + 10d$$

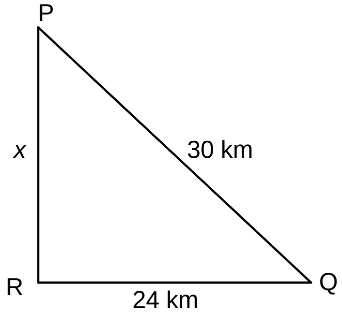
$\therefore$ The formula for the sum of the first five terms of the arithmetic progression is given by: $S_5 = 5A_1 + 10d$

8 (b)	solution:
	Given: $G_1 = 3$, $G_2 = 9$.
	from: $r = \frac{G_2}{G_1}$
	$r = \frac{9}{3} = 3$.
(i)	from:
	$G_3 = G_1 r^2$
	$G_3 = 3 \times 3^2$
	$G_3 = 3 \times 9$
	$\therefore G_3 = 27$.
	Then:
	$G_4 = G_1 r^3$
	$G_4 = 3 \times 3^3$
	$G_4 = 3 \times 27$
	$\therefore G_4 = 81$.
	Then;
	$G_5 = G_1 r^4$
	$G_5 = 3 \times 3^4$
	$G_5 = 3 \times 81$
	$\therefore G_5 = 243$
(ii)	solution.
	case I:
	$S_5 = G_1 + G_2 + G_3 + G_4 + G_5$
	$S_5 = 3 + 9 + 27 + 81 + 243$
	$S_5 = 363$.
	Case II:
	from: $S_n = G_1 \frac{r^n - 1}{r - 1}$
	$S_5 = 3 \frac{(r^5 - 1)}{5 - 1}$
(ii)	Then: $S_5 = 3 \frac{(r^5 - 1)}{r - 1}$
	$S_5 = 3 \frac{(3^5 - 1)}{3 - 1}$
	$S_5 = 3 \frac{(243 - 1)}{2}$
	$S_5 = \frac{3 \times 242}{2}$
	$S_5 = 3 \times 121$
	$S_5 = 363$.
	Then; since the results obtained in case I and case II is equal $= 363$.
	$S_n = G_1 \frac{r^n - 1}{r - 1}$ Hence Proved.

Extract 8.2: A sample response of a candidate who had adequate knowledge and skills in using the formulae for nth term and sum of the first n terms of arithmetic and geometric progressions.

2.9 Question 9: Pythagoras Theorem and Trigonometry

This question comprised parts (a) and (b). In part (a), the candidates were required to find the distance $\overline{PR}$ from the following figure given that $\overline{PR}$ and $\overline{RQ}$ are perpendicular.



In part (b), they were required to find to the nearest cm, the length of the shadow of a flag pole whose height is 5m and angle of elevation of the sun is $60°$.

Figure 9 shows that, 61,322 (17.0%) candidates who attempted this question scored from 2 to 6, implying that candidates' performance was weak. The figure also shows that, 298,904 (83.0%) candidates scored low marks ranging from 0 to 1.5 and among them, 278,061 (77.2%) candidates scored 0 mark.

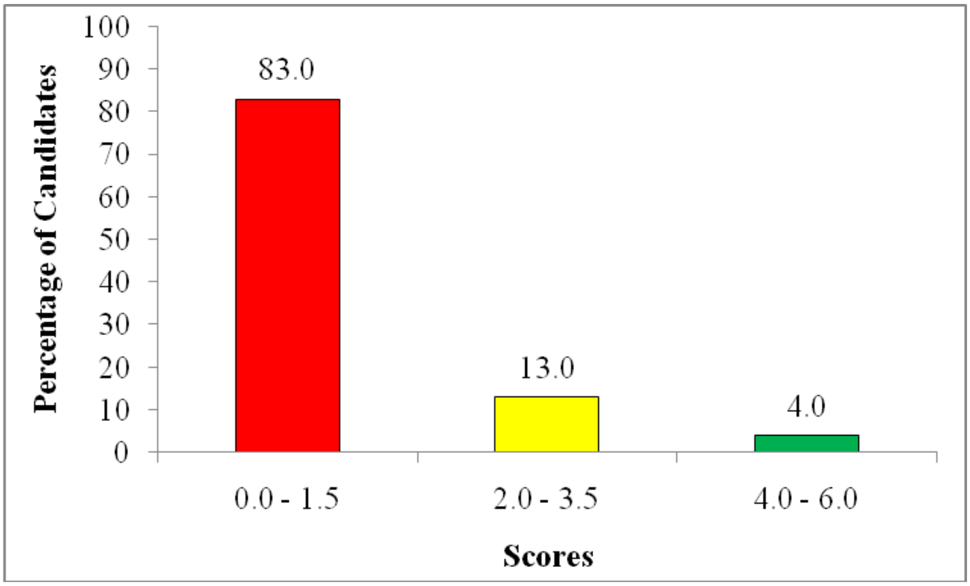


***Figure 9**: Candidates' Performance in Question 9.*

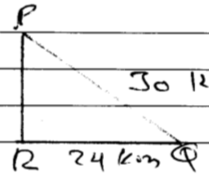
In part (a), the candidates who scored zero failed to apply the Pythagoras' theorem or the definitions of trigonometric ratios. For example, some wrote $\overline{PQ}^2 + 30^2 = 24^2$, $24^2 + 30^2 = \overline{PR}^2$ instead of $\overline{PR}^2 + 24^2 = 30^2$.

It was noted that, there were few candidates who correctly wrote $\overline{PR}^2 + 24^2 = 30^2$ leading to $\overline{PR} = \sqrt{324}$ but failed to evaluate $\sqrt{324}$ while others were able to state the Pythagoras theorem but did not proceed to next steps as required.

In part (b), the majority of candidates were unable to correctly represent the given information in a right angled triangle which was an important step in arriving at the required answer. For example, some could not make a distinction between the angle of elevation and the angle of depression. Others interchanged the height of the flagpole and the length of the shadow. Extract 9.1 shows a sample response of a candidate who failed to answer this question correctly.

It was further noted that, few candidates represented the given information correctly in a right angled triangle but could not use the definitions of trigonometrical ratios to find the length of the shadow, that is, $l = \dfrac{500}{\tan 60°} \cong 289\,\text{cm}$ as required. Some of the incorrect responses include: $\sin 60° = \dfrac{5}{l}$, $\tan 60° = \dfrac{l}{5}$ and $\sin 60° = \dfrac{5}{l}$ suggesting that these candidates did not understand well how to apply the concepts of trigonometric ratios.

9 a)



Distance of $\overline{PR}$

by using pythagoras theorem

$$\overline{RQ}^2 + \overline{QP}^2 = \overline{PR}^2$$

$$24^2 + 30^2 = \overline{PR}^2$$

$$576 + 900 = \overline{PR}^2$$

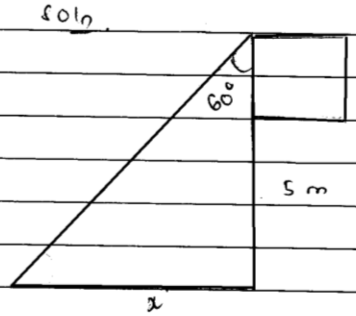
$$1470 = \overline{PR}^2$$

$$\sqrt{1470} = \sqrt{\overline{PR}^2}$$

$$PR = \sqrt{1470}$$

∴ The distance of PR is $\sqrt{1470}$ km

9. b) soln.



$$\tan \theta = O/A.$$

$$\tan 60° = \frac{x}{5}$$

$$x = \tan 60° \times 5$$

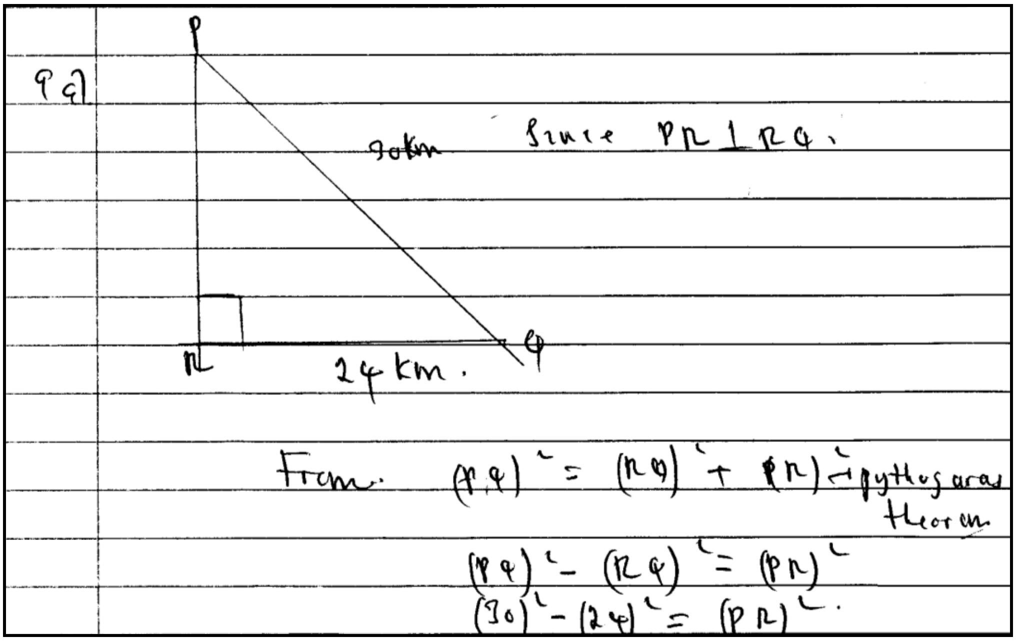
$$x = 1.7321 \times 5$$

$$x = 8.6605 \text{ m.}$$

∴ The length of the shadow $= 8.6605$ m.

Extract 9.1: A sample response of a candidate who could not correctly apply the Pythagoras' theorem and represent the given information diagrammatically.

Despite the weak performance, 694 (0.2%) candidates applied correctly the Pythagoras' theorem to find the distance $\overline{PR}$. Likewise, these candidates were able to sketch a correct diagram representing the given information and calculated the required length as shown in Extract 9.2.



From. $(PQ)^2 = (RQ)^2 + (PR)^2$ → pythagoras theorem

$(PQ)^2 - (RQ)^2 = (PR)^2$

$(30)^2 - (24)^2 = (PR)^2$.

9a) $900 - 576 = (\overline{PR})^2$

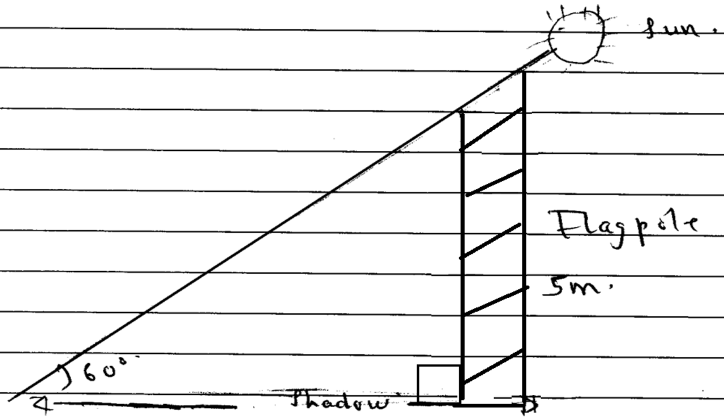
$(\overline{PR}) = 324$

$(PR) = \sqrt{324}$

$(PR) = 18km.$

$\overline{PR} = 18km$

9b)



From $\tan \theta = \frac{\text{opposite}}{\text{Adjacent}}$

$\tan 60° = \frac{5m}{\text{Adjacent}}$

$\text{Adjacent} = \frac{5m}{\tan 60°}$

$\text{Adjacent} = \frac{5m}{1.732}$

Adjacent = 2.89 m

Adjacent = 2.89m (2.d.p)

but

1m = 100cm.

2.89m = x

x = 289cm.

The length of its shadow is 289cm.

Extract 9.2: A sample response of a candidate who answered this question correctly.

2.10 Question 10: Quadratic Equations

The question had parts (a) and (b). In part (a), the candidates were required to use factorization method to solve $x^2 - 9x + 14 = 0$. In part (b), they were required to find the values of x that satisfy the equation $\frac{1350}{x} - \frac{1350}{(x+3)} = 5$.

The analysis of performance scores revealed that, candidates' performance was weak. As indicated in Figure 10, only 39,369 (10.3%) candidates scored from 2 to 6 marks. The figure also shows that, 320,843 (89.1%) candidates scored low marks ranging from 0 to 1.5 and among them, 299,101 (83.0%) candidates scored 0 mark.

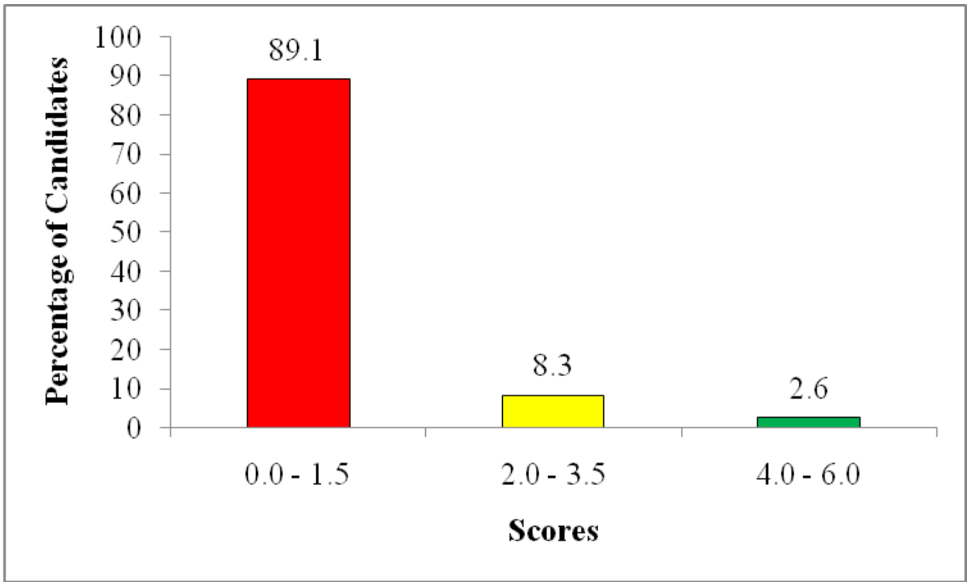


***Figure 10**: Candidates' Performance in Question 10.*

The candidates who scored zero in part (a) were unable to factorize the given quadratic equation particularly by splitting the middle term. Most of them were unable to identify two numbers whose product is 14 and whose sum is -9. Other candidates did not follow the given instructions. They applied the quadratic formula instead of using the factorization method.

In part (b), most of the candidates who scored zero could not put the given equation into the same denominator and perform correct algebraic operations in order to obtain the quadratic equation $x^2 + 3x - 810 = 0$. Other candidates were able to get the equation correctly but could not apply the quadratic formula, the factorization method or the method of completing the square to solve for x. These candidates committed errors while working out the solution. Extract 10.1 shows a sample response of a candidate who performed poorly in this question.

10 a) $x^2 - 9x + 14 = 0$

$x(x-9) + 14 = 0$

$x - 9 = 0$

$x_1 = 9$

$x + 14 = 0$

$x_2 = -14$

$\therefore$ The values of x are 9 and -14.

10 b) $\frac{1350}{x} - \frac{1350}{(x+3)} = 5$

$\frac{1350}{x} - \frac{1350}{x+3} = 5$

$\frac{4050 - 1350}{x+3} = 5$

$4050 - 1350 = 5(x+3)$

$2700 = 5x + 15$

$5x = 2700 - 15$

$\frac{5x}{5} = \frac{\cancel{2697}}{5} \quad \frac{2685}{5}$

$\frac{5x}{5} = \frac{2685}{5}$

$x = 537$

Extract 10.1: A sample response of a candidate who lacked the basic algebraic skills in solving the given equations.

On the other hand, 3,428 (1.0%) candidates managed to attempt the question correctly. In part (a), the candidates correctly factorized the given equation as $(x-7)(x-2)=0$ and eventually obtained $x=7$ and $x=2$ as required. In part (b), the candidates multiplied both sides of the given equation by $x(x+3)$, re-arranged the resulting equation into $x^2+3x-810=0$ and solved it to obtain $x=27$ and $x=-30$. Extract 10.2 shows how one of the candidates answered this question correctly.

10 a). $x^2 - 9x + 14 = 0$

$a = -2$

$b = -7$

$x^2 - 7x - 2x + 14 = 0$

$x(x-7) - 2(x-7) = 0$

$(x-2)(x-7) = 0$

$x - 2 = 0$ or $x - 7 = 0$

$x = 2$ or $x = 7$

$\therefore$ The value of x is 2 or 7

10 b). $\frac{1350}{x} - \frac{1350}{(x+3)} = 5$

$\frac{1350x + 4050 - 1350x}{x^2 + 3x} = \frac{5}{1}$

$\left(1350x + 4050 - 1350x = 5x^2 + 15x\right) \times \frac{1}{5}$

$270x + 810 - 270x = x^2 + 3x$

$810 = x^2 + 3x$

$x^2 + 3x - 810 = 0$

Recall general formular

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $a = 1$

$b = 3$

$c = -810$

$x = \frac{-3 \pm \sqrt{3^2 - 4(1)(-810)}}{2}$

$x = \frac{-3 \pm \sqrt{9 + 3240}}{2}$

$x = \frac{-3 \pm \sqrt{3249}}{2} \Rightarrow x = \frac{-3 \pm 57}{2}$

$x = \frac{-3 + 57}{2}$ or $\frac{-3 - 57}{2}$

$x = 27$ or -30

$\therefore x = 27$ or -30

Extract 10.2: A sample response of a candidate who showed good understanding on how to solve quadratic equations.

2.11 Question 11: Linear Programming

The question was as follows;

"A farmer needs to buy up to 25 cows for a new herd. He can buy either brown cows at 50,000/= each or black cows at 80,000/= each and he can spend a total of not more than 1,580,000/=. He must have at least 9 cows of each type. On selling the cows he makes a profit of 5,000/= on each brown cow and 6,000/= on each black cow. How many cows of each type he should buy to maximize profit?"

Figure 11 shows that, 76,316 (65.3%) candidates scored marks ranging from 0 to 2.5 while 40,510 (34.7%) candidates scored marks from 3 to 10, out of which 698 (0.6%) candidates managed to attempt the question as required. Generally, candidates had average performance in this question.

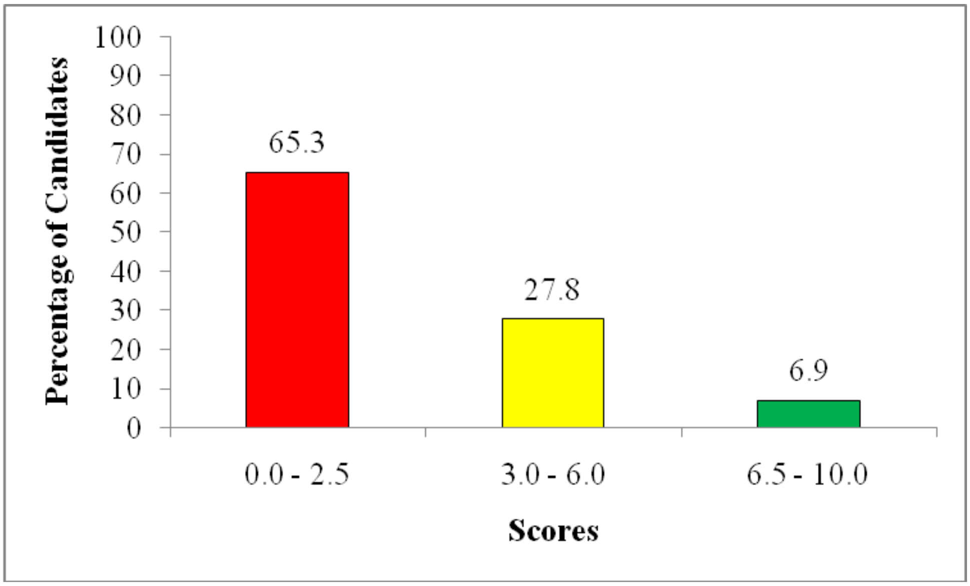


***Figure 11**: Candidates' Performance in Question 11.*

The candidates who answered this question correctly were able to: formulate the objective function $f(x, y) = 5000x + 6000y$ and the linear inequalities $x + y \leq 25$, $5x + 8y \leq 158$, $x \geq 9$ and $y \geq 9$, where x is the number of brown cows and y is the number of black cows; draw a graph of the inequalities; identify the corner points of the feasible region; and finally determine the number of brown cows and black cows to maximize profit as shown in Extract 11.1.

11\. Let x be number of brown cows

y be number of black cows

Constraints

$x + y \leqslant 25$

$50{,}000x + 80{,}000y \leqslant 1{,}580{,}000$

$5x + 8y \leqslant 158$

$x \geqslant 9$

$y \geqslant 9$

Objective function

maximize $(x, y) = 5000x + 6000y$

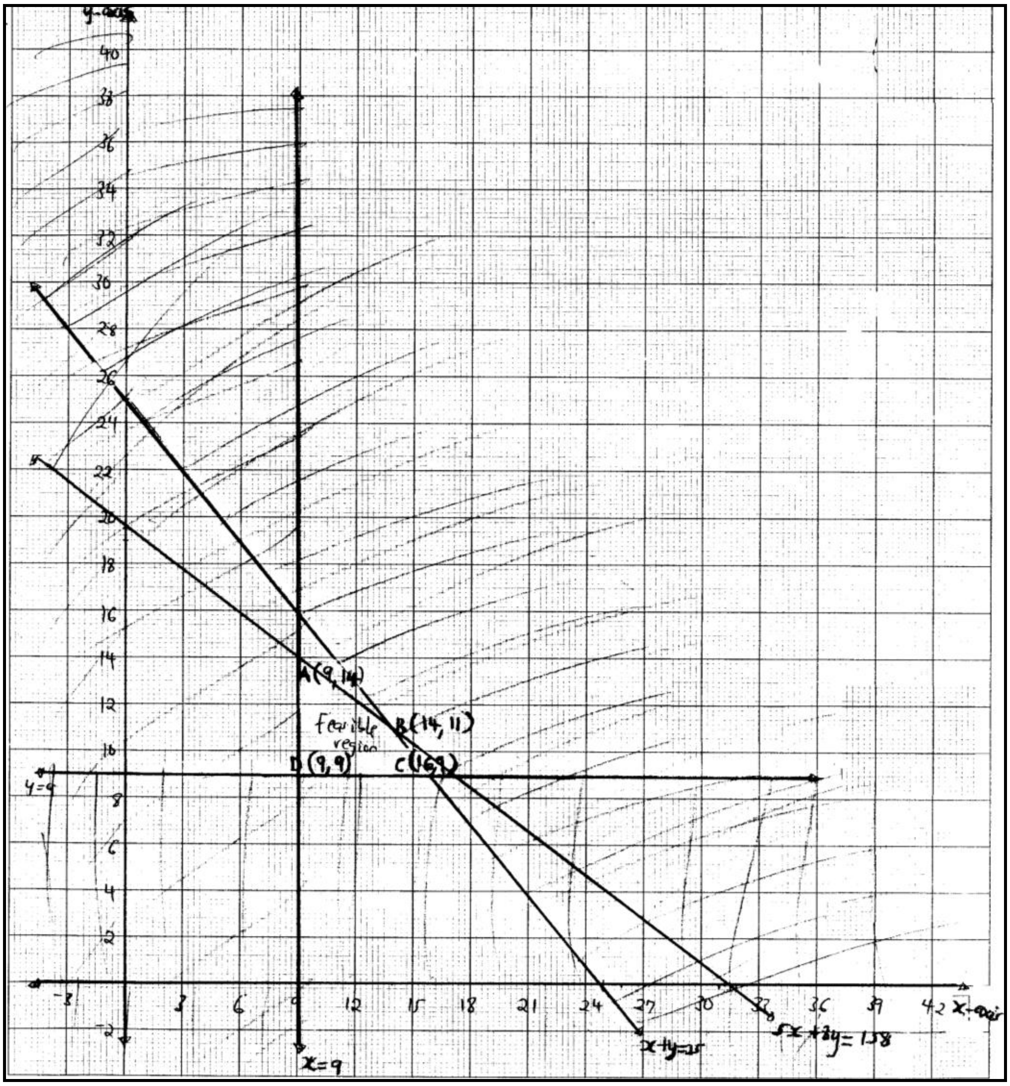
Tables of values

$x + y = 25$

x	0	25
y	25	0

$5x + 8y = 158$

x	0	31.6
y	19.75	0



corner points	objective function= 5000x + 6000y	value
A (9, 14)	5000 (9) + 6000(14)	129,000
B (14, 11)	5000 (14) + 6000 (11)	136,000
C (16, 9)	5000 (16) + 6000(9)	134,000
D(9, 9)	5000 (9) + 6000 (9)	99,000

∴ In order to maximize profit, he should buy 14 brown cows and 11 black cows.

Extract 11.1: A sample response of a candidate who answered this question correctly.

On the other hand, the analysis shows that, 36,862 (31.6%) candidates scored 0 mark as they failed to answer this question correctly due to various reasons, including the following:

Basically, the candidates failed to identify the decision variables. Instead of writing "Let *x* be the number of brown cows and *y* be the number of black cows", they wrote wrong statements like "Let *x* be brown cows and *y* be black cows ".

These candidates formulated the incorrect linear inequalities including: $9x + 9y \geq 25$ instead of $x + y \leq 25$;
$x + y \geq 9$ instead of $x \geq 9$ and $y \geq 9$;
$50{,}000x + 80{,}000y \geq 1{,}580{,}000$ or $80{,}000x + 50{,}000y \leq 1{,}580{,}000$ instead of $50{,}000x + 80{,}000y \leq 1{,}580{,}000$. The candidates lacked knowledge and skills on formulating the inequalities from the word problem.

Also, the candidates formulated the incorrect objective functions including; $f(x) = 50000x + 80000y$, $f(x, y) = 6000x + 5000y$ and $f(x) = x + y$ instead of $f(x, y) = 5000x + 6000y$.

With these reasons, the candidates drew incorrect graphs and consequently ended up with incorrect feasible regions and subsequently got incorrect number of cows and maximum profit.

Extract 11.2 shows a sample response of a candidate who failed to answer this question correctly.

11 x - y intercepts.

$5x + 8y = 158$

x	31.6
y	19.7

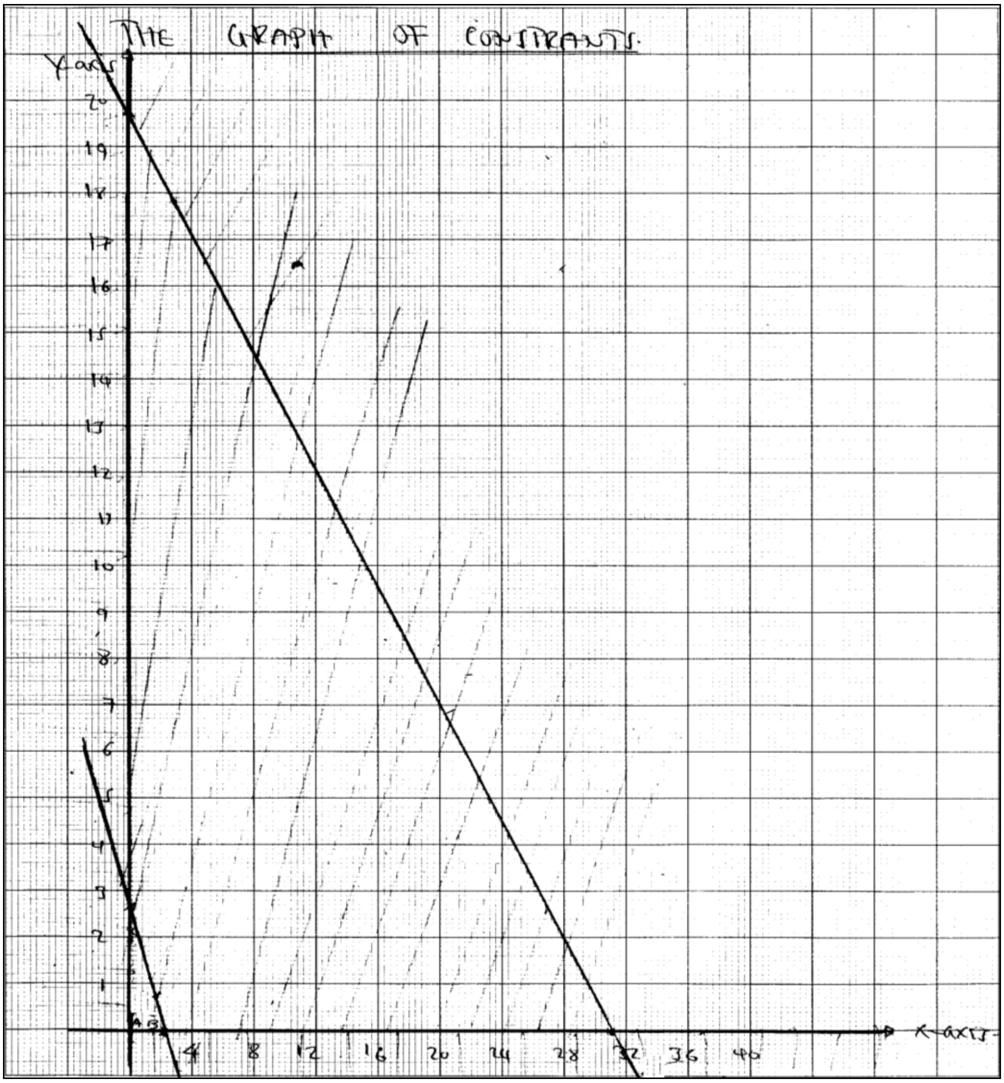
$9x + 9y = 25$

x	2.7
y	2.7

THE DECISION TABLE.

Corner point	obj f(x,y): 5,000x + 6,000y = Max.
A (0, 0)	0 + 0 = 0
B (2.7, 0)	5,000(2.7) + 0 = 13,500
C (0, 2.7)	0 + 6,000(2.7) = 16,200

∴ A farmer should buy ~~black~~ 3 black cows only inorder to ~~make~~ maximize profit.



Extract 11.2: A sample response of a candidate who failed to formulate the required linear inequalities, as a result ended up with incorrect graph.

2.12 Question 12: Statistics

The candidates were given that; the scores of a Civics test taken by 45 pupils were recorded as follows:

30	65	50	62	40	35	64	32	28
68	46	48	73	92	54	46	63	75
61	71	36	64	80	61	64	76	64
59	60	82	35	76	73	24	35	63
58	43	71	70	64	46	72	27	28

In part (a), they were required to construct a frequency distribution table of the given data, taking equal class intervals 21 – 40, 41 – 60, … In part (b), they were required to calculate the mean score, and in part (c) they were required to draw the cumulative frequency curve and use it to estimate the median.

Figure 12 represents the candidates' performance in this question. The figure reveals that, 139,913 (49.9%) candidates scored marks ranging from 0 to 2.5. The figure also shows that, 140,474 (50.1%) candidates scored marks from 3 to 10, out of which 4,532 (1.6%) candidates managed to attempt the question as required. Candidates had average performance in this question.

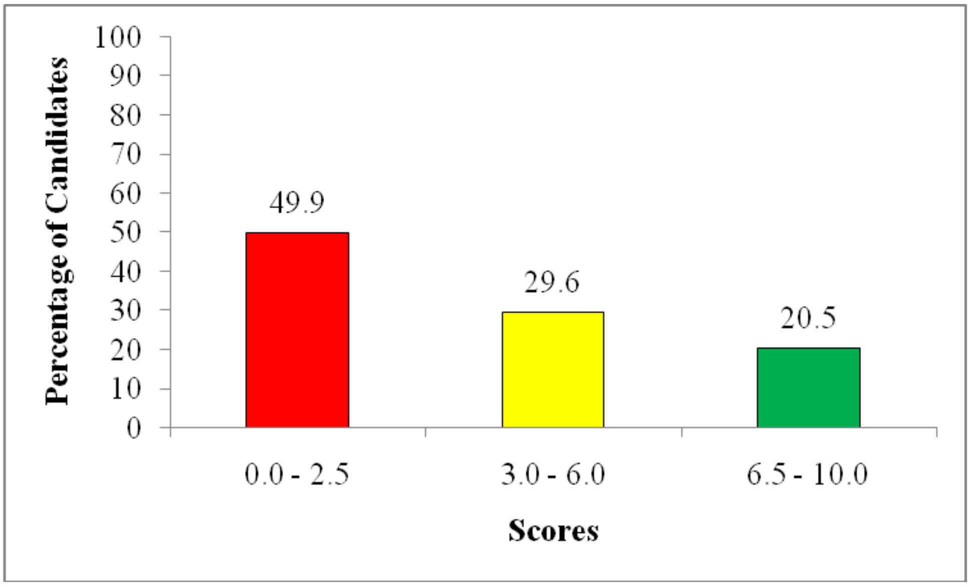


***Figure 12**: Candidates' Performance in Question 12.*

The candidates who answered this question correctly were able to: construct a frequency distribution table for the given data; calculate the mean score using the correct formula; and draw a cumulative frequency curve that they eventually used to get the correct estimate of the median as shown in Extract 12.1.

12. a) FREQUENCY DISTRIBUTION TABLE

Class Intervals	Tally marks	frequency (f)	Class mark (x)	fx	Cumulative frequency C.F
21 – 40	~~////~~ ~~////~~ /	11	30.5	335.5	11
41 – 60	~~////~~ ~~////~~	10	50.5	505.0	21
61 – 80	~~////~~ ~~////~~ ~~////~~ ~~////~~ //	22	70.5	1551.0	4~~2~~3
81 – 100	//	2	90.5	181.0	45
				2572.5	

b. Mean score $(\bar{x}) = \frac{\Sigma fx}{N}$

N = Total number of frequency

$N = 45$

$\Sigma fx = 2572.5$

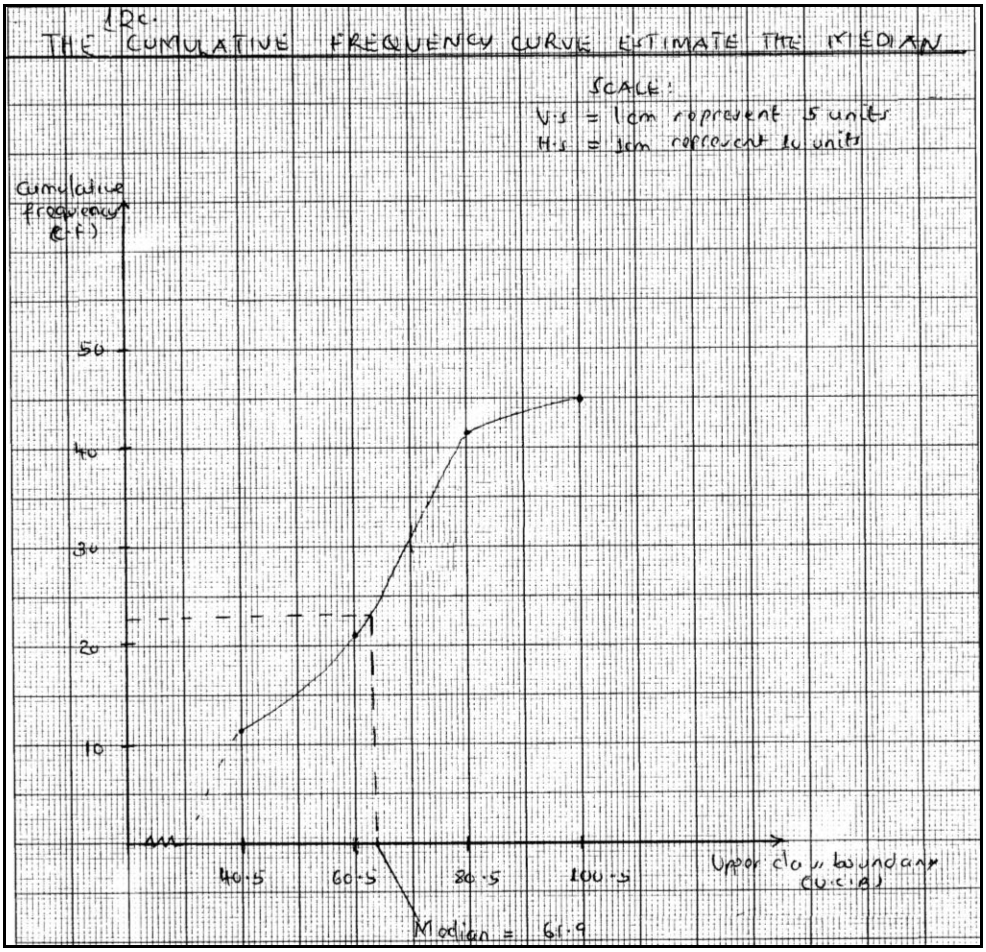
$\bar{x} = \frac{2572.5}{45}$

$\bar{x} = 57.17$

∴ Mean score is 57.17

c.

Cumulative frequency C.F	Upper class boundary (U.C.B)
11	40.5
21	60.5
43	80.5
45	100.5



Extract 12.1: A sample response of a candidate who had adequate knowledge and skills in finding the mean for grouped data and estimating the median from the graph.

On the other hand, further analysis shows that, 59,427 (21.2%) candidates were not able to attempt the question as required. In part (a), these candidates failed to construct a correct frequency distribution table. Some of them categorized classes like 61–70 or 81–90 instead of 61–80 or 81–100. These candidates had no understanding on the concept of "equal class size". Also, other candidates could not tally the given values to get the frequency of each interval. Failure to prepare the correct frequency distribution table led to wrong mean and cumulative frequency curve.

In part (b), the candidates were unable to get the required mean due to various reasons, including the following:

Using or citing the wrong formulae like: $\text{Mean} = L + \left(\frac{t_1}{t_1 + t_2}\right)i$ which is the formula for finding the mode; $\text{Mean} = \frac{\sum fx}{fx}$ or $\text{Mean} = \frac{\sum x}{N}$ instead of $\text{Mean} = \frac{\sum fx}{\sum f}$.

Inability to find the correct class marks, which was an important step for finding the mean. These candidates did not understand that the class marks were obtained by taking the average of the lower and upper class limits, that is, $\frac{21+40}{2}$, $\frac{41+60}{2}$, $\frac{61+80}{2}$ and $\frac{81+100}{2}$ that could give the correct class marks 30.5, 50.5, 70.5 and 90.5 respectively.

In part (c), the candidates failed to draw a correct cumulative frequency curve. Some candidates used the incorrect cumulative frequencies and the incorrect upper class boundaries. Few of them drew the cumulative frequency curves by using a ruler instead of using a free hand. Others drew histograms and frequency polygons contrary to the given instructions. These candidates lacked knowledge and skills on how to draw the cumulative frequency curve.

Failure to draw the correct graph led to wrong estimation of the median. However, few candidates were able to draw a correct graph but could not use it to estimate the median. Also, there were other candidates who calculated the median by using the formula contrary to the given instructions. Extract 12.2 shows a sample response of a candidate who failed to answer this question correctly.

FREQUENCY . DISTRIBUTION TABLE

12 a)

C. Intv	C. mark	F	C. F	Fx	L.L	up
21 - 40	30.5	11	11	805	20.5	40.5
41 - 60	50.5	9	20	495	40.5	60.5
61 - 70	70.5	12	32	410	60.5	70.5
71 - 80	90.5	9	41	845	70.5	80.5
81 - 90	110.5	1	42	1105	80.5	90.5
91 - 100	130.5	1	43	1301	90.5	100.5
		N=43		ΣFx 6361		

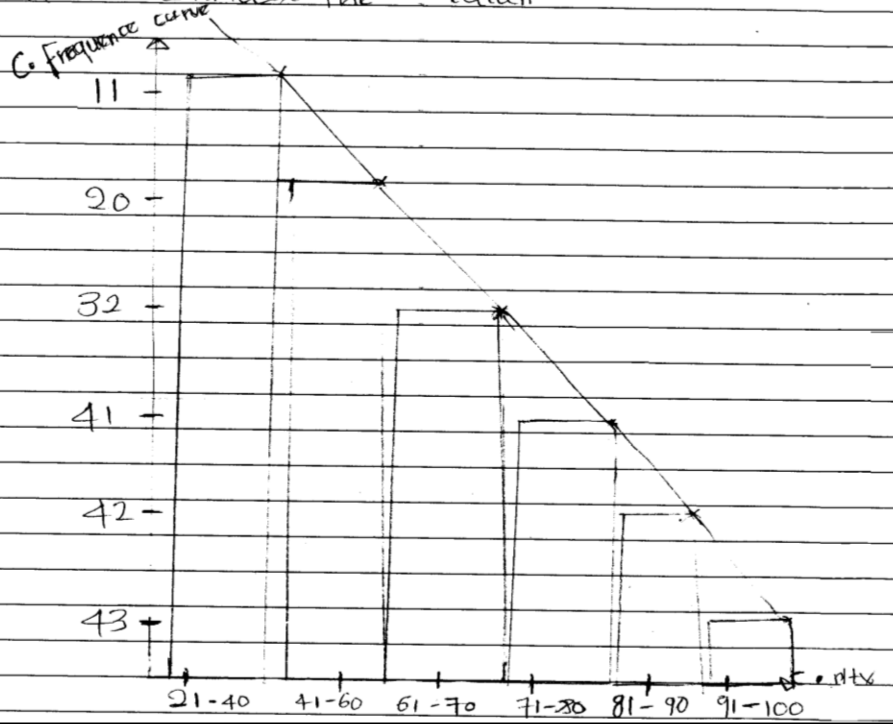
B) Calculate the mean score

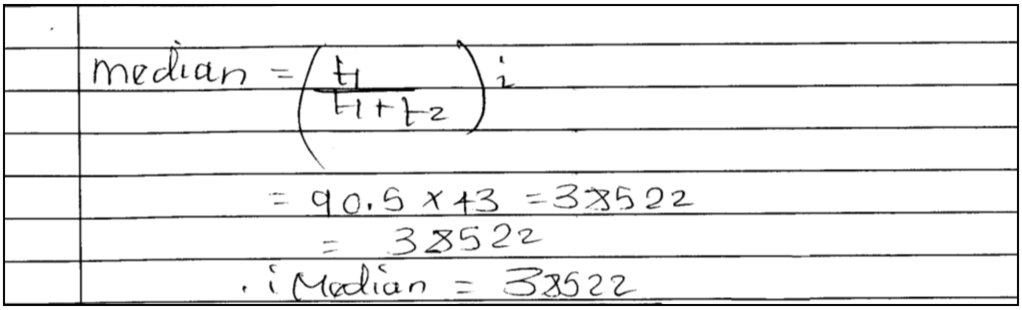
$$\text{mean} = \frac{\Sigma Fx}{N}$$

$$= \frac{6361}{43}$$

$\therefore$ Mean = 2.18

C) Draw the cumulative frequency and use it to estimate the median





$$\text{median} = \left(\frac{f_1}{f_1+f_2}\right) i$$

$= 90.5 \times 43 = 38522$

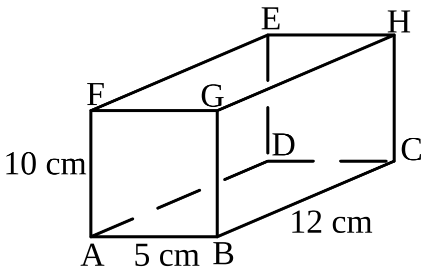
$= 38522$

$\therefore$ Median $= 38522$

Extract 12.2: A sample response of a candidate who wrote the wrong class intervals and drew a bar chart instead of cumulative frequency curve.

2.13 Question 13: Three Dimensional Figures and Circles

This question had parts (a) and (b). In part (a), the candidates were given the cuboid with $\overline{AB} = 5\,\text{cm}$, $\overline{BC} = 12\,\text{cm}$ and $\overline{BG} = 10\,\text{cm}$.



They were required to calculate (i) the length of AH correct to one decimal place and (ii) the angle CAH. In part (b), the candidates were required to find $A\hat{C}D$, $A\hat{D}B$, $D\hat{A}T$ and $C\hat{A}O$ in the following figure, where A, B, C and D lie on the circle with centre O, $\overline{BD}$ is the diameter, PAT is the tangent of the circle at A, $A\hat{B}D = 59°$ and $C\hat{D}B = 35°$.

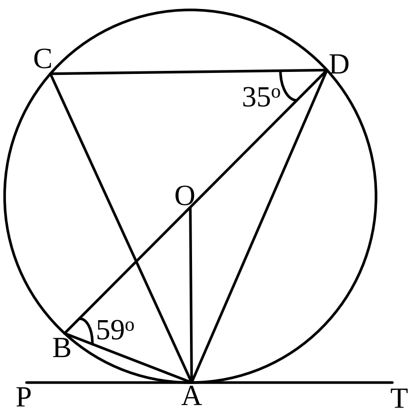


Figure 13 shows that, 61,635 (92.5%) candidates scored marks ranging from 0 to 2.5, out of which 51,836 (77.8%) candidates scored 0 mark. The figure also shows that, 5,021 (7.5%) candidates scored marks from 3 to 10. In general, the performance was weak.

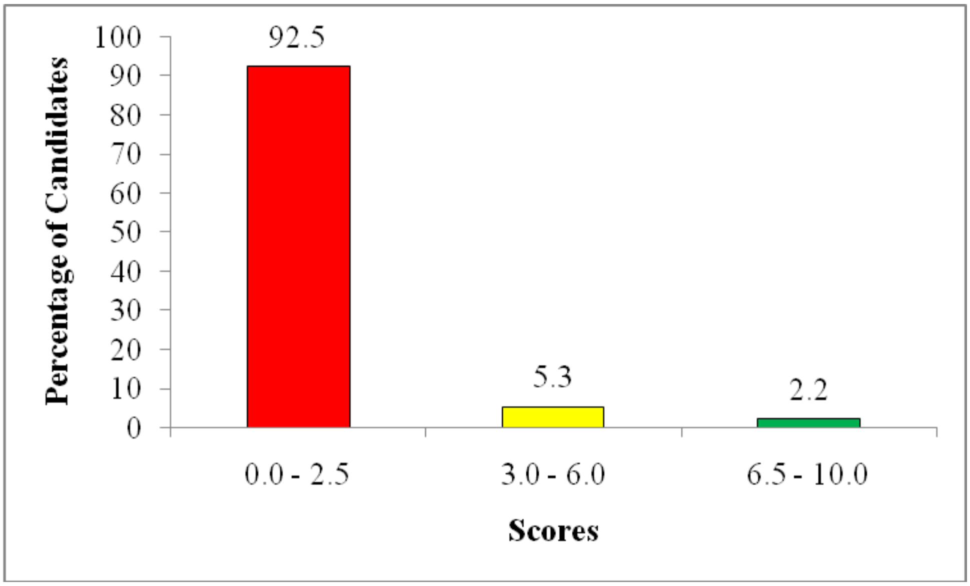


Figure 13: *Candidates' Performance in Question 13.*

In part (a), the candidates failed to find the length of $\overline{AH}$. Firstly, they were unable to get the length of $\overline{AC}$ and then the length of $\overline{AH}$ that could be obtained by using Pythagoras' theorem in the right angled triangle ACH. The majority calculated $\overline{AH}$ by using the sides $\overline{AH}$, $\overline{AB}$ and $\overline{AF}$ which do not form a right angled triangle. Basically, these candidates lacked knowledge and skills in constructing the right angled triangles *ABC* and *ACH* from the planes *ABCD* and *ACHF* respectively in the given cuboid.

In finding angle CAH, the candidates did not understand that they were supposed to apply trigonometric ratios involving any two sides in the right angled triangle ACH. Instead, most of them calculated the angle between the diagonal $\overline{AG}$ or $\overline{BH}$ and the base *ABCD*, instead of finding the angle between $\overline{AH}$ and the base *ABCD*.

In part (b), most of the candidates showed weaknesses in using the angle properties of a circle and other theorems to find the angles $A\hat{C}D$, $A\hat{D}B$,

$D\hat{A}T$ and $C\hat{A}O$. The majority were unable to identify that $A\hat{C}D$ and $A\hat{B}D$ are equal as they are subtended by an arc in the same segment $\overline{AD}$ or $m(O\hat{A}T) = 90°$ as $\overline{PT}$ is perpendicular to $\overline{OA}$ at *A*. This shows that these candidates lacked knowledge and skills on angle properties and circle theorems. Extract 13.1 shows some common mistakes from one of the candidates' responses.

13 (a) AH (i) $a^2+b^2 = c^2$

(i) $AH = (10)^2+(5)^2 = c^2$

$AH = 100+25 = c^2$

$AH = \sqrt{125} = \sqrt{c^2}$

$AH = 5\times5\times5$

$AH = 5\cdot5$

$AH = 5.5cm$

(ii) $C\hat{A}H = 90°$

13 b) $B\hat{A}D = 59°$ (angles on the same chord).

$\therefore A\hat{D}B = 59+59+x = 180°$

$118+x = 180°$

$x = 180-118$

$x = 62°$

$A\hat{D}B = 62°$

$A\hat{C}D = 35°+62$

$A\hat{C}D = 97°$ (angles on the same chord)

$\therefore A\hat{C}D = 97°$

$D\hat{A}T = C\hat{D}A$

$D\hat{A}T = 97°$

$\therefore D\hat{A}T = 97°$

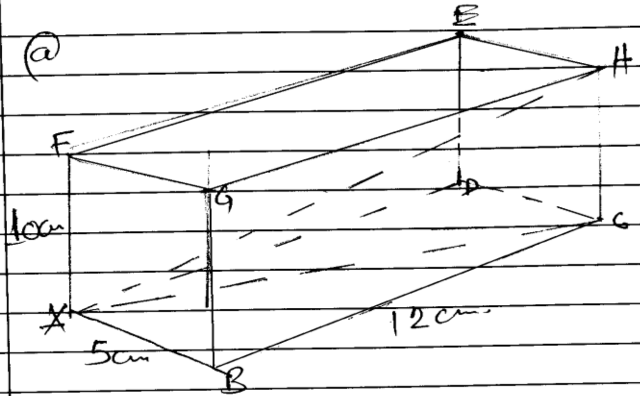
$C\hat{A}O = 97$

Extract 13.1: A sample response of a candidate who lacked enough knowledge and skills in application of Pythagoras' theorem, trigonometric ratios, circle theorems and angle properties.

Despite the weak performance, 336 (0.5%) candidates answered this question correctly. These candidates were able to calculate the length of $\overline{AH}$ by using Pythagoras' theorem, the angle between the length of $\overline{AH}$ and the plane *ABCD* and the required angles of the figure inscribed in a circle. Extract 13.2

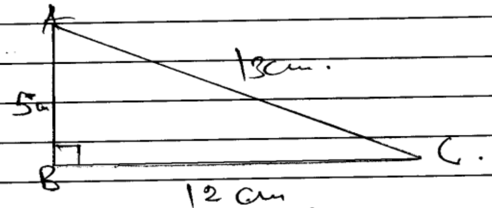
shows a sample response of a candidate who answered this question correctly.

13 (a)



(i) Length AH.

Consider the triangle ABC



$(AC)^2 = (12)^2 + (5)^2.$

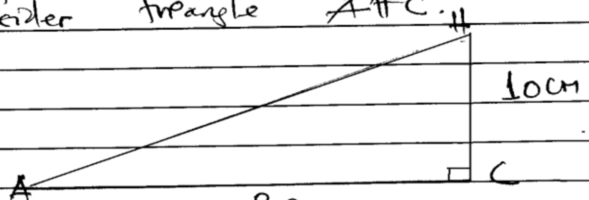
$(AC)^2 = 144 + 25$

$\sqrt{(AC)^2} = \sqrt{169}$

$AC = 13cm$

Solve for AH

Consider triangle AHC.



$(AH)^2 = (13cm)^2 + (10cm)^2$

13 (a) (i) $(AH)^2 = 169cm^2 + 100cm^2$

$(AH)^2 = \sqrt{269 cm^2}$

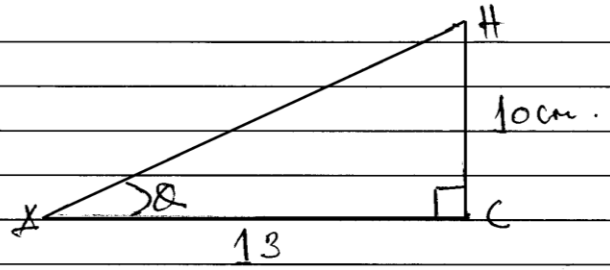
$AH = 16.4$ cm

∴ The length AH = 16.4 cm.

(ii) The angle CAH.

Soln

Consider Triangle CAH.



from

$\tan\theta = \frac{Opp}{Adj}$

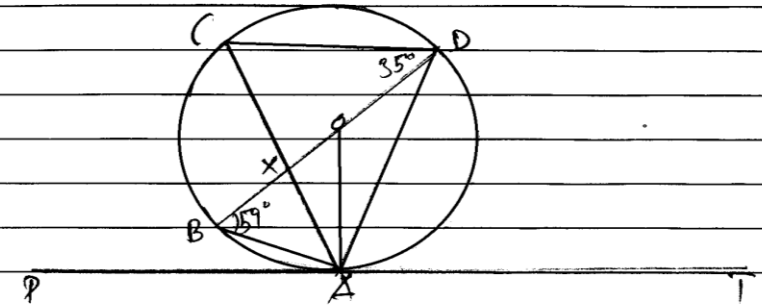
$\tan \hat{CAH} = \frac{10}{13} = 0.7692.$

$\hat{CAH} = \tan^{-1} 0.7692$

$\hat{CAH} = 37° 34'$

∴ The angle CAH $= 37°34'$.

(b)



13 (b) (i) $A\hat{C}D$

from $A\hat{O}D = 2(B\hat{A}O)$

$A\hat{O}D = 2(59°)$.

$A\hat{O}D = 118°$

Also

$A\hat{C}D = \frac{1}{2}(A\hat{O}D)$

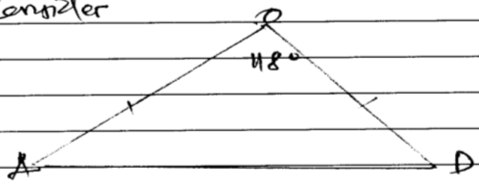
$A\hat{C}D = \frac{1}{2} \times 118°$

$A\hat{C}D = 59°$

$\therefore A\hat{C}D = 59°$.

(ii) $A\hat{D}B$

Consider



$A\hat{D}O = \hat{A}DB = DAO$.

Thus $A\hat{D}B + D\hat{A}O + A\hat{O}D = 180$

$A\hat{D}B + A\hat{D}B + 118° = 180°$

$2A\hat{D}B = 180° - 118°$

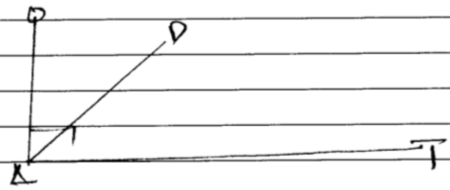
$\frac{2A\hat{D}B}{2} = \frac{62°}{2}$

$A\hat{D}B = 31°$

$\therefore A\hat{D}B = 31°$

(iii) $D\hat{A}T$

Consider



13 (b) (iii) Since $D\hat{A}O = A\hat{D}B$

$D\hat{A}O = 31°$.

But $T\hat{A}O = 90°$

$D\hat{A}O + D\hat{A}T = 90°$

$31° + D\hat{A}T = 90°$

$D\hat{A}T = 90° - 31°$

$D\hat{A}T = 59°$

$\therefore D\hat{A}T = 59°$.

(iv) $C\hat{A}O$

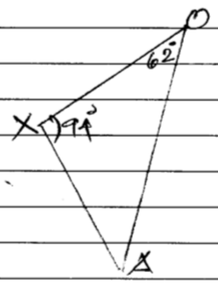
soln

from $A\hat{C}D = 59°$, $B\hat{D}C = 35°$.

Thus $O\hat{X}A = 59° + 35°$

$O\hat{X}A = 94°$.

Now Consider



$X\hat{A}O = C\hat{A}O$.

$C\hat{A}O + 94° + 62° = 180°$

$C\hat{A}O + 156° = 180°$

$C\hat{A}O = 180° - 156$

$C\hat{A}O = 24°$

$\therefore C\hat{A}O = 24°$.

Extract 13.2: A sample response of a candidate who applied the appropriate theorems in answering question 13.

2.14 Question 14: Accounts

In this question, the candidates were given that; Mwanne commenced business on 1st April, 2015 with capital in cash 200,000/=

April 2 bought goods for cash 100,000/=

3 bought goods for cash 300,000/=

4 purchased shelves for cash 230,000/=

5 sold goods for cash 400,000/=

9 paid wages for cash 50,000/=

12 purchased goods for cash 70,000/=

13 sold goods for cash 600,000/=

16 paid rent for cash 100,000/=
20 bought goods for cash 60,000/=
25 sold goods for cash 300,000/=
27 paid salary for cash 70,000/=

They were required to prepare (a) cash account and (b) trial balance.

Figure 14 shows that, 184,807 (77.7%) candidates scored marks from 3 to 10 and among them, 39,491 (16.6%) candidates scored 10 marks. Overall, candidates' performance was good.

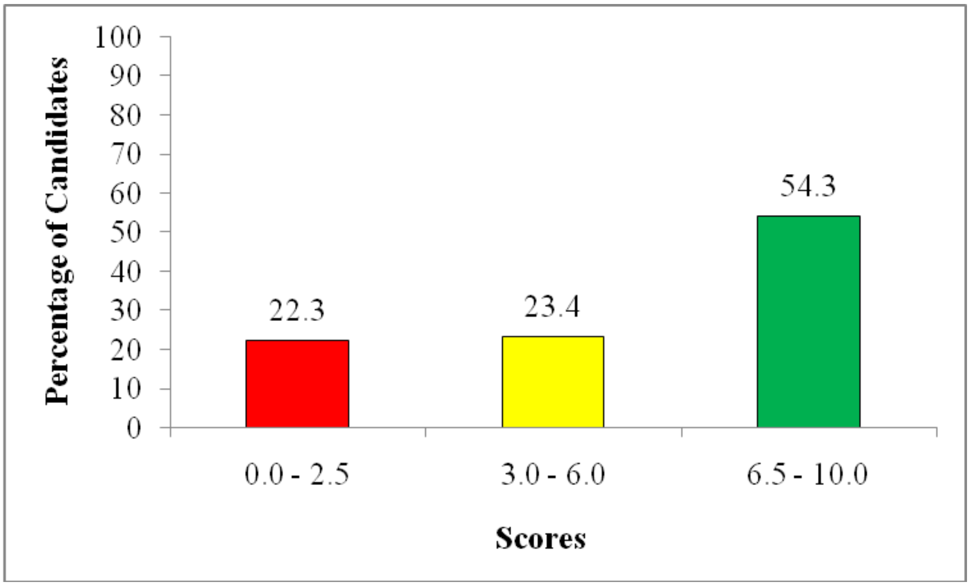


Figure 14: *Candidates' Performance in Question 14.*

Most of the candidates who attempted this question were able to prepare the cash account and trial balance by using the given transactions as illustrated in Extract 14.1.

14. (a)

DR CASH ACCOUNT CR

Date	Particulars	Folio	Amount	Date	Particulars	Folio	Amount
2015				2015			
1.4	capital		200,000	2.4	Purchases		100,000
5.4	Sales		400,000	3.4	Purchases		300,000
13 4	Sales		600,000	4.4	Shelves		230,000
25.4	Sales		300,000	9.4	Wages		50,000
				12.4	Purchases		70,000
				16.4	Rent		100,000
				20.4	Purchases		60,000
				27.4	Salaries		70,000
				30.4	Balance	c/d	520,000
			1,500,000				1,500,000
1.5	Balance	b/d	520,000				

MWANNE

(b) Trial balance as at 30th. April. 2015

S/NO	Particulars	Folio	Debit	Credit
1	cash		520,000	
2	Capital			200,000
3	Sales			1,300,000
4	Purchases		530,000	
5	Shelves		230,000	
6	Wages		50,000	
7	Rent		100,000	
8	Salaries		70,000	
	TOTAL		1,500,000	1,500,000

Extract 14.1: A sample response of a candidate who was able to prepare the cash account and trial balance as required.

Although the majority of candidates performed well, 30,287 (12.7%) candidates were not able to attempt the question as required. In part (a), some of them failed to identify the transactions that were to be posted in the debit side and credit side of the cash account. Others failed to present the standard format of the cash account, and wrongly named the columns of debit side and credit side. Similarly, other candidates did not indicate the debit side and credit side. Also, there were some candidates who confused between cash account and trading, profit and loss account. Unexpectedly, some candidates prepared the ledgers for each transaction which were not the requirements of the question.

In part (b), most of the candidates were unable to prepare the correct trial balance. Some candidates got incorrect balances (totals) due to wrong posting of entries, omission of entries or error in addition. Other candidates did not write the title of the trial balance as required. These candidates lacked knowledge and skills on how to prepare cash account and trial balance. Extract 14.2 shows a sample response of a candidate who failed to answer this question correctly.

CASH ACCOUNT

Date	Particular	F	Amount	Date	Particular	F	Amount
4 Apr	Purchases	L1	230,000	5. April	sold	L1	400,000
2	Cash	L2	100,000	13	Sold	L1	600,000
3	Cash	L2	300,000	16	rent	L2	100,000
~~5~~	~~Sold~~	~~L3~~	~~400,000~~	20	bough good	L3	60,000
12	Purchased.	L1	70,000	25	Sold good	L1	300,000
				27	Salary	L4	700,000
		b/d	110,000,000			c/d.	110,000,000

MWANNE BUSINESS ON 1st APRIL 2015

CAPITAL ACCOUNT

Date	Particular	F	Amount	Date	Particular	F	Amount
				1 April	Capital	L5	220,000

TRIAL BALANCE

Date	Particular	F	Amount	Date	Particular	F	Amount
April 5	sold	L1	400,000	April 1	Purchases	L1	230,000
13	sold	L1	600,000	2	cash	L2	100,000
16	rent	L2	100,000	3	cash	L2	300,000
20	bough	L3	60,000	12	Purchased	L1	700,000
25	Sold good	L1	300,000				
27	Salary	L4	700,000				
		c/d	133,000			b/d	133,000

Extract 14.2: A sample response of a candidate who failed to prepare the correct cash account and trial balance.

2.15 Question 15: Matrices and Transformations

This question had parts (a), (b) and (c). In part (a), the candidates were required to find the point $P(x, y)$ if $\begin{pmatrix} 2 & 3 \\ 4 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -23 \\ -11 \end{pmatrix}$. In part (b), they were given a translation T maps the point $P(x, y)$ obtained in part (a) into $(3, 2)$, and they were required to find where T takes the point $(7, 4)$. In part (c), the candidates were required to find the image of the point obtained in part (b) under a rotation of 90^{o} followed by another rotation of 180^{o} anticlockwise.

The data in Figure 15 show that, 123,419 (79.7%) candidates scored marks from 0 to 2.5 while 31,482 (20.3%) candidates scored marks from 3 to 10. Generally, candidates' performance was weak.

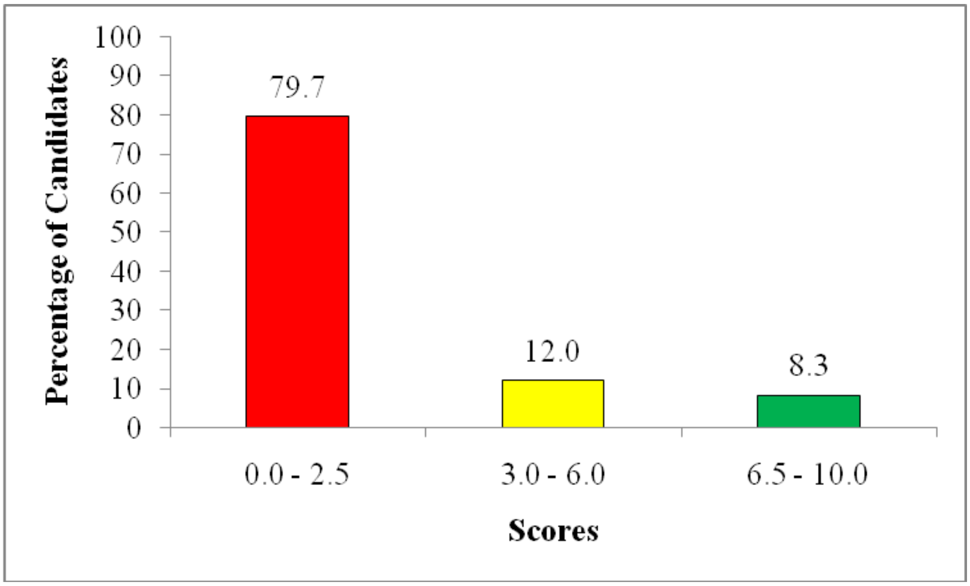


***Figure 15**: Candidates' Performance in Question 15.*

The weak performance in this question was attributed by several factors. In part (a), most of the candidates who used the inverse matrix method, incorrectly calculated the determinant of the coefficient matrix $A = \begin{pmatrix} 2 & 3 \\ 4 & -1 \end{pmatrix}$. For example, some candidates wrote $|A| = (4\times3) - (2\times-1) = 14$ and $|A| = (2\times1) - (4\times3) = -10$. They ended up with incorrect values of x and y.

Instead, they were supposed to write $|A| = (2\times -1) - (4\times 3) = -14$. Also, some candidates correctly got $\begin{vmatrix} 2 & 3 \\ 4 & -1 \end{vmatrix} = -14$ but calculated the absolute value of the answer, that is, $|-14| = 14$. These candidates confused between the notation of determinant of a matrix and the absolute value of a number.

It was noted that some candidates also made errors while interchanging the elements in the leading diagonal and changing the signs of the elements in the main diagonal when finding the inverse matrix.

In part (b), some candidates did not realize that they were required to use the point obtained in part (a) as an object and the given point $(3, 2)$ as an image to find the translation vector. They wrongly treated $(3, 2)$ and $(7, 4)$ as object and image respectively to obtain $\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} + \begin{pmatrix} 7 \\ 4 \end{pmatrix} = \begin{pmatrix} 10 \\ 6 \end{pmatrix}$. Instead, they were supposed to find the translation vector as follows: $\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} - \begin{pmatrix} -4 \\ -5 \end{pmatrix} = \begin{pmatrix} 7 \\ 7 \end{pmatrix}$ and finally the required image: $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 7 \\ 4 \end{pmatrix} + \begin{pmatrix} 7 \\ 7 \end{pmatrix} = \begin{pmatrix} 14 \\ 11 \end{pmatrix}$.

In part (c), most of the candidates were unable to recall the correct formula for rotation of a point. For example, some of them incorrectly wrote $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$ instead of $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$. Also, other candidates did not use the image obtained in part (b) to answer part (c) as well as the image obtained under rotation of 90^0 to find the image under rotation of 180^0. Furthermore, other candidates were unable to find the values of $\sin 90^0$, $\sin 180^0$, $\cos 90^0$ and $\cos 180^0$. Besides, the majority showed weaknesses in performing arithmetic operations of integers in all parts of this question. Some of the noted errors are shown in Extract 15.1.

15a $\begin{pmatrix} 2 & 3 \\ 4 & ^-1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} ^-23 \\ ^-11 \end{pmatrix}$

$$\underset{A}{\begin{pmatrix} 2 & 3 \\ 4 & ^-1 \end{pmatrix}} \underset{B}{\begin{pmatrix} x \\ y \end{pmatrix}} = \underset{C}{\begin{pmatrix} -23 \\ -11 \end{pmatrix}}$$

$$AB = C$$
$$A^{-1}AB = A^{-1}C$$
$$IB = A^{-1}C$$
$$B = A^{-1}C$$

$$A \begin{pmatrix} 2 & 3 \\ 4 & ^-1 \end{pmatrix}$$
$$= 3 \times 4 - 2 \times ^-1$$
$$= 12 - ^-2$$
$$= 24$$

$$\cancel{I}A = \frac{1}{|A|} \times \text{Adjoint matrix}$$

$$\frac{1}{-24} \begin{pmatrix} ^-1 & ^-3 \\ ^-4 & 2 \end{pmatrix}$$

$$\begin{pmatrix} 1/24 & ^+1/8 \\ 1/6 & -1/12 \end{pmatrix} \begin{pmatrix} ^-23 \\ ^-11 \end{pmatrix}$$

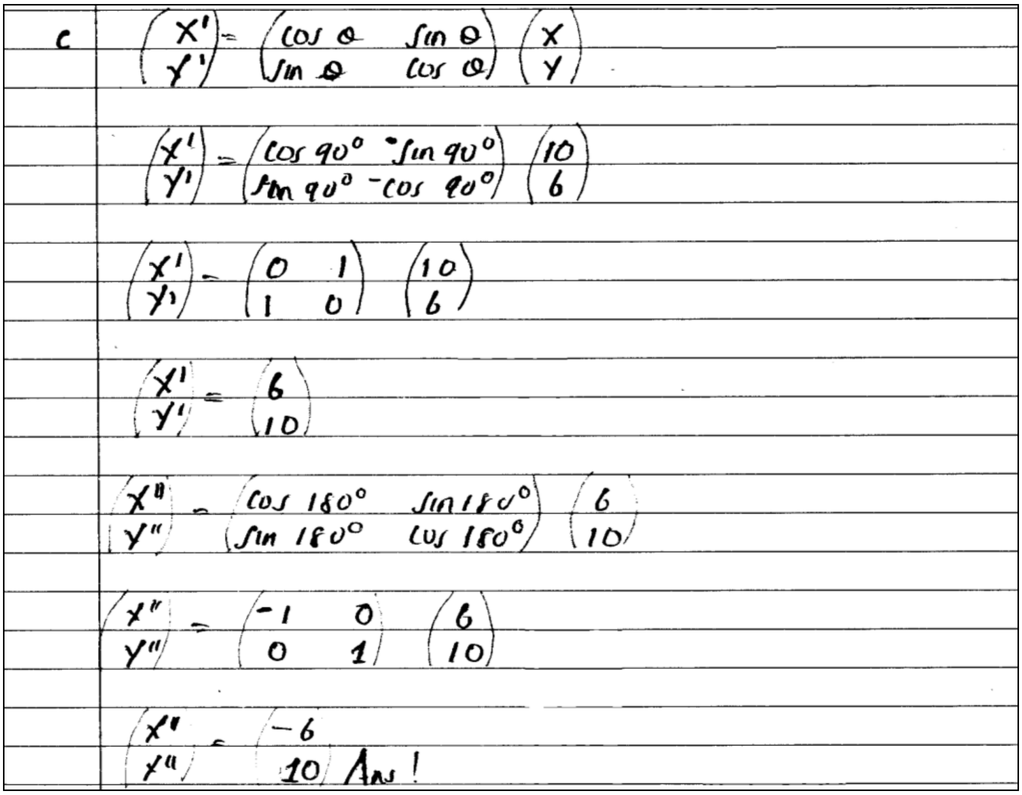
$$\begin{pmatrix} ^-23/24 + 11/8 \\ ^-23/6 + 11/12 \end{pmatrix}$$

$$\begin{pmatrix} 12/24 \\ 12/12 \end{pmatrix} \quad \begin{pmatrix} 1/2 \\ 1 \end{pmatrix} \text{Ans!}$$

b $(T\ a,b) = (x, y) + (x', y')$

$(T(a,b)) = (3, 2) + (7, 4)$

$(T\ a,b) = (10, 6)$ Ans!



c $\begin{pmatrix} X' \\ Y' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix}$

$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos 90^\circ & -\sin 90^\circ \\ \sin 90^\circ & -\cos 90^\circ \end{pmatrix} \begin{pmatrix} 10 \\ 6 \end{pmatrix}$

$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 10 \\ 6 \end{pmatrix}$

$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 6 \\ 10 \end{pmatrix}$

$\begin{pmatrix} x'' \\ y'' \end{pmatrix} = \begin{pmatrix} \cos 180^\circ & \sin 180^\circ \\ \sin 180^\circ & \cos 180^\circ \end{pmatrix} \begin{pmatrix} 6 \\ 10 \end{pmatrix}$

$\begin{pmatrix} x'' \\ y'' \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 6 \\ 10 \end{pmatrix}$

$\begin{pmatrix} x'' \\ y'' \end{pmatrix} = \begin{pmatrix} -6 \\ 10 \end{pmatrix}$ Ans !

Extract 15.1: A sample response of a candidate who failed to get the determinant, inverse matrix, and ended up with wrong answers.

On the other hand, 6,758 (4.4%) candidates were able to attempt this question as required. The good performance was mainly contributed by ability of the candidates to solve the linear equations that were presented in matrix form as well as finding the images of the points under translation and rotation. Extract 15.2 illustrates a sample response from one of the candidates.

15 a) $\begin{pmatrix} 2 & 3 \\ 4 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -23 \\ -11 \end{pmatrix}$

soln.

let $\begin{pmatrix} 2 & 3 \\ 4 & -1 \end{pmatrix}$ be A. and find determinant of A.

$|A| = 2\times{-1} - 4\times 3$

$|A| = -2 - 12$

$|A| = -14$

$A^{-1} = \frac{1}{|A|}\begin{bmatrix} -1 & -3 \\ -4 & 2 \end{bmatrix}$

$A^{-1} = \frac{1}{-14}\begin{bmatrix} -1 & -3 \\ -4 & 2 \end{bmatrix}$

$A^{-1} = \begin{pmatrix} -1/-14 & -3/-14 \\ -4/-14 & 2/-14 \end{pmatrix}$

$A^{-1} = \begin{bmatrix} 1/14 & 3/14 \\ 2/7 & -1/7 \end{bmatrix}$

Then premultiply by the inverse both sides of the matrix equation.

$\begin{pmatrix} 1/14 & 3/14 \\ 2/7 & -1/7 \end{pmatrix}\begin{pmatrix} 2 & 3 \\ 4 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1/14 & 3/14 \\ 2/7 & -1/7 \end{pmatrix}\begin{pmatrix} -23 \\ -11 \end{pmatrix}$

but

$AA^{-1} = I$

$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -23/14 & -33/14 \\ -46/7 & 11/7 \end{pmatrix}$

$\begin{pmatrix} x + 0 \\ 0 + y \end{pmatrix} = \begin{pmatrix} -56/14 \\ -35/7 \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -4 \\ -5 \end{pmatrix}$

$\therefore\ x = -4$ and $y = -5$

$P(x, y) = P(-4, -5)$

b) $P(x, y) = P(^-4, ^-5) \longrightarrow P'(3, 2)$

From

Translation $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix} + \begin{pmatrix} x \\ y \end{pmatrix}$

$x_1, y_1 = 3, 2 \quad x, y = ^-4, ^-5 \quad a, b = ?$

$\begin{pmatrix} 3 \\ 2 \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix} + \begin{pmatrix} -4 \\ ^-5 \end{pmatrix}$

$\begin{pmatrix} 3 \\ 2 \end{pmatrix} - \begin{pmatrix} -4 \\ -5 \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix}$

$\begin{pmatrix} 7 \\ 7 \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix}$

$\therefore$ Translation vector is $(7, 7)$

Hence $x', y' = ?$ $T(7, 7)$ and $(x, y) = (7, 4)$

Then

$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix} + \begin{pmatrix} x \\ y \end{pmatrix}$

$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 7 \\ 7 \end{pmatrix} + \begin{pmatrix} 7 \\ 4 \end{pmatrix}$

$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 14 \\ 11 \end{pmatrix}$

$\therefore$ It will take point $(7, 4)$ to $(14, 11)$

c) Point $(14, 11)$

R_{+90}

Matrix of rotation $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$

$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos 90 & -\sin 90 \\ \sin 90 & \cos 90 \end{pmatrix} \begin{pmatrix} 14 \\ 11 \end{pmatrix}$

$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 14 \\ 11 \end{pmatrix}$

$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 0 & -11 \\ 14 & 0 \end{pmatrix} = \begin{pmatrix} -11 \\ 14 \end{pmatrix}$

$\therefore$ After rotation through 90° image is $(-11, 14)$

Then through 180°

$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$

$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos 180 & -\sin 180 \\ \sin 180 & \cos 180 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$

$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} -11 \\ 14 \end{pmatrix}$

$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 11 & 0 \\ 0 & -14 \end{pmatrix}$

$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 11 + 0 \\ 0 + -14 \end{pmatrix} = \begin{pmatrix} 11 \\ -14 \end{pmatrix}$

$\therefore$ The image under R_{+90} and R_{+180° is $(11, -14)$

Extract 15.2: A sample response of a candidate who answered question 15 correctly.

2.16 Question 16: Functions and Probability

This question had parts (a) and (b). In part (a), the candidates were required to determine the probability of picking (i) three blue shirts, (ii) two white shirts and one blue shirt and (iii) one white shirt and two blue shirts, when three shirts are picked at random one after another with replacement from a bag containing 6 white shirts and 3 blue shirts. In part (b), they were required to (i) sketch the graph of f and (ii) use the graph to determine the domain and range of f, where f was defined by $f(x)=\begin{cases} -2 \text{ if } x<-1 \\ 0 \text{ if } x=-1 \\ x+2 \text{ if } x\geq -1 \end{cases}$.

The candidates' performance in this question is summarized in Figure 16. It is evident from the figure that, candidates' performance was weak because only 17,238 (8.3%) candidates managed to score from 3 to 10 marks. Also, 189,772 (91.7%) candidates scored low marks from 0 to 2.5 and among them, 64,318 (31.1%) candidates scored 0 mark.

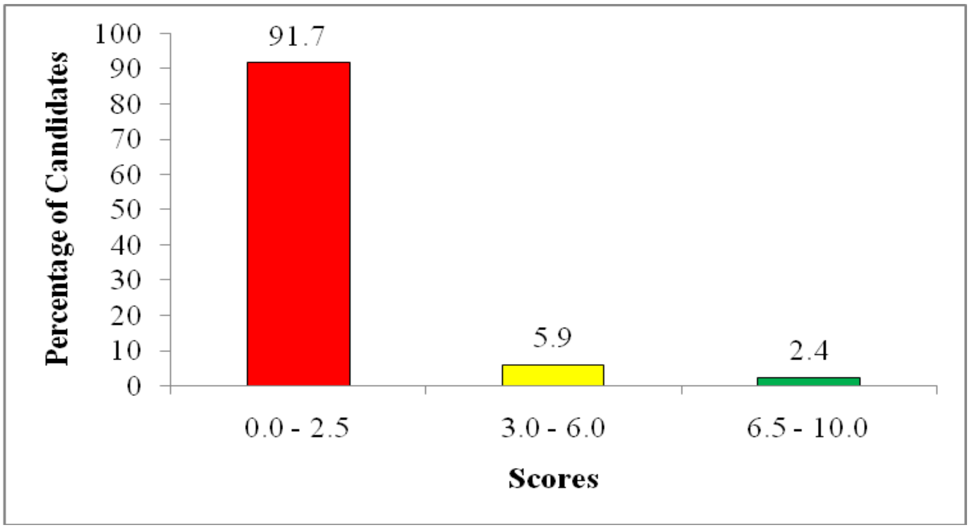
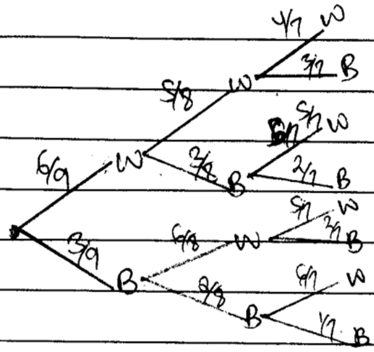


***Figure 16**: Candidates' Performance in Question 16.*

In part (a), the majority of candidates could not correctly present the given information using a tree diagram. Even when they sketched tree diagrams they failed to assign the correct probabilities. Some assumed that the picking of shirts was done without replacement, which was contrary to the requirements of the question. Other candidates were not able to find the probabilities of three combined events. This indicates that the candidates lacked adequate knowledge and skills on solving probability problems by using a tree diagram and application of probability rules.

In part (b), many candidates lacked the skills to draw a graph of a function defined on sub intervals (sub-domains). Some candidates were not able to interpret the signs "$<$", "$=$" and "$\geq$" when sketching the graph. As a result they could not use the symbols "●" for included values and "○" for excluded values. Also, the candidates faced difficulties to get the required range of the function. For example, many of them wrote range is {all real numbers}, instead of $\{y: y=-2,\ y=0 \text{ and } y \geq 1\}$. Extract 16.1 represents a sample response of a candidate who was unable to attempt this question.

16. a)



let w represent white shirts

let B represent blue shirts.

i) $\frac{3}{9} \times \frac{2}{8} \times \frac{1}{7}$

$= \frac{3}{9} \times \frac{2}{8}.$

$= \frac{6}{72} \times \frac{1}{7}.$

$= \frac{6}{504}$

$= \frac{1}{84}.$

$\therefore$ The probability that All three shirts are blue in colour is $\frac{1}{84}$.

ii) $\frac{6}{9} \times \frac{5}{8} \times \frac{3}{7}$

$= \frac{6}{9} \times \frac{5}{8}$

$= \frac{30}{72} \times \frac{3}{7}$

$= \frac{90}{504}$

$= \frac{5}{28}$

$\therefore$ The probability that two shirts are white and one shirt is blue is $5/28$.

iii) $\frac{6}{9} \times \frac{3}{8} \times \frac{2}{7}$

$= \frac{6}{9} \times \frac{3}{8}$

$= \frac{18}{72} \times \frac{2}{7}$

$= \frac{36}{504}$

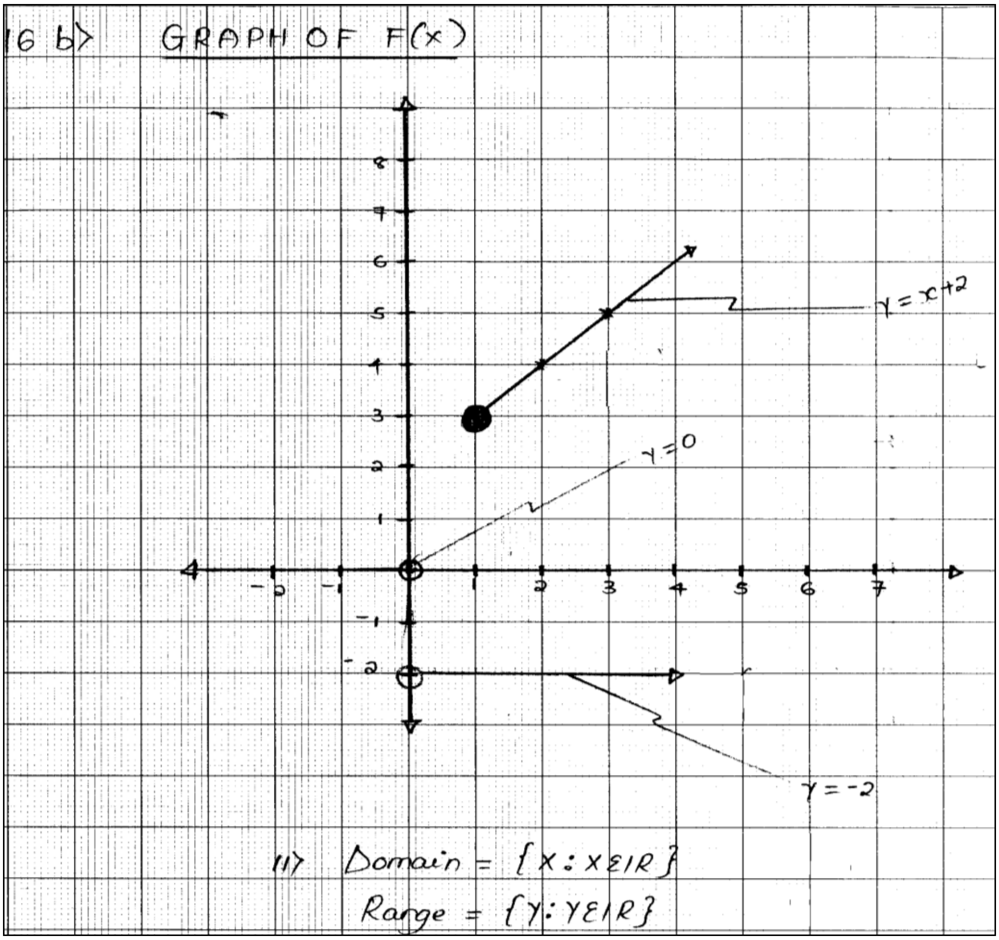
$\therefore$ The probability that one shirt is white and two shirts are blue is $1/14$.

16 b) $F(x) = \begin{cases} -2 & \text{when } x < -1 \\ 0 & \text{if } x = -1 \\ x+2 & \text{if } x \geq -1 \end{cases}$

Table value

$y = x + 2$

x	1	2	3
y	3	4	5



Extract 16.1: A sample response of a candidate who had no understanding on the concept of "replacement" in probability experiments and the use of subdomains in drawing the graph.

On the other side, 475 (0.2%) candidates managed to attempt this question as intended. They were able to sketch the correct tree diagram and determine the required probabilities. They were also able to sketch the correct graph of the given function and eventually determine the domain and range as shown in Extract 16.2.

16(a) Let W – set of white shirts

B – set of blue shirts.

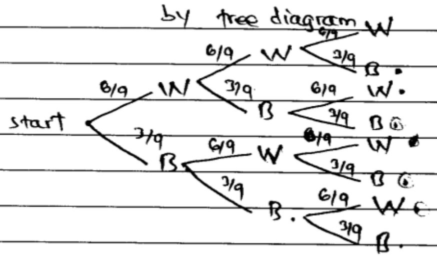
Given $n(W) = 6$

$n(B) = 3$

$n(\xi) = 9.$

(i) Probability that all are blue.

by tree diagram



Probability of all three having blue colour $= 3/9 \times 3/9 \times 3/9 = \frac{27}{729} = 1/27.$

$\therefore = \cancel{27/729}$. $\therefore = 1/27.$

(ii) Two shirts are white and One blue = Event (E).

From tree diagram above.

$P(E) = (6/9 \times 6/9)(3/9) + (6/9 \times 3/9 \times 6/9) + (3/9 \times 6/9 \times 6/9).$

$= \frac{108}{729} + \frac{108}{729} + \frac{108}{729} = \frac{108+108+108}{729}$

$= \frac{324}{729} = 4/9.$

$\therefore$ The probability is $4/9$.

(iii) Let E = One shirt is white two shirts are blue.

$P(E) = (6/9 \times 3/9 \times 3/9) + (6/9 \times 3/9 \times 3/9) + (3/9 \times 3/9 \times 6/9)$

$= \frac{54}{729} + 54/729 + 54/729 = \frac{162}{729} = 2/9$

16 (a)(ii) The probability for two blue shirts and one white is $2/9$.

(b) Tables of values for.

-2 if $x < -1$.

x	−1	−2	−3	−4.
Y	−2	−2	−2	−2.

0 if $x = -1$

x	−1.
y	0

$x+2$ if $x \geq -1$.

x	−1	0	1	2	3.
Y	1	2	3	4	5.

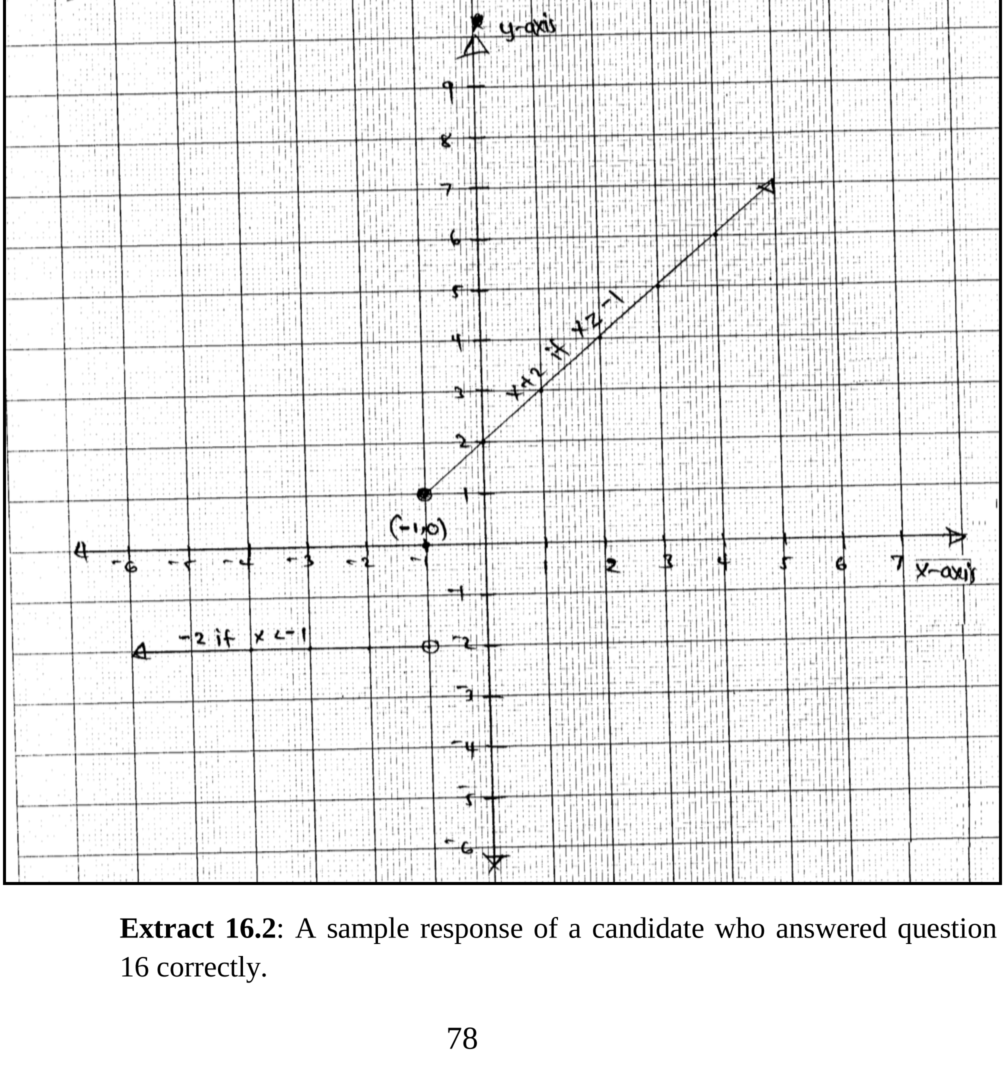
(i) Graph on graph paper.

(ii) From the graph:

Domain = $\{x : x = \mathbb{R}\}$

Range = $\{y : y = 0, y = -2\ y \geq 1\}$

16 (b) (i)



Extract 16.2: A sample response of a candidate who answered question 16 correctly.

3.0 CONCLUSION AND RECOMMENDATIONS

3.1 Conclusion

The candidates' performance was analyzed on each topic that was tested. The percentage of candidates who passed is indicated in the brackets. Out of **twenty four** (**24**) topics that were tested, candidates had good performance on **one** (**01**) topic of *Accounts* (77.7%). They performed averagely on **five** (**05**) topics of: *Statistics* (50.1%); *Linear Programming* (34.7%); and *Numbers, Fractions* and *Decimals* (33.3%).

The candidates had weak performance on the remaining **eighteen** (**18**) topics. The topics were: *Rates and Variations* (23.1%); *Matrices and Transformations* (20.3%); *Ratios, Profit and Loss* (18.1%)*; Sets* and *Algebra* (17.3%); *Pythagoras' Theorem* and *Trigonometry* (17.0%); *Sequences and Series* (12.3%); *Quadratic Equations* (10.9%); *Exponents* and *Logarithms* (10.2%); *Functions* and *Probability* (8.3%); *Circles* and *Three Dimensional Figures* (7.5%); *Vectors* and *Coordinate Geometry* (5.3%); and *Perimeters and Areas* (3.4%). The performance of the candidates on each topic is shown in the Appendix.

The weak performance in these topics was contributed by several factors including the candidates' inability to:

(a) use laws, formulae, theorems and other mathematical concepts in answering the questions.
(b) identify the requirements of the questions that led to use wrong methods in solving the questions.
(c) formulate expressions, equations and inequalities from the word problems.
(d) sketch the correct diagrams and graphs which were useful in answering the questions.
(e) perform mathematical operations correctly. Candidates made mistakes or errors mainly on integers and the use of brackets.

3.2 Recommendations

In order to improve the candidates' performance in the future, it is recommended that:

(a) The students should;
    (i) use various learning materials such as books and journals in order to improve their competence in Mathematics.

(ii) form discussion groups and participate effectively in solving questions.
(iii) consult their teachers for any concept that need more explanation in order to improve their understanding.

(b) The teachers should;
(i) teach according to the syllabus by using participatory methods and cover all topics on time.
(ii) provide regular exercises so that students can apply the formulae and other mathematical concepts in answering questions.
(iii) incorporate real life related practices when teaching including using different teaching aids to enhance the students' understanding especially in the topics with weak performance.
(iv) assign students classroom projects as a part of teaching and learning strategies in order to master the lessons/subject.
(v) assess students according to their learning abilities and advise a mechanism to assist them appropriately.

(c) The government should;
(i) facilitate in-house trainings for teachers in order to update their knowledge and skills in Mathematics subject.
(ii) make follow-ups in schools mainly on the effective and timely implementation of the syllabus so as to ensure that all topics are well covered.

Appendix

ANALYSIS OF THE CANDIDATES' PERFORMANCE TOPIC-WISE CSEE 2018

S/N	Topic/Subtopic	Question Number	The Percentage of Candidates who Passed	Remarks
1	Accounts	14	77.7	Good
2	Statistics	12	50.1	Average
3	Linear Programming	11	34.7	Average
4	Numbers, Fractions and Decimals	1	33.3	Average
5	Rates and Variations	6	23.1	Weak
6	Matrices and Transformations	15	20.3	Weak
7	Ratios, Profit and Loss	7	18.1	Weak
8	Sets and Algebra	3	17.3	Weak
9	Pythagoras Theorem and Trigonometry	9	17.0	Weak
10	Sequences and Series	8	12.3	Weak
11	Quadratic Equations	10	10.9	Weak
12	Exponents and Logarithms	2	10.2	Weak
13	Functions and Probability	16	8.3	Weak
14	Circles and Three Dimensional Figures	13	7.5	Weak
15	Vectors and Coordinate Geometry	4	5.3	Weak
16	Perimeters and Areas	5	3.4	Weak