THE UNITED REPUBLIC OF TANZANIA
MINISTRY OF EDUCATION, SCIENCE AND TECHNOLOGY
NATIONAL EXAMINATIONS COUNCIL OF TANZANIA

CANDIDATES' ITEM RESPONSE ANALYSIS REPORT ON THE CERTIFICATE OF SECONDARY EDUCATION EXAMINATION (CSEE) 2022

BASIC MATHEMATICS

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041 BASIC MATHEMATICS

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FOREWORD

This report presents the Candidates' Item Response Analysis (CIRA) on the Secondary Education Basic Mathematics National Examination which was conducted in November 2022. The report aims at providing feedback to all educational stakeholders on the factors that contributed to the candidates' performance in Basic Mathematics.

The Certificate of Secondary Education Examination (CSEE) intends to evaluate candidates' skills and knowledge gained in their four years of secondary education and provides feedback to teachers, students and other educational stakeholders for improving teaching and learning. This analysis shows justification for the candidates' performance in the Basic Mathematics subject. The candidates who scored low marks faced various challenges including inability to: recall and apply correct formulae, rules, theorems, properties and postulates, formulate mathematical inequalities, expressions and equations from word problems, use appropriate procedures when performing calculations and failure to sketch and interpret figures, graphs and diagrams correctly.

This report will help students to identify their strengths and weaknesses to improve learning before sitting for the Certificate of Secondary Education Examination (CSEE). It will also help teachers to identify the challenging areas and take appropriate measures during teaching and learning.

The National Examinations Council of Tanzania (NECTA) expects that the feedback provided in this report will be useful to teachers, students and other education stakeholders in overcoming the challenges that contributed to low performance. It is also expected that the feedback provided will guide education stakeholders through taking appropriate measures to improve teaching and learning of the Basic Mathematics subject. Consequently, students will acquire knowledge, skills and competence indicated in the syllabus for better performance in future examinations.

The Council appreciates the contribution of all those who prepared this report.

Dr. Said Ally Mohamed
EXECUTIVE SECRETARY

1.0 INTRODUCTION

This report is based on the candidates' item response analysis on the Certificate of Secondary Education Examination (CSEE) 2022 in the Basic Mathematics subject. The analysis mainly describes the areas where the candidates faced difficulties and areas where they performed well in answering the questions.

The data analysis shows that out of 520,332 candidates who sat for Basic Mathematics examination in CSEE 2022, only 104,488 (20.08%) candidates passed, where in CSEE 2021, only 94,677 (19.54%) out of 484,439 candidates who sat for the examination passed. Therefore, by comparison, the CSEE 2022 performance has increased by 0.54 per cent.

According to the Basic Mathematics examination format, the paper consisted of two sections, A and B. Each question in Section A carried six (6) marks while each question in Section B carried 10 marks and the candidates were required to answer all questions from both sections.

The candidates' performance in each question was considered *Good, Average* or *Weak* given the percentage of candidates who scored at least 30 per cent is 65 – 100, 30 – 64 or 0 – 29 per cent, respectively.

2.0 ANALYSIS OF THE CANDIDATES' PERFORMANCE ON EACH QUESTION

This part addresses the analysis of the candidates' performance in each question. The analysis includes the description of the requirements of the question, the summary on how candidates attempted the questions and the sample extracts of the candidates' correct and incorrect responses. It further indicates the reasons for the success and failure of the candidates to get the correct responses to each question. The analysis of the candidates' performance and graphical presentation were done based on the score intervals of 6.0 – 4.0, 3.5 – 2.0 and 1.5 – 0.0 out of 6 marks in Section A; and 10 – 6.5, 6.0 – 3.0 and 2.5 – 0.0 out of 10 marks in Section B in each question to mean *Good*, *Average* and *Weak* performance, respectively.

2.1 Question 1: Numbers, Fractions, Percentages and Logarithms

The question had two parts, (a) and (b). In part (a), the candidates were examined whether they could find the percentage of numbers which are multiples of 5 in one decimal place from the set $\{1, 2, 3, 4, ..., 52\}$. Part (b)(i) intended to examine whether they were able to arrange the fractions $\frac{1}{2}$, $\frac{2}{9}$, $\frac{3}{8}$, $\frac{1}{12}$ and $\frac{2}{5}$ in ascending order. Part (b)(ii) examined whether they were able to simplify $\frac{7\times10^4}{0.000035}$ and write the answer in the standard form.

The analysis shows that this question was attempted by 521,886 (100%) candidates where a total of 116,853 (22.4%) candidates passed. This shows that the candidates' performance on this question was weak. Further analysis indicates that, 13,434 (2.6%) candidates scored all the allotted marks while 330,868 (63.4%) scored zero. Figure 1 summarizes the candidates' performance on question 1.

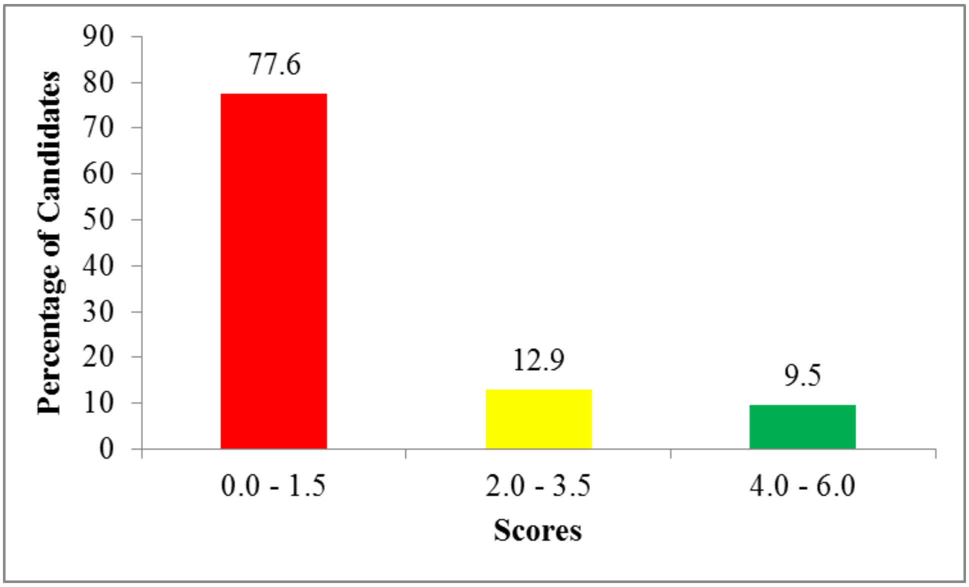


Figure 1: *Candidates' performance on question 1*

The analysis of responses shows that, the candidates who failed to answer this question correctly lacked knowledge and skills in the concepts of numbers, decimals and percentages. In part (a), the candidates were not able to find the percentage of numbers which are multiple of 5 from the set $\{1, 2, 3, ..., 52\}$. They failed to meet the requirement of the question. For instance, some of the candidates presented the numbers in a Venn diagram.

Further analysis shows that some of the candidates calculated the LCM and GCF of $\{1, 2, 3, 4, 5\}$ which was incorrect. Also, other candidates took 5 as a sample space and some numbers which were sorted out in the question as number of events. They failed to recognize the given set of natural numbers. Furthermore, other candidates were able to identify the elements in the set of natural numbers from 1 to 52 and the multiple of 5 from it. However, they failed to use a proper formula to get the correct answer.

In part (b)(i), the candidates who failed to arrange the given fractions in the ascending order lacked competences in the basic concepts of fractions. The candidates multiplied the given fractions by the Greatest Common Factor (GCF) instead of the Lowest Common Multiple (LCM). This could not help them to arrange the fractions in ascending order. Further analysis shows that some of the candidates arranged the fractions by comparing the denominators. For instance, they arranged as $\frac{1}{2}, \frac{2}{5}, \frac{3}{8}, \frac{2}{9}, \frac{1}{12}$ which was incorrect.

Moreover, some candidates were able to convert the fractions into percentages but failed to arrange the fractions in the ascending order. In part (b)(ii), the candidates failed to write the number in the denominator in the standard form. They wrote 0.000035 as 35×10000, while others wrote 3.5×10^{5} instead of 3.5×10^{-5}. Furthermore, some candidates were able to evaluate the expression, that is $\frac{7\times10^{4}}{0.000035}=2{,}000{,}000{,}000$, but they failed to write the final answer in standard form. Extract 1.1 shows a sample of a response of a candidate who answered this question incorrectly.

1\. a) Find the percentage Of numbers which are multiples of 5 from {1, 2, 3, 4 ..... up to 52

Soln

1 2 3 4

$\underline{5\ 2}$

2 8 6

$\underline{\quad 1}$

$\underline{2\ 8\ 7}$

b) Franction in ascending order $\frac{1}{2}, \frac{2}{9}, \frac{3}{8}, \frac{1}{12}$ and $\frac{2}{5}$

Soln

$\frac{1}{2} + \frac{2}{9} = \frac{3}{11} + \frac{3}{8} = \frac{6}{19}$

$\frac{6}{19} + \frac{2}{5} = \frac{8}{24}$

$24 \div 8 = 17$

$\underline{= 17}$

ii) Simplify the expression $\frac{7 \times 10^4}{0.000035}$ hence write the answer in standard form.

Soln

$\frac{7 \times 10 + 10 + 10 + 10}{0.000039}$

$7 \times 10 = 80 + 10 = 90 + 10 = 100 + 10 = 110$

$\frac{11\cancel{0}\,\cancel{0}\cancel{0}0}{\cancel{0}.\cancel{0}\cancel{0}\,\cancel{0}\cancel{0}39}$

$\underline{110}$

39

$= 149$

Extract 1.1: A sample of the candidate's incorrect responses to question 1

Extract 1.1 shows that in part (a), the candidate failed to recognize the set of natural numbers S from 1 to 52 and the multiples of 5 in it. The candidate added the given numbers in the set. In part (b)(i), the candidate lacked the knowledge of the ascending order of numbers. In part (b)(ii), the candidate failed to apply the laws of exponents to compute the given mathematical expression which involved multiplication and division of numbers.

On the other hand, the response analysis shows that the candidates who answered the question correctly were able to apply knowledge and skills gained after learning the concepts of numbers, decimals, and percentages. In part (a), the candidates identified correctly the sets S of natural numbers from 1 to 52 and the set E of multiples of 5 in S. That is, $\mathrm{S}=\{1, 2, 3, ..., 52\}$ and $\mathrm{E}=\{5, 10, 15, 20, 25, 30, 35, 40, 45, 50\}$ such that $n(\mathrm{S})=52$ and $n(\mathrm{E})=10$ respectively.

Thereafter, they calculated the percentage of numbers which are multiples of 5 by using the formula $\%n(\mathrm{E})=\dfrac{n(\mathrm{E})}{n(\mathrm{S})}\times 100\%$. That is, $\%n(\mathrm{E})=\dfrac{10}{52}\times 100\%=19.2307\%$. The answer was correctly approximated to one decimal place to get 19.2% as required. This shows that, the candidates gained the expected competence from learning the basic concepts of percentages, decimals, and approximation.

In part (b)(i), the candidates who scored all the allotted marks were able to write the given fractions in ascending order, that is from the lowest to the largest fraction. This was achieved by using different approaches. Some of the candidates multiplied the denominators of the fractions by their LCM. They multiplied all the fractions by the LCM of the denominators to obtain the whole numbers representing the given fractions. The candidates used the whole numbers obtained to arrange the fractions in the ascending order. Thus, the ascending order of the fractions was $\dfrac{1}{12}, \dfrac{2}{9}, \dfrac{3}{8}, \dfrac{2}{5}, \dfrac{1}{2}$. Other candidates converted the fractions into percentages. They multiplied each fraction by 100% as follows: $\dfrac{1}{2}\times 100\%$, $\dfrac{2}{9}\times 100\%$, $\dfrac{3}{8}\times 100\%$, $\dfrac{1}{12}\times 100\%$, and $\dfrac{2}{5}\times 100\%$. Thereafter the candidates used the resulting percentages to arrange the given fractions in the ascending order.

Moreover, the analysis shows that some candidates converted the fractions into decimals which enabled them to compare the results to arrange the fractions in ascending order. In part (b)(ii), the candidates who scored all the marks applied knowledge and skills gained from learning the basic concepts of logarithms and standard form of numbers. They correctly computed the given mathematical expression which involved multiplication and division of numbers. The candidates also applied the laws of exponents and expressed the final answer in standard form. That is,

$\frac{7\times10^4}{0.000035}=\frac{7\times10^4}{3.5\times10^{-5}}=2\times10^9$. Extract 1.2 shows a sample of a response by a candidate who answered this question correctly.

1. (a) From the set given

{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52}

Multiples of 5 are as follow,
{5, 10, 15, 20, 25, 30, 35, 40, 45, 50}

$\%$ of numbers $= \frac{\text{numbers which are multiples of 5}}{\text{Total number in the set}} \times 100\%$

$= \frac{10}{52} \times 100\%$

$= 0.1923 \times 100\%$

$= 19.23\%$

$\approx 19.2\%$

$\therefore$ the percentage of numbers which are multiples of 5 from the set given $\approx 19.2\%$

(b) (i) Given,

$1/2$, $2/9$, $3/8$, $1/12$ and $2/5$

To find the L.C.M of the ~~numb~~ denominators

	2	9	8	12	5
2	1	9	4	6	5
2	1	9	2	3	5
2	1	9	1	3	5
3	1	3	1	1	5
3	1	1	1	1	5
5	1	1	1	1	1

L.C.M $= 2\times2\times2\times3\times3\times5$

$= 4\times6\times15$

$= 360$

1.	(b)(i) Then,
	i) $\frac{1}{2} \times 360 = 180$
	ii) $\frac{2}{9} \times 360 = 80$
	iii) $\frac{3}{8} \times 360 = 136$
	iv) $\frac{1}{12} \times 360 = 30$
	v) $\frac{2}{5} \times 360 = 144$
	Therefore the numbers in ascending order is as; $1/12$, $2/9$, $3/8$, $2/5$ and $1/2$
	(ii) $\frac{7 \times 10^4}{0.000035}$
	$= \frac{7 \times 10^4}{3.5 \times 10^{-5}}$
	$= \frac{7}{3.5} \times 10^{4-(-5)}$
	$= 2 \times 10^9$.

Extract 1.2: A sample of the candidate's correct responses to question 1

Extract 1.2 shows that, in part (a), the candidate correctly listed the elements of the set of natural numbers from 1 to 52, that is, $\{1, 2, 3, \ldots, 52\}$. Then, the candidate formed a set of multiples of 5, which is $\{5, 10, 15, 20, 25, 30, 35, 40, 45, 50\}$. Thereafter, the candidate correctly calculated the percentage of numbers which are multiples of 5 and approximated the result to one decimal place to get 19.2% as required. Moreover, the candidate converted the fractions into whole numbers so as to compare and arrange the fractions in the ascending order. Moreover, the candidate performed correct mathematical computations that involved multiplication and division of numbers. The result was expressed in the standard form.

2.2 Question 2: Exponents, Radicals and Logarithms

The question had two parts, (a) and (b). In part (a), the candidates were examined on whether they could find the value of x in the equation $8^{x-1}=16$. In part (b)(i), the candidates were required to simplify the expression $\log_a\sqrt{a}+\log_a\left(a^2\right)$ while in (b)(ii), they were examined if they are able to rationalize the denominator of the expression $\dfrac{5+\sqrt{2}}{\sqrt{6}-\sqrt{2}}$.

The data analysis indicates that, this question was attempted by 521,886 (100%) candidates, out of whom only 136,305 (26.1%) candidates passed. This indicates that the candidates' performance on this question was weak. It was also noted that, 18,988 (3.6%) candidates scored all the allotted marks while 337,279 (64.6%) scored zero. The summary of the candidates' performance on question 2 is presented in Figure 2.

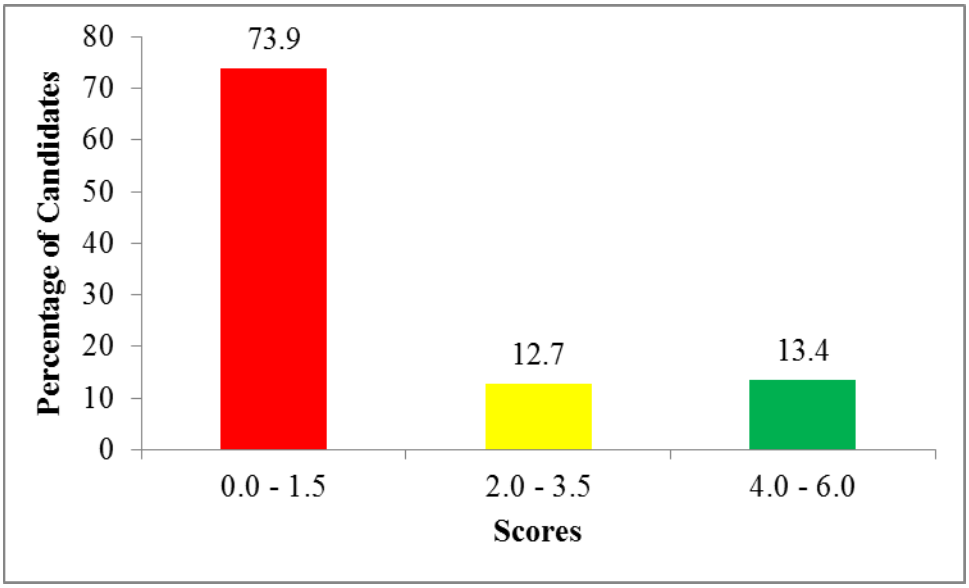


Figure 2: *Candidates' performance on question 2*

The analysis shows that majority of the candidates scored low marks on this question. In part (a), the candidates lacked adequate knowledge on the application of the laws of exponents. For instance, some candidates expressed 16 as 8^2 instead of 2^4. They performed wrong mathematical computations to find the value of x, which was $x=16-18+1$. Although some of the candidates correctly simplified the equation to $2^{3(x-1)} = 2^4$, they failed to solve it after equating the exponents, that is, $3(x-1) = 4$. They could not rearrange the equation to get the value of x. They wrote $3x = 4-3$ instead of $3x = 4+3$. Other candidates just divided 16 by 8 and obtained $x-1 = 2$. They solved for x to get $x = 3$ which was incorrect. In part (b)(i), most of the candidates were unable to express $\log_a \sqrt{a} + \log_a\left(a^2\right)$ into $\log_a\left(\sqrt{a} \times a^2\right)$, which was the essential step in simplifying the expression. For instance, the candidates factored out $\log_a$ from the expression $\log_a \sqrt{a} + \log_a\left(a^2\right)$ to get $\log_a\left(\sqrt{a} + a^2\right)$ which violated the laws of logarithms. Also, some candidates were able to express $\log_a \sqrt{a} + \log_a\left(a^2\right)$ as $\log_a\left(\sqrt{a} \times a^2\right)$, but they failed to apply the laws of exponents to simplify the expression $\sqrt{a} \times a^2$. For instance, the candidates expressed $\log_a\left(\sqrt{a} \times a^2\right)$ as $\log_a 2\sqrt{a}$ instead of $\log_a a^{\frac{5}{2}}$. Other candidates expressed $\sqrt{a}$ as $(a)^{\frac{1}{3}}$ instead of $(a)^{\frac{1}{2}}$. Moreover, there were candidates who managed to express $\log_a\left(\sqrt{a} \times a^2\right)$ as $\log_a a^{\frac{5}{2}}$ but they failed to realize that $\log_a a = 1$.

In part (b)(ii), majority of the candidates failed to rationalize the denominator of the expression $\dfrac{5+\sqrt{2}}{\sqrt{6}-\sqrt{2}}$ for different reasons. The analysis shows that some of the candidates were not able to determine the rationalizing factor. They wrote $\sqrt{6}-\sqrt{2}$ instead of $\sqrt{6}+\sqrt{2}$. Other candidates were not competent to perform basic operations on radicals after introducing the rationalizing factor. For instance, they wrote $\left(5+\sqrt{2}\right)\left(\sqrt{6}+\sqrt{2}\right)$ as $\sqrt{30}+\sqrt{4}$ instead of writing $\left(5+\sqrt{2}\right)\left(\sqrt{6}+\sqrt{2}\right)$ as $5\sqrt{6}+5\sqrt{2}+\sqrt{12}+2$. Moreover, some of the candidates incorrectly simplified the expression $5\sqrt{6}+5\sqrt{2}$ and got $10\sqrt{8}$. Extract 2.1 provides a sample of a response of a candidate who failed to answer the question correctly.

2 a) $8^{x-1} = 16$

from the laws of exponents.

$2^{3x-1} = 2^4$

Compare exponents.

$3x-1 = 4$

$\frac{2x}{2} = \frac{4}{2}$

$x = 2$

$\therefore$ The value of x is 2

b) (i) Solution

Given that,

$\log_a \sqrt{a} + \log_a (a^2)$

Now,

$\log_a \sqrt{a} + \log_a (a^2)$

$a \log a + \log_a a^2$

$a \log a + a^2 \log a$

$a \log a + 2 \log_a a$

from law of logarithm

$a^{-1} = a^0$

$a^a = 1$

Now,

$\therefore \log_a \sqrt{a} + \log_a (a^2) = 2$

$\therefore \log_a \sqrt{a} + \log_a (a^2) = 2$

2	(b) (ii) Solution
	Data given
	$\frac{5+\sqrt{2}}{\sqrt{6}-\sqrt{2}}$
	from
	$\frac{5+\sqrt{2}}{\sqrt{6}-\sqrt{2}} + \frac{\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}$
	$= \frac{5\sqrt{6}+5\sqrt{2}+3+2}{6+\cancel{3}+\cancel{3}+2}$
	$= \frac{5\sqrt{6}+5\sqrt{2}+3+2}{6+2}$
	$\therefore = \frac{5\sqrt{6}+5\sqrt{6}+5}{8}$

Extract 2.1: A sample of the candidate's incorrect responses to question 2

Extract 2.1 shows that, in part (a), the candidate failed to apply the laws of exponents to find the value of x. In part (b), the candidate violated the laws of logarithms that were required to simplify the logarithmic expression and failed to rationalize the denominator.

The analysis further shows that the candidates who scored all the marks on this question had applied knowledge and skills on the concepts of exponent, radicals and logarithms. In part (a), the candidates were able to use the laws of exponents to rewrite the exponential equation such that the terms had the same base. That is, $8^{x-1} = 16$ was written as $2^{3(x-1)} = 2^4$. The exponents were compared so as to solve for the value of x. That is, $3(x-1) = 4$ such that $x = \frac{7}{3}$. In part (b)(i), the candidates were able to apply the laws of logarithms and exponents to simplify the logarithmic expression, $\log_a \sqrt{a} + \log_a (a^2)$ to get $\log_a (\sqrt{a} \times a^2)$, which was further simplified to $\frac{5}{2}$. In part (b)(ii), the candidates were able to rationalize the denominator after identifying the rationalizing factor, $\sqrt{6}+\sqrt{2}$, that is, $\frac{5+\sqrt{2}}{\sqrt{6}-\sqrt{2}} \times \frac{\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}$. Then, they simplified the expression to get

$\dfrac{5\sqrt{6}+5\sqrt{2}+2\sqrt{3}+2}{4}$. Extract 2.2 shows a sample of responses of a candidate who answered the question correctly.

2\. (a): $x = ?$, If $8^{x-1} = 16$.

soln:

$\Longrightarrow 8^{x-1} = 16$

$(2^3)^{x-1} = 2^4$

$2^{3x-3} = 2^4$

Compare:

$\Longrightarrow$ The same bases.

$3x - 3 = 4$

$3x = 4 + 3$

$3x = 7$

$x = 7/3$:

$\therefore$ The value of $x = 7/3$:

(b): (i): $\mathrm{Log}_a \sqrt{a} + \mathrm{Log}_a (a^2)$; simplify

soln.

$\Longrightarrow \mathrm{Log}_a \sqrt{a} + \mathrm{Log}_a (a^2) = \mathrm{Log}_a a^{1/2} + \mathrm{Log}_a a^2$

$= \mathrm{Log}_a (a^{1/2} \cdot a^2)$

$= \mathrm{Log}_a (a^{1/2+2})$

$= \mathrm{Log}_a (a^{5/2})$

$= 5/2\ \mathrm{Log}_a a$

$= 5/2\ (1)$:

$= 5/2$

$\therefore \mathrm{Log}_a \sqrt{a} + \mathrm{Log}_a (a^2) = 5/2$:

2: (b) (ii): Required to rationalize the denominator of $\frac{5+\sqrt{2}}{\sqrt{6}-\sqrt{2}}$

soln.

$\Longrightarrow \frac{5+\sqrt{2}}{\sqrt{6}-\sqrt{2}}$ R.F $= \sqrt{6}+\sqrt{2}$.

$\Longrightarrow \frac{(5+\sqrt{2})}{(\sqrt{6}-\sqrt{2})} \times \frac{(\sqrt{6}+\sqrt{2})}{(\sqrt{6}+\sqrt{2})}$

$= \frac{5\sqrt{6}+5\sqrt{2}+\sqrt{12}+2}{6+\sqrt{12}-\sqrt{12}-2}$

$= \frac{5\sqrt{6}+5\sqrt{2}+\sqrt{12}+2}{6-2}$

$= \frac{5\sqrt{6}+5\sqrt{2}+\sqrt{12}+2}{4}$

$= \frac{5\sqrt{6}+5\sqrt{2}+\sqrt{12}+2}{4}$

$\therefore \frac{5+\sqrt{2}}{\sqrt{6}-\sqrt{2}} = \frac{5\sqrt{6}+5\sqrt{2}+\sqrt{12}+2}{4}$.

Extract 2.2: A sample of the candidate's correct responses to question 2

Extract 2.2 shows that, in part (a), the candidate managed to use the laws of exponents to find the value of x. In part (b)(i), the candidate was able to use the laws of logarithms to simplify the logarithmic expression and in part (b)(ii), the candidate was able to rationalize the denominator correctly.

2.3 Question 3: Sets and Probability

This question had parts (a) and (b). In part (a)(i), the candidates were required to find $P \cap Q$, given that, $P = \{\text{all multiples of 5 less than 35}\}$ and $Q = \{\text{all odd numbers between 14 and 30}\}$. In part (a)(ii), the candidates were given the information that “In a village of 50 farmers, 25 grow cashew nut, 16 grow both cashew nut and maize and 10 grow neither cashew nut nor maize”. They were required to find the number of farmers who grow maize only without using a Venn diagram. In part (b), the candidates were given the information that, “A farmer was given three seeds to germinate in a nursery. The probability that a seed will germinate is $\frac{1}{3}$”. By using a tree diagram, they were required to find the probability that at least two seeds would germinate.

The analysis reveals that, the question was attempted by 521,886 (100%) candidates, out of whom 65,498 (12.6%) candidates passed, indicating that the candidates’ performance on this question was weak. The analysis also shows that, only 1,089 (0.2%) candidates scored all the allotted marks and 356,343 (68.3%) scored zero. Figure 3 indicates a summary of the candidates’ performance on question 3.

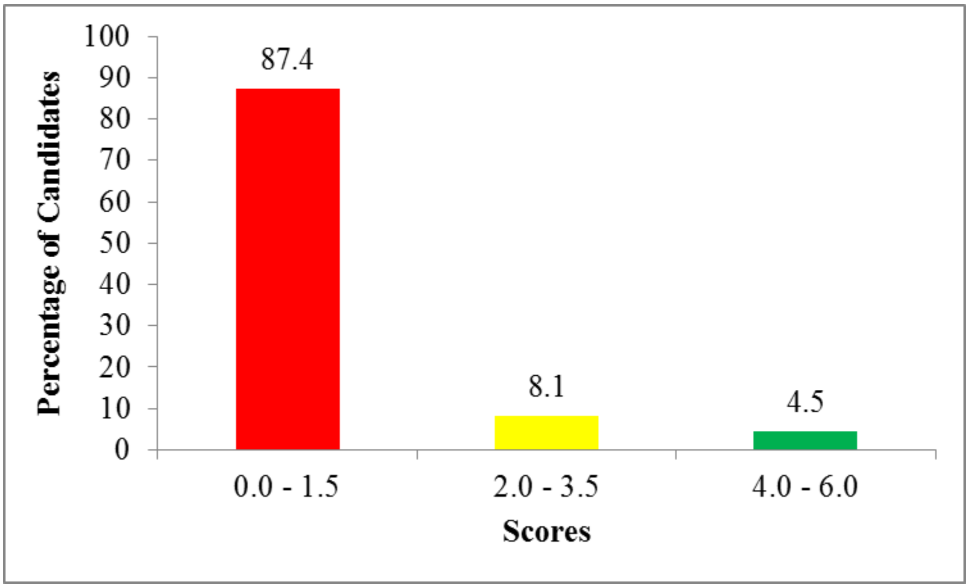


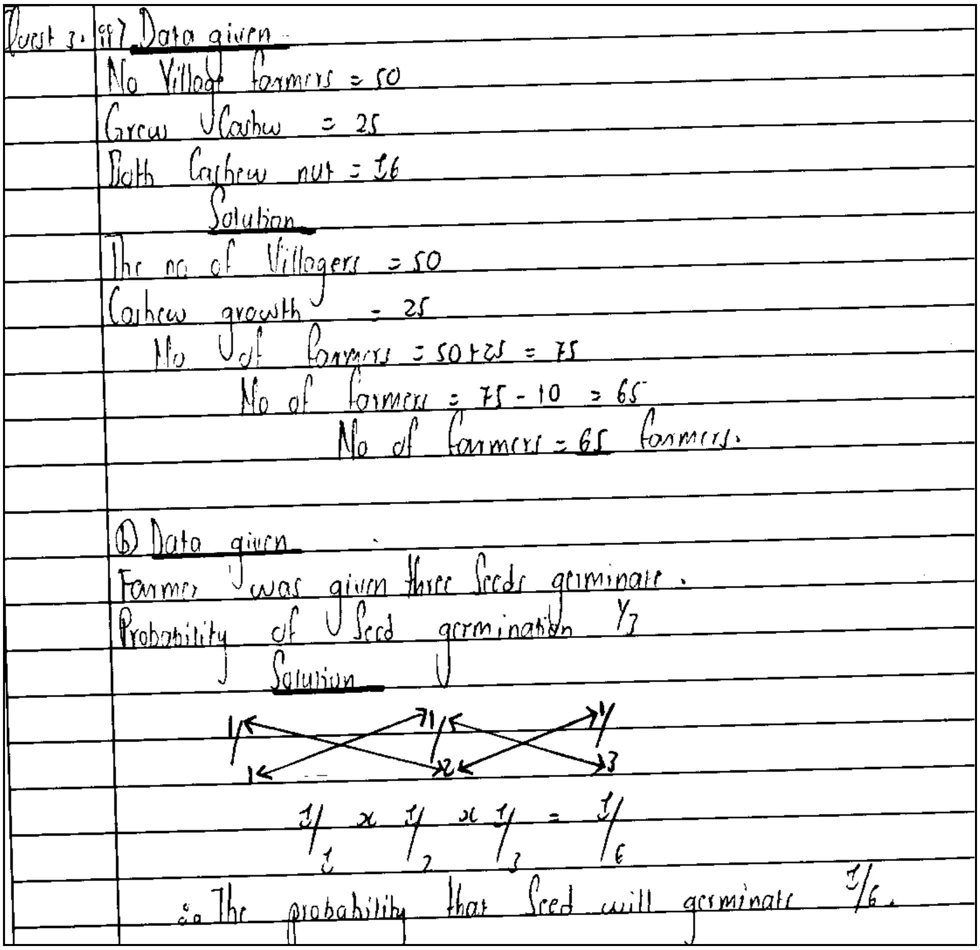
Figure 3: *Candidates’ performance on question 3*

The analysis shows that majority of the candidates performed poorly on this question. Those candidates lacked knowledge and skills on the basic concepts of sets and probability. In part (a), the candidates failed to list the elements in the sets P, Q and $P \cap Q$. For instance, they listed the sets P and

Q as $P=\{5,7\}$ and $Q=\{17, 19, 23, 29\}$ instead of $P=\{5, 10, 15, 20, 25, 30\}$ and $Q=\{15, 17, 19, 21, 23, 25, 27, 29\}$. This shows that, the candidates were not able to identify the multiples of 5 and odd numbers. Also, they lacked knowledge of set theory as they were not able to apply relevant formulae to find the number of farmers who grow maize only. For instance, some of the candidates obtained $n(C \cup M) = 50$ instead of $n(U) = 50$. Some candidates failed to make proper substitution in the formulae to get the correct answer.

Furthermore, the analysis shows that some of the candidates failed to follow the instructions as they used Venn diagrams instead of using the formulae. In part (b), the candidates were not able to present the information in a tree diagram, hence they could not determine the sample space and the outcomes of the events. Extract 3.1 is a sample of a response of a candidate who answered the question incorrectly.

Qust 3.	Data given
	i) Multiples of 5 less than 35
	ii) Q odd numbers between 14 and 30
	find $P \cap Q$
	$P \cap Q = 35 + 14 + 30$
	$P \cap Q = 49 + 30$
	$P \cap Q = 79$
	Answer $P \cap Q = 79$


Qust 3. (a) Data given
No Village farmers = 50
Grew Cashew = 25
Both Cashew nut = 16
Solution
The no of Villagers = 50
Cashew growth = 25
No of farmers = 50 + 25 = 75
No of farmers = 75 - 10 = 65
No of farmers = 65 farmers.

(b) Data given
Farmer was given three seeds germinate.
Probability of seed germination 1/3
Solution

$\frac{1}{1} \times \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$

∴ The probability that seed will germinate 1/6.

Extract 3.1: A sample of the candidate's incorrect responses to question 3

As Extract 3.1 shows, in part (a), the candidate could not identify the elements in the sets P and Q, thus failed to get the elements in $P \cap Q$. The candidate could not write the formulae to be used to get the number of farmers who grew maize only. In part (b), the candidate lacked competence in using a tree diagram to determine the probabilities of independent events.

On the other hand, the analysis shows that the candidates who scored all the marks allotted to this question had sufficient knowledge regarding the concepts of sets and probability. In part (a)(i), they were able to list the elements of sets P and Q which were $P = \{5, 10, 15, 20, 25, 30\}$ and $Q = \{15, 17, 19, 21, 23, 25\}$. They correctly obtained $P \cap Q = \{15, 25\}$. In part (a)(ii), the candidates were able to apply the formulae in set theory to arrive at a correct answer. That is, $n(\mathrm{U}) = n(\mathrm{C}\cup\mathrm{M}) + n(\mathrm{C}\cup\mathrm{M})'$ where $n(\mathrm{U}) = 50$ and $n(\mathrm{C}\cup\mathrm{M})' = 10$ to get $n(\mathrm{C}\cup\mathrm{M}) = 40$. The formula $n(\mathrm{C}\cup\mathrm{M}) = n(\mathrm{C}) + n(\mathrm{M}) - n(\mathrm{C}\cap\mathrm{M})$, where $n(\mathrm{C}) = 25$ and $n(\mathrm{C}\cap\mathrm{M}) = 16$ was used to get $n(\mathrm{M}) = 31$. Finally, the candidates were able to find the number of farmers who grew maize only. That is, $n(\mathrm{M}) - n(\mathrm{C}\cap\mathrm{M}) = 15$. In part (b), the candidates who performed well were able to use the tree diagram to present the given information. The candidates obtained the sample space of the experiment and the outcomes of the event that at least two seeds would germinate. That is, $\mathrm{S} = \{\mathrm{GGG, GGG', GG'G, GG'G', G'GG, G'GG', G'G'G, G'G'G'}\}$ and $\mathrm{E} = \{\mathrm{GGG, GGG', GG'G, G'GG}\}$. The candidates were able to realize that the events were independent such that $\mathrm{P(E)} = \mathrm{P(GGG)} + \mathrm{P(GGG')} + \mathrm{P(GG'G)} + \mathrm{P(G'GG)}$. They performed proper computations to get $\mathrm{P(E)} = \left(\frac{1}{3}\times\frac{1}{3}\times\frac{1}{3}\right) + \left(\frac{1}{3}\times\frac{1}{2}\times\frac{2}{3}\right) + \left(\frac{1}{3}\times\frac{2}{3}\times\frac{1}{3}\right) + \left(\frac{2}{3}\times\frac{1}{3}\times\frac{1}{3}\right) = \frac{7}{27}$. This indicates that, the candidates were competent in sets and probability. Extract 3.2 is a sample of a response of a candidate who answered the question correctly.

3ai) $P = \{5, 10, 15, 20, 25, 30\}$

$Q = \{15, 17, 19, 21, 23, 25, 27, 29\}$

$P \cap Q = \{15, 25\}$

$\therefore P \cap Q = \{15, 25\}$

3aii) $n(\mathcal{U}) = 50$ whereby C = cashewnuts.

$n(C) = 25$ M = Maize.

$n(C \cap M) = 16$

$n(C \cup M)' = 10$

$n(M) = ?$

$n(C \cup M) + n(C \cup M)' = n(\mathcal{U})$

$n(C \cup M) = 50 - 10 = \underline{40}$.

3aii) $n(C \cup M) = n(C) + n(M) - n(C \cap M)$.

$40 = 25 + n(M) - 16$

$40 - 9 = n(M)$

$n(M) = 31$.

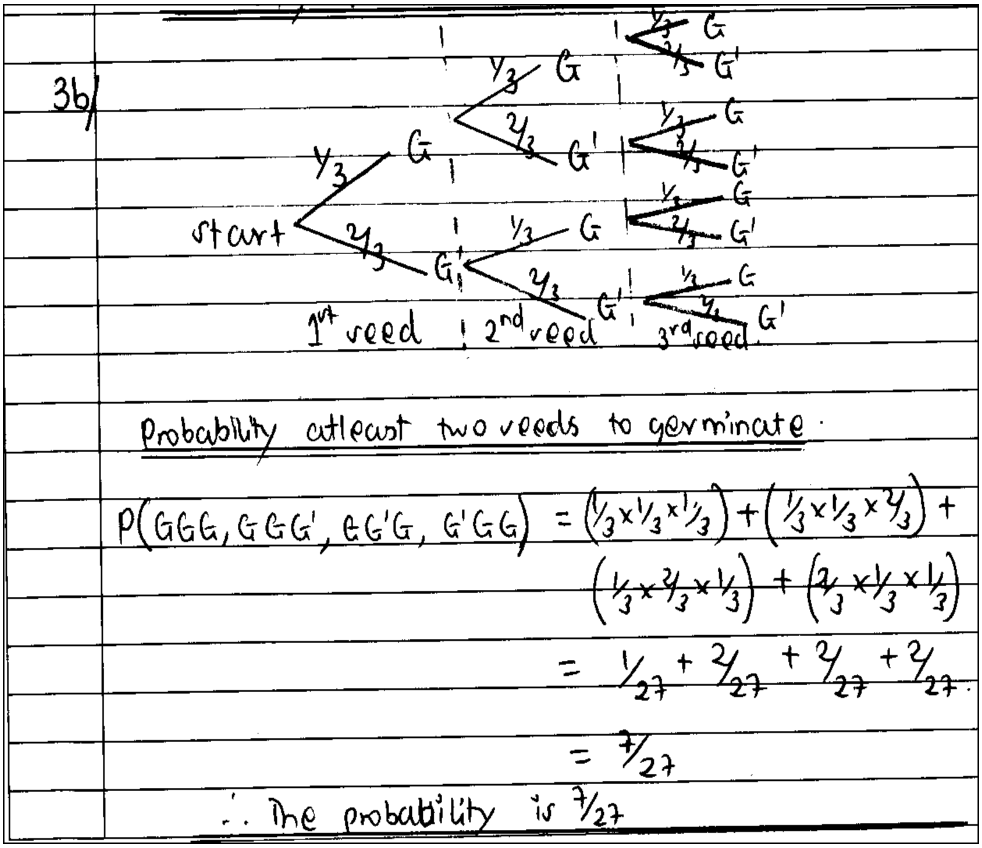
$\therefore$ Number of farmers who grow maize only

$= n(M) - n(C \cap M)$

$= 31 - 16$

$= \underline{15}$

$\therefore$ Number of farmers who grow maize only is 15



Probability atleast two seeds to germinate.

$P(GGG, GGG', GG'G, G'GG) = (1/3 \times 1/3 \times 1/3) + (1/3 \times 1/3 \times 2/3) + (1/3 \times 2/3 \times 1/3) + (2/3 \times 1/3 \times 1/3)$

$= 1/27 + 2/27 + 2/27 + 2/27$

$= 7/27$

$\therefore$ The probability is $7/27$

Extract 3.2: A sample of the candidate's correct responses to question 3

Extract 3.2 shows that, in part (a), the candidate was able to list the elements of the sets P, Q and $P \cap Q$ correctly. The candidate used relevant formulae in set theory to determine the number of farmers who grew maize only. In part (b), the candidate was able to use a tree diagram to find the probability of independent events.

2.4 Question 4: Vectors and Coordinate Geometry

This question consisted of two parts, (a) and (b). In part (a), the candidates were given the vectors $\underset{\sim}{a}=(4,3)$, $\underset{\sim}{b}=(-4,1)$ and $\underset{\sim}{c}=(2,5)$. They were required to determine which of the vectors $\underset{\sim}{a}+2\underset{\sim}{b}$ and $3\underset{\sim}{a}+\underset{\sim}{c}$ is longer than the other. In part (b), the candidates were given the point $(4,2)$ and they were required to find the equation of a line passing through the point and was perpendicular to the line $2x+3y+14=0$.

The analysis shows that out of 521,886 (100%) candidates who attempted this question, only 84,563 (16.2%) candidates passed. Generally, the candidates' performance on this question was weak. Further analysis shows that 7,572 (1.5%) candidates scored all the allotted marks whereas 390,700 (74.9%) scored 0 marks. Figure 4 shows the candidates' performance on question 4.

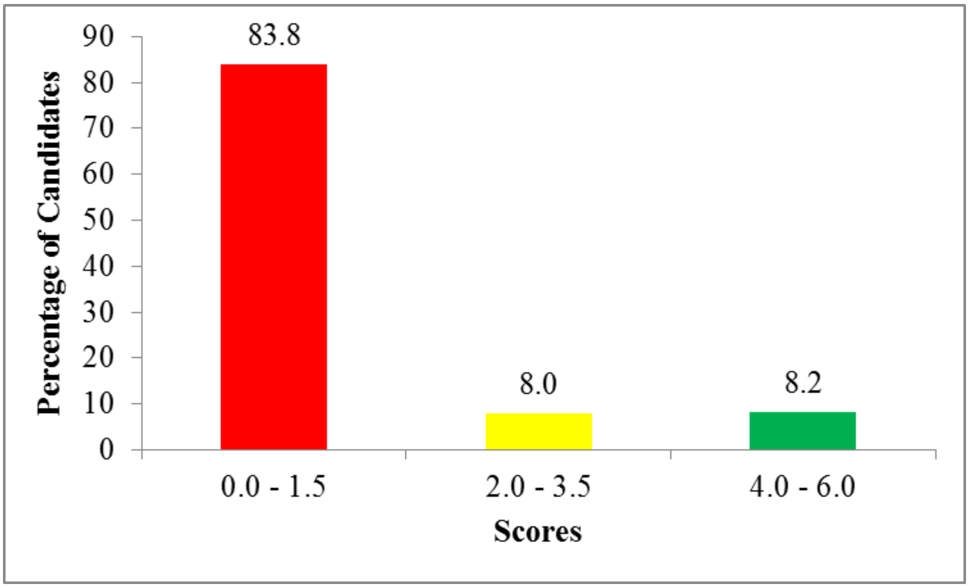


Figure 4: *Candidates' performance on question 4*

The response analysis shows that majority of the candidates scored less than 2 marks. Those candidates had insufficient competence in the concepts of vectors and coordinate geometry. In part (a), the candidates could not perform addition and scalar multiplication of the given position vectors. Also, they failed to determine the lengths (moduli) of the given vectors. Some of them used wrong formulae such as $r=\sqrt{x^2-y^2}$ or $r=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$.

Furthermore, in part (b), the candidates failed to write the equation of the line $2x+3y+14=0$ in the form $y=mx+c$ to get its slope. For instance, the candidates wrote $2x+3y+14=0$ as $y=\frac{2}{3}x+\frac{14}{3}$ to obtain $m_1=\frac{2}{3}$ which was incorrect. This error propagated to an incorrect value of m_2. Also, there were some candidates who managed to get the slope of the line

$2x+3y+14=0$ but failed to find the equation of the line passing through point $(4,2)$. Moreover, other candidates used the condition $m_1 = m_2$ for parallel lines instead of $m_1 m_2 = -1$ for perpendicular lines. Extract 4.1 shows a sample of incorrect responses from one of the candidates who attempted the question.

4. a) $a = (4,3)$, $b = (-4,1)$ and $c = (2,5)$

Soln.

$a + 2b = 2ab$

$3a + c = 3ac$

$\therefore$

$\therefore = 2ab$ and $3ac$.

b) $(4,2)$, $2x+3y+14=0$

Soln.

$2x+3y+14=0$

$= 2x+3y = 5xy+14=0$

$= 2x+3y = 19xy = 0$

$= 2x+3y = 19xy = 5x-y$

$= 2x+3y = 19xy = 5x-y = 5x$

$\therefore = 2x+3y = 19xy = 0$.

Extract 4.1: A sample of the candidate's incorrect responses to question 4

Extract 4.1 shows that, in part (a), the candidate could not perform addition and scalar multiplication of the given position vectors, hence the candidate failed to determine and compare the lengths of vectors $\underset{\sim}{a}+2\underset{\sim}{b}$ and $3\underset{\sim}{a}+\underset{\sim}{c}$.

In part (b), the candidate failed to express y in terms of x, thus could not proceed to the next step.

Although majority of the candidates performed poorly on this question, there were candidates who scored all the allotted marks. Those candidates had sufficient competence on the basic concepts of vectors and coordinate geometry. In part (a), they were able to apply the properties of vectors relating to addition and scalar multiplication. First, they obtained the two vectors $\underset{\sim}{a}+2\underset{\sim}{b}$ and $3\underset{\sim}{a}+\underset{\sim}{c}$ and computed their magnitudes to obtain $|\underset{\sim}{a}+2\underset{\sim}{b}|=\sqrt{41}$ and $|3\underset{\sim}{a}+\underset{\sim}{c}|=\sqrt{392}$. Finally, they were able to conclude based on the magnitudes of the two vectors, that was the vector $3\underset{\sim}{a}+\underset{\sim}{c}$ is longer than $\underset{\sim}{a}+2\underset{\sim}{b}$. In part (b), the candidates obtained the slope m_1 of the line $2x+3y+14=0$, that was $m_1=-\frac{2}{3}$. They used the condition $m_1 m_2=-1$ for perpendicular lines to get the slope $m_2=\frac{3}{2}$ of the line passing through the point $(4,2)$. Finally, the candidates applied the formula $y-y_0=m_2(x-x_0)$ to find the equation of the line passing through the point $(x_0,\ y_0)=(4,\ 2)$. Extract 4.2 shows a sample of a response by one of the candidates who answered this question correctly.

4 a) $3a + c = 3(4i + 3j) + (2i + 5j)$

$= 12i + 9j + 2i + 5j$

$= 12i + 2i + 9j + 5j$

$3a + c = 14i + 14j$

$|3\underline{a} + \underline{c}| = \sqrt{(14)^2 + (14)^2}$

$= \sqrt{392}$

$= 19.8$ units

$|\underline{a} + 2\underline{b}| = \sqrt{(4)^2 + (5)^2}$

$= \sqrt{41}$

$= 6.4$ units

19.8 units is longer than 6.4 units

$\therefore$ vector $3\underline{a} + \underline{c}$ is longer than vector $a + 2\underline{b}$

4 b) $2x + 3y + 14 = 0$

$\frac{3y}{3} = \frac{-2x - 14}{3}$

$y = \frac{-2x}{3} - \frac{14}{3}$

$y = mx + c$

$M_1 = -2/3$

$M_1 M_2 = -1$

$-2/3 \; M_2 = -1$

$M_2 = \frac{-1}{-2/3}$

$M_2 = 3/2$

Extract 4.2: A sample of the candidate's correct responses to question 4

Extract 4.2 shows that, in part (a), the candidate was able to determine the vectors $\underset{\sim}{a}+2\underset{\sim}{b}$ and $3\underset{\sim}{a}+\underset{\sim}{c}$ and their moduli $|\underset{\sim}{a}+2\underset{\sim}{b}|$ and $|3\underset{\sim}{a}+\underset{\sim}{c}|$, respectively. The candidate compared the values of the moduli of $\underset{\sim}{a}+2\underset{\sim}{b}$ and $3\underset{\sim}{a}+\underset{\sim}{c}$ and concluded correctly that, the vector $3\underset{\sim}{a}+\underset{\sim}{c}$ is longer than $\underset{\sim}{a}+2\underset{\sim}{b}$. This shows that the candidate had sufficient knowledge and skills in the concepts of vectors and their applications. In part (b), the candidate arranged the equation, $2x+3y+14=0$ in standard form, $y=mx+c$ such that $y=-\frac{2}{3}x-\frac{14}{3}$ and obtained its slope, $m_1=-\frac{2}{3}$. The candidate used the condition $m_1m_2=-1$ to get $m_2=\frac{3}{2}$ which is the slope of the line passing though the point $(4,2)$. Finally, the candidate was able to determine the equation of the line passing through the point $(4,2)$.

2.5 Question 5: Similarity, Perimeters and Areas

This question consisted of two parts, (a) and (b). The question was intended to examine the candidates' ability to apply the Side-Side-Side (SSS) similarity theorem. Also, it examined the ability of candidates to recall and apply relevant formulae. Part (a) stated that "The sides of a triangle are $4\,\text{cm}$, $5\,\text{cm}$ and $6\,\text{cm}$. If the longest side of a similar triangle is $18\,\text{cm}$, find the lengths of the other sides". Parts (b)(i) required the candidates to find the radius of a circle inscribing a regular hexagon with perimeter of $72\,\text{cm}$. Part (b)(ii) stated that, the area of a triangle ABC is $70\,\text{cm}^2$. If $\overline{\text{AB}}=14\,\text{cm}$ and $\overline{\text{AC}}=20\,\text{cm}$, find the angle BAC.

The analysis reveals that, a total of 521,886 (100%) candidates attempted the question and among them, 64,649 (12.4%) passed. This marks the weak performance by the candidates on this question. It was further noted that, 16,187 (3.1%) candidates scored all the allotted marks while 419,881 (80.5%) candidates scored zero. Figure 5 indicates the summary of the candidates' performance on question 5.

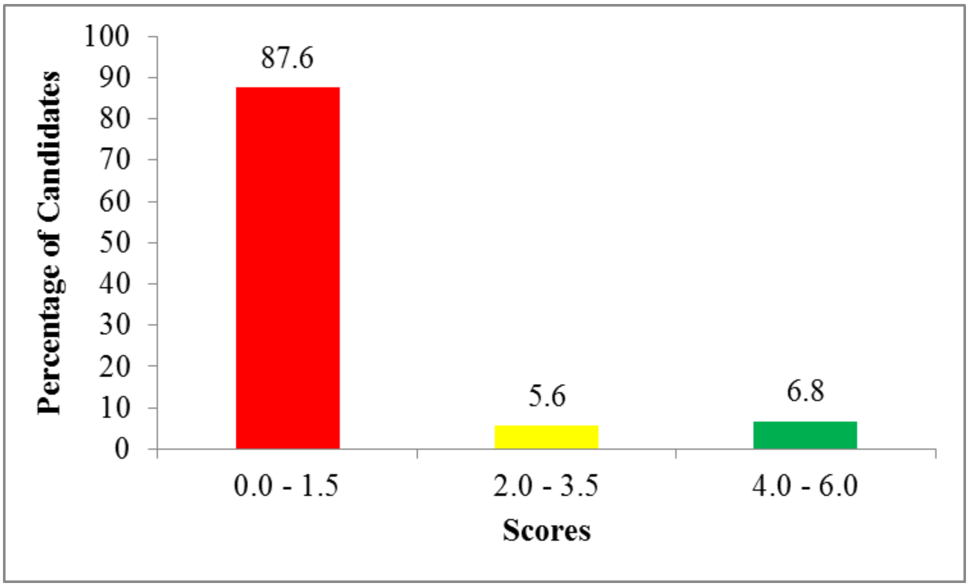


Figure 5: *Candidates' performance on question 5*

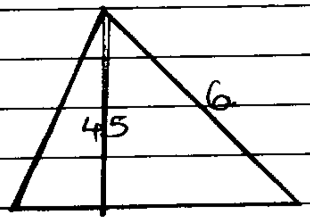
The analysis of responses shows that, the candidates who performed poorly on this question lacked competence in the basic concepts of similarities, perimeters and areas. In part (a), majority of the candidates could not meet the requirement of the question. Some of the candidates applied the sine rule, $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ which could not lead to getting the correct answer. Other candidates summed up the given lengths of the sides, that is $\text{length} = 4\text{cm} + 5\text{cm} + 6\text{cm}$ to get $\text{length} = 15\text{cm}$ which was incorrect. Also, the analysis shows that some of the candidates applied the formula for area of right-angled triangle, $\text{Area} = \frac{1}{2} \times \text{base} \times \text{height}$ to obtain $\text{Area} = \frac{1}{2} \times 4 \times 6 = 12\,\text{cm}$ which was incorrect.

In part (b)(i), majority of the candidates were not able to recall and apply the correct formula. They used irrelevant formulae and got incorrect answers. For instance, some candidates confused the formula for perimeter with that of the area of a regular polygon inscribed in a circle. Thus, they used the formula, $\text{Perimeter} = \frac{1}{2} r^2 \sin\left(\frac{360°}{n}\right)$ instead of

$\text{Perimeter} = 2nr\sin\left(\frac{180°}{n}\right)$. Furthermore, other candidates applied the formula for the circumference of a circle to find the radius. That is, $\mathrm{C} = 2\pi r,\ 72 = 2\pi r$ to get $r = 11.46\ \text{cm}$. Moreover, other candidates applied the formula for perimeter of a rectangle, that is $\text{Perimeter} = 2(\text{length} + \text{width})$.

In part (b)(ii), the candidates who performed poorly failed to recognize the appropriate formula to be used. They used the formulae that were irrelevant to the given question. For instance, some candidates used the Pythagoras' theorem and cosine of an angle. That is, $c^2 = a^2 + b^2$ to get $\mathrm{c} = 24.4\ \text{cm}$ and $\cos\theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}$ to obtain $\theta = 55°$. Extract 5.1 is a sample of a response of a candidate who answered the question incorrectly.

5. a)



18cm

soln

$\frac{1}{2}(b \times h)$

$\frac{1}{2}(4 \times 5)$

$\frac{1}{2}(20)$

$\frac{20}{2} = 10.$

$= 20 \times 5 \times 10 = 20$

$=$

b) soln

$\pi r^2 = R$

$\frac{22}{7} \times 72 \times 72$

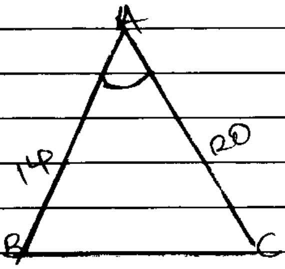
$\frac{22}{7} \times 5184.$

$\frac{22}{7} \times 5184 = \frac{114048}{7}$

5b) $\frac{72}{72} = \frac{144}{72}$

$= 5cm$

ii)



$AB = 14$ Ac

$Ac = 20.$

$Bc = x$

from $\frac{SO}{H} \Big| \frac{TO}{x} \Big| \frac{CA}{H}$

$70cm^2 = AB = AC$

$Bc = BC$

$= \frac{14}{x} = \frac{20}{x}$

$= \frac{14}{14} = \frac{20}{14}$

$BAC = 20cm.$

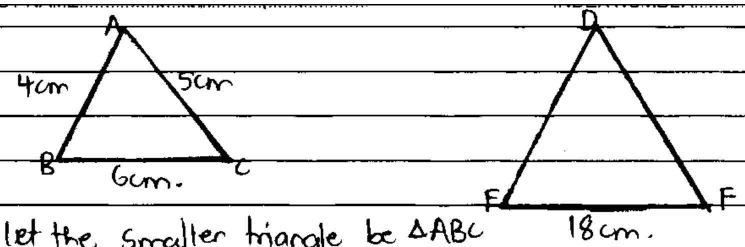
Extract 5.1: A sample of the candidate's incorrect responses to question 5

Extract 5.1 shows that, in part (a), the candidate was not able to recall and apply the appropriate formula. The candidate applied a wrong formula to obtain an incorrect answer. In part (b)(i), the candidate failed to answer the question correctly due to failure to grasp the concept of perimeter. The candidate applied the formula for the area of a circle to find the radius. In part (b)(ii), the candidate performed irrelevant computations contrary to the requirement of the question

Further analysis shows that the candidates who performed well on this question were able to apply the competences developed in the basic concepts of similarity, perimeter and areas. In part (a), the candidates demonstrated adequate knowledge and skills by identifying two similar triangles whose sides are proportional. They were able to recall and apply the SSS-similarity theorem which states that, “If the correspondence between two triangles is such that the corresponding sides are proportional, then the triangles are similar”. Thus, the candidates considered the two triangles as ΔABC and ΔPRT, such that $\Delta\text{ABC} \sim \Delta\text{PRT}$. Then, they were able to get $\frac{\overline{\text{AB}}}{\overline{\overline{\text{PR}}}} = \frac{\overline{\text{BC}}}{\overline{\overline{\text{RT}}}} = \frac{\overline{\text{AC}}}{\overline{\overline{\text{PT}}}}$ or $\frac{a}{a'} = \frac{b}{b'} = \frac{c}{c'} = k$. They computed the lengths of the other two sides after making correct substitutions. Thus, the lengths of the two sides are $12\,\text{cm}$ and $15\,\text{cm}$.

In part (b)(i), the candidates applied the appropriate formula to determine the radius of the circle when the perimeter of a regular hexagon inscribed in a circle is known. That is, the perimeter of the polygon is given by, $\text{Perimeter} = 2nr\sin\left(\frac{180°}{n}\right)$. They were able to recognize that the regular polygon has 6 sides of equal lengths such that $n = 6$. They substituted the value of n and calculated the radius of the circle. That is, $72\text{ cm} = 2(6)r\sin\left(\frac{180°}{6}\right)$ to get $r = 12\text{ cm}$. In part (b)(ii), the candidates applied the formula for area of a triangle which involves sine of an angle and simplified it to get the correct answer. That is, $\text{Area} = \frac{1}{2}ab\sin\text{C}$. The formula was used correctly to obtain the measure of angle $\text{BAC} = 30°$. Extract 5.2 is a sample of a response of a candidate who answered this question correctly.

5A.



let the smaller triangle be ΔABC
the bigger triangle be DEF

$K = \cancel{AB} \quad \frac{DE}{AB} = \frac{DF}{AC} = \frac{EF}{BC}$

$K = \frac{18}{6} = \frac{DF}{5cm} \qquad K = 3.$

$3 = \frac{DF}{5cm}$

$DF = 15cm.$

$3 = \frac{18\,cm}{6\,cm} = \frac{DE}{4cm.}$

$DE = \frac{18 \times 4\,cm}{6}$

$DE = 12cm.$

$\therefore$ The lengths of other sides are 12cm and 15cm.

---

let r be the radius

5B. I

$\text{Perimeter} = 2nr\sin\left(\frac{\theta}{2}\right) \qquad \text{where } \theta = \frac{360^\circ}{n}$

$\text{Perimeter} = 2nr\sin\left(\frac{360}{2n}\right)$

$\text{Perimeter} = 2 \times 6 \times r \times \sin\left(\frac{360^\circ}{2 \times 6}\right)$

5B I

Perimeter of the hexagon = 72cm.

$72cm = 12 \times r \times \sin(30°)$

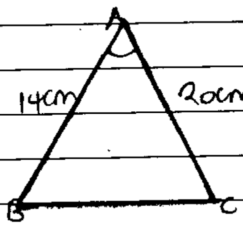
$72 cm = 12 \times r \times 0.5$

$\frac{72 cm}{6} = \frac{6r}{6}$

$r = 12cm$

∴ The radius of the circle = 12cm.

II.



let the angle be θ.

Area = 70cm^2

$Area = \frac{1}{2} \times \overline{AB} \times \overline{AC} \times \sin\theta$.

$70cm^2 = \frac{1}{2} \times 14cm \times 20cm \times \sin\theta$.

$70cm^2 = 140cm \times \sin\theta$

$\sin\theta = \frac{70cm^2}{140cm^2}$

$\sin\theta = 0.5$

$\theta = \sin^{-1}(0.5)$

$\theta = 30°$

∴ The angle $B\hat{A}C$ is 30°

Extract 5.2: A sample of the candidate's correct responses to question 5

Extract 5.2 shows that, in part (a), the candidate followed the necessary steps correctly to arrive at the correct answer. The candidate applied appropriately the SSS similarity theorem. In part (b), the candidate used correctly the formulae for perimeter and area of a hexagon inscribed in a circle and a triangle.

2.6 Question 6: Units, Rates and Variations

This question comprised two parts, (a) and (b). In part (a), the candidates were required to find the distance (in metres) that Anna walks in two days, if she walks 24 km every day. In part (b), the candidates were informed that "A dealer sales mattresses whose cost price (C) is directly proportional to the selling price (s). The selling price and cost price of one mattress are $\text{Tsh. } 20,000$ and $\text{Tsh. } 18,000$, respectively". They were required to determine the constant of proportionality and the corresponding equation.

The analysis shows that a total of 521,886 (100%) candidates attempted this question, out of whom 276,863 (53.1%) scored zero. Although more than two thirds of the candidates scored below 2 marks, a total of 170,626 (32.7%) candidates passed, out of whom, 20,018 (3.8%) scored all the allotted marks. This shows that the candidates' performance on this question was average. Figure 6 summarizes the candidates' performance on question 6.

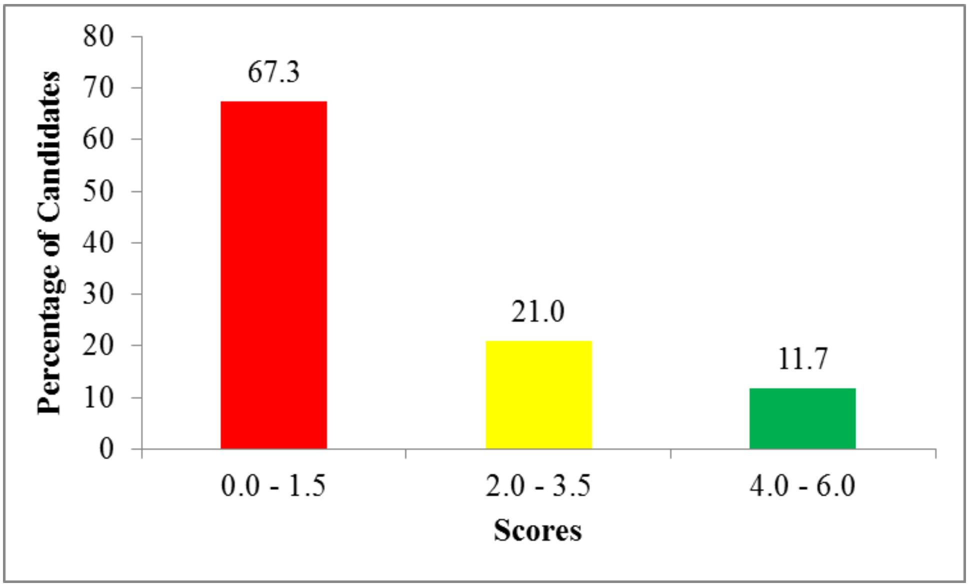


Figure 6: *Candidates' performance on question 6*

The response analysis shows that the candidates who scored all the allotted marks had sufficient knowledge and skills in units, rates and variations. They were able to apply the competence developed to solve the given problems. In part (a), the candidates multiplied 24km by 2 to obtain 48km, thereafter they converted the resulting answer into metres as required using the conversion, $1\text{km} = 1000\text{m}$. Some of the candidates converted 24km into metres to get $24{,}000\text{m}$ and then they multiplied by 2 to get the correct answer.

In part (b)(i), the candidates were able to interpret the information on direct variation. They presented the information in mathematical statement which was $C \propto s$ implying that $C = ks$, where k is the constant of proportionality. They substituted the values, $C = 18,000$ and $s = 20{,}000$ to obtain $k = 0.9$. In part (b)(ii), the candidates substituted $k = 0.9$ in the equation $C = ks$ to get the equation that relates the cost price and the selling price, that is $C = \frac{9}{10}s$. Extract 6.1 is a sample of a response of a candidate who answered this question correctly.

6a Soln

Given

1 day = 24km
2 days = x

$x = \frac{2 \text{ days} \times 24\text{ km}}{1 \text{ day}}$

$x = 48\text{km}$

1km = 1000m
48km = ?

$\frac{48\text{km} \times 1000\text{m}}{1\text{km}}$

$= 48000\text{ m}$

$\therefore$ In 2days she walks 48000m

6b i) Soln

Given

Selling price = 20,000
Cost price = 18000

6b ii) from

Cost price $\propto$ Selling price

$C \propto S$

$C = KS$

given $C = 18{,}000$ Tsh
$S = 20{,}000$ Tsh

$K = \frac{C}{S}$

$K = \frac{18,\cancel{000}}{20,\cancel{000}}$

$K = 9/10$.

∴ The constant of proportionality is $9/10$ ... Ans

ii) Soln

Let the cost price be C .

Selling price be S .

$C \propto S$.

$C = kS$

$C = \frac{9}{10} S$.

∴ The equation is Cost price = $\frac{9}{10}$ x Selling price

Extract 6.1: A sample of the candidate's correct responses to question 6

Extract 6.1 shows that, in part (a), the candidate related the two quantities, that is distance and time correctly. The candidate was able to convert the measurements into metres. In part (b), the candidate was able to formulate the problem on direct variation. The candidate calculated correctly the constant of proportionality and determined the equation relating the two variables.

On the other hand, the candidates who failed to answer this question lacked knowledge and skills in the concepts of units, rates and variations. In part (a), the candidates failed to convert metric units of length. For instance, the candidates used wrong conversions such as $1\,\text{km}=100\,\text{m}$ instead of $1\,\text{km}=1000\,\text{m}$. In addition, other candidates added the units of length and time, that is $24\,\text{km}+2\,\text{days}=26\,\text{km}$ which was incorrect. Also, some of the candidates divided the distance by time, that is $\frac{24\,\text{km}}{2}=12\,\text{km}$. Furthermore, the analysis shows that the candidates multiplied $24\,\text{km}$ by 2 to get $48\,\text{km}$, but failed to convert the answer into metres.

In part (b)(i), the candidates failed to formulate the required variation equation. They used the concept of inverse variation instead of direct variation. Thus, they presented the problem as $C \propto \frac{1}{s}$ instead of $C \propto s$. Furthermore, some other candidates applied the wrong formula to calculate the value of the constant of proportionality which was $k = \frac{\text{selling price}}{\text{cost price}}$. Also, there were some candidates who wrongly interpreted the question as an accounting problem. Thus, they opened the cash account. In part (b)(ii), the candidates could not use the results obtained in (b)(i) to write the corresponding equation. Some of the candidates substituted $k = 0.9$, $C = 18{,}000$ and $s = 20{,}000$ in the equation $C = ks$ to obtain $18{,}000 = 18{,}000$ which was incorrect. Moreover, other candidates formulated the equation of the form $\text{Cost price} - \text{Selling price} = \text{Constant}$. The responses of the candidates who performed poorly suggested that they had not developed the expected competence in the concepts of units, rates and variations. Extract 6.2 is a sample of a response of a candidate who answered this question incorrectly.

6a.	24 km every day ? 2 days
	1cm = 1000 km
	24 = x
	1cm × x 24km × ~~1000km~~
	1000km ~~1000km~~
	24 km × 2 days
	= 72 km
	∴ Anna walk in 2day 72 km^2
b i.	
	cost prices of one mattress Tsh. 200,00
	Respectively 18,000
	200,00 − 18000
	= 2000.
	The proportionality = 2000.
ii	Cost price 20000 cost price + selling price / 2.
	selling price 18000.
	20000 cost price + 18000 selling price / 2.
	38000 / 2 = 19000.

Extract 6.2: A sample of the candidate's incorrect responses to question 6

Extract 6.2 shows that, in part (a), the candidate failed to convert kilometres into metres. In part (b), the candidate could not present the information in mathematical statement. Such candidate computed the proportionality constant by finding the average of the cost price and the selling price to get an incorrect answer.

2.7 Question 7: Ratios, Profit, Loss and Accounts

This question consisted of two parts, (a) and (b). In part (a), the candidates were given the information that, "A damaged table that costs Tshs. 20,000 was sold at a loss of 15%. They were examined whether they were able to find the loss made and the selling price. In part (b), the candidates were examined if they were able to extract a trial balance from the following Mabala's cash account.

Debit				**Credit**			
Date	**Particular**	**Folio**	**Amount**	**Date**	**Particular**	**Folio**	**Amount**
5/8/2018	Capital		100,000	5/8/2018	Purchases		80,000
9/8/2018	Sales		43,000	15/8/2018	Telephone bills		28,000
11/8/2018	Sales		47,000				
				31/8/2018	Balance	c/d	82,000
			190,000				**190,000**
1/9/2018	Balance	b/d	82,000				

The analysis shows that this question was attempted by 521,886 (100%) candidates, out of whom, 140,509 (26.9%) candidates passed. This shows that, the candidates' performance on this question was weak. Further analysis indicates that, 35,669 (6.8%) candidates scored all the marks allotted to this question whereas 329,021 (63.0%) scored zero. Figure 7 shows the candidates' performance on question 7.

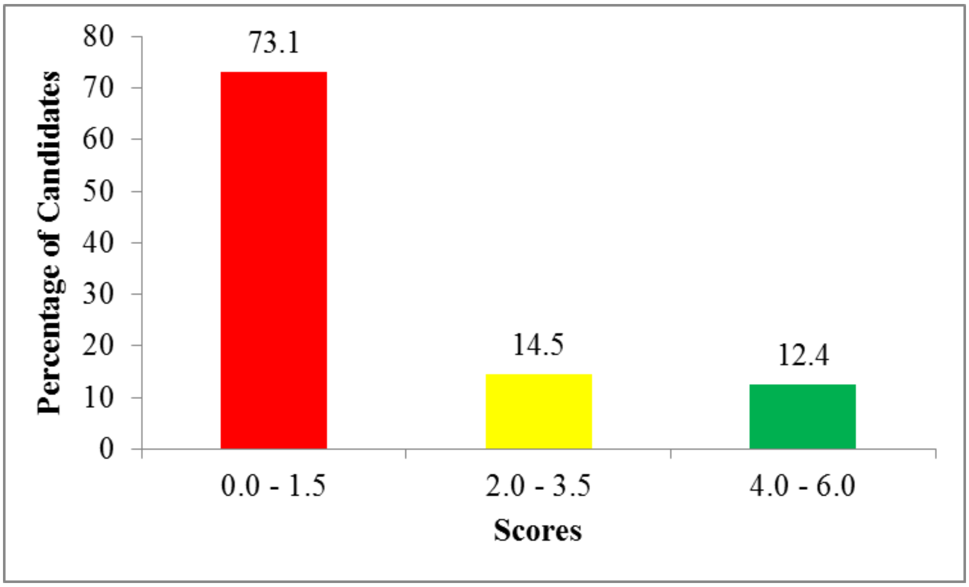


Figure 7: *Candidates' performance on question 7*

The analysis reveals that, the candidates who failed to answer the question correctly lacked competence in the basic concepts of ratio, profit and loss and accounts. In part (a), majority of the candidates applied wrong formulae such as $\text{Percentage loss} = \frac{\text{Loss made}}{\text{Selling price}} \times 100\%$ or $\text{Percentage loss} = \frac{\text{Selling price}}{\text{Buying price}} \times 100\%$. Moreover, some of the candidates performed irrelevant calculations such as $20{,}000 \div 15\%$ or $20{,}000 - 15\%$.

In part (b), most of the candidates were unable to extract the trial balance from Mabala's cash account. They were not able to make proper posting of the accounts. For instance, the candidates posted the debit balances to the side of the credit balances, and vice versa. Further analysis shows that some of the candidates constructed a trading account instead of a trial balance, which was contrary to the requirement of the question. Moreover, some candidates added or subtracted the total and the balances in the given cash account. For instance, the candidates expressed the difference in debit side as $190{,}000 - 82{,}000 = 108{,}000$ and the sum on the credit side as $190{,}000 + 82{,}000 = 272{,}000$ to get incorrect answers. Extract 7.1 is a sample of a response of a candidate who answered the question incorrectly.

7·a) A damaged table that costs Tshs· 20,000 was sold at a loss of 15%· Find the loss made and the selling price·

Soln·

Tshs· 20,000 = 15%

20,000 × 15%

∴ The loss made and the selling price is 300,000

7 b) Mr MABALA's CASH ACCOUNT

TRIAL BALANCE ACCOUNT

ACCOUNT NAM	Dr	Cr
Capital	100,000	
Sales	43,000	
Sales	47,000	
Purchases	190,000	
Telephone bills		50,000
TOTAL		25,000
		105,000

Extract 7.1: A sample of the candidate's incorrect responses to question 7

Extract 7.1 shows that, in part (a), the candidate performed irrelevant multiplication of the cost price by the percentage loss. Such a candidate could not use the appropriate formulae. In part (b), the candidate extracted the incorrect trial balance from the given cash account.

On the other hand, the analysis of responses shows that, the candidates who scored all the marks allotted to this question applied properly the competence developed through learning the basic concepts of ratios, profit and loss and accounts. In part (a), the candidates applied the correct formula, $\text{Percentage loss} = \frac{\text{Loss made}}{\text{Cost price}} \times 100\%$. They extracted the correct data from the question and substituted them in the formula. That is, they substituted $\text{percentage loss} = 15\%$ and $\text{cost price} = \text{Tshs. } 20,000$ to obtain the $\text{Loss made} = \text{Tshs.} 3,000$. Also, they were able to compute the selling price by using the formula, $\text{Selling price} = \text{Cost price - Loss made}$. That is,

$\text{Selling price} = \text{Tshs.}\,20{,}000 - \text{Tshs.}\,3{,}000 = \text{Tshs.}\,17{,}000$. Further analysis shows that some of the candidates were able to use the formula, $\text{Selling price} = (100\% - 15\%)$ of the buying price. That is, $\text{Selling price} = \frac{85}{100} \times 20{,}000 = \text{Tshs.}17{,}000$. They applied the formula, $\text{Loss made} = \text{Cost price} - \text{Selling price}$ to calculate the loss made which was Tshs. 3,000. In part (b), the candidates managed to prepare a correct trial balance from the given cash account. They posted each of the particulars appropriately in the debit and credit sides of the trial balance. Lastly, they balanced the accounts by carrying down a balance of Tsh. 82,000 in the credit side. Extract 7.2 is a sample of a response of a candidate who answered the question correctly.

7.(a)	porcet loss = 15%, Cost = Tsh 20,000/=
	loss made - if 20,000/= = 100%
	? = 15%
	$= \frac{15\% \times 20{,}000/=}{100\%}$
	= Tsh 3,000
	Selling price = Cost - loss made
	= 20,000/= - 3000/= = 17,000/=
	∴ loss made was Tsh 3000
	Selling price was Tsh 17,000

7(b)	TRIAL BALANCE AS AT 31/12/2018,		
S/NO	particulars	debit	credit
1	balance	82,000	-
2	Capital		100,000
3	sales		90,000
4	Purchases	80,000	
5	Telephone bills	28,000	
		190,000	190,000

Extract 7.2: A sample of the candidate's correct responses to question 7

Extract 7.2 shows that, in part (a), the candidate was able to apply appropriate formulae to compute the loss made and selling price. In part (b), the candidate extracted correctly the trial balance from the given cash account.

2.8 Question 8: Sequences and Series

This question consisted of two parts, (a) and (b). Part (a) stated that "The fifth and eleventh terms of an arithmetic progression are 8 and -34, respectively". The candidates were examined whether they were able to recall and apply appropriate formulae to determine the sum of the first ten terms of the progression. Part (b) stated that "A school wishes to invest $\text{Tshs.}\ 100,000,000$ in a bank which pays an interest rate of 2% compounded annually". The candidates were required to: (i) find the total amount of money that will be accumulated after two years and (ii) calculate the interest earned after two years.

The analysis shows that a total of 521,886 (100%) candidates attempted this question and among them, 61,417 (11.8%) candidates passed, showing that the candidates' performance on this question was weak. Further analysis shows that 10,639 (2.0%) candidates scored all the allotted marks whereas 425,794 (81.6%) scored zero. Figure 8 portrays a summary of the candidates' performance on question 8.

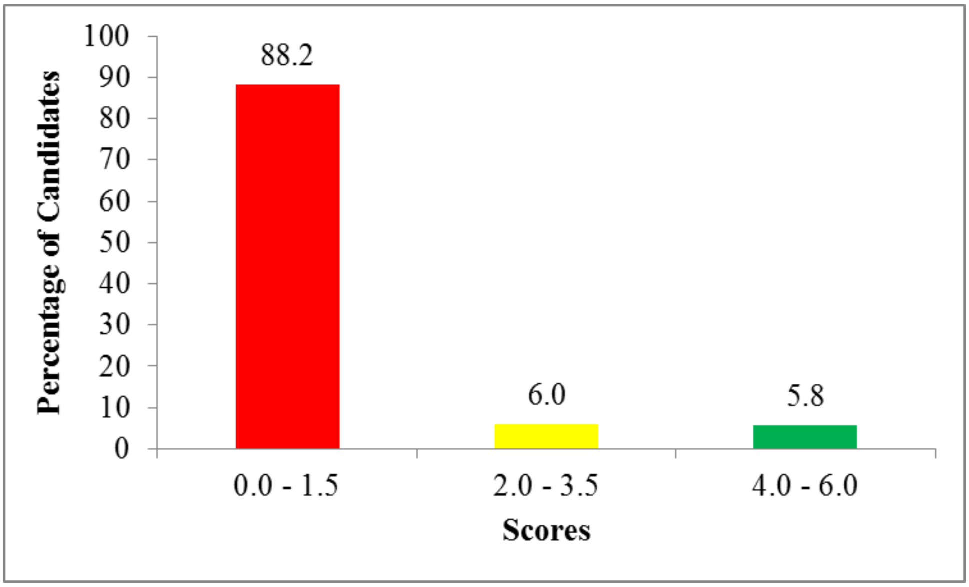


Figure 8: *Candidates' performance on question 8*

The analysis of responses shows that, majority of the candidates performed poorly on this question. The candidates lacked the intended competence in the basic concepts of sequences and series. In part (a), the candidates were not able to formulate the equations for A_5 and A_{11}. Thus, they could not get the correct values of A_1 and d so as to substitute them in the equation for sum of the first ten terms. For instance, the candidates formulated wrong equations such as $A_5 = A_1 + 5d = 8$ and $A_{11} = A_1 + 11d = -34$. The analysis also shows that, some of the candidates were able to formulate the equations for A_5 and A_{11} but they lacked computation skills to arrive at the correct answer.

In part (b), the analysis shows that majority of the candidates failed to apply the appropriate formula for compound interest to obtain the total amount of money accumulated after two years. This shows that, the candidates were not able to use relevant formulae. In this case, some of the candidates used wrong formulae for amount accumulated such as $A_n = P\left(1+\frac{r}{100}\right)^{nt}$ or $A_n = P\left(1+\frac{r}{100}\right)^{n}$. Moreover, some of the candidates applied the formula for simple interest, that is $A_n = \frac{PRT}{100}$. Further analysis shows that majority of the candidates failed to find the interest earned after two years. Such candidates wrongly applied the formula for simple interest, $I = \frac{PRT}{100}$ instead of $\text{Interest} = \text{Amount} - \text{Principle}$. The substitution of the given values in the formula for simple interest resulted into a wrong answer. That is, $\text{Interest} = \frac{100{,}000{,}000 \times 2 \times 2}{100} = \text{Tshs. } 4{,}000{,}000$ which is incorrect. Extract 8.1 is a sample of incorrect responses from one of the candidates who failed to answer the question correctly.

8. (a) solution.

$A_5 + d = A_6$

A_5

$8 + d = A_6$

$S_{10} = A_5(1-d)$

$S_{10} = 8(1-d)+34$

$S_{10} = 8 - 8d + -34$

$S_{10} = \frac{-8d}{-8} \pm \frac{-26}{-8}$

$d = 3$

(b). (i). $I = \left(\frac{P \times R \times T}{100}\right)$

$\frac{100,000,000 \times 2 \times 2}{100}$

4×1000000

$= 4,000,000/=$

Total amount will be 4,000,000

(ii) The Interest is 4,000,000/= after two years.

Extract 8.1: A sample of the candidate's incorrect responses to question 8

Extract 8.1 shows that, the candidate could not formulate the correct equations for A_5 and A_{11} in part (a). As a result, the candidate failed to find the sum of the first ten terms of the progression. In part (b), the candidate lacked competence on the concept of compound interest, thus failed to find the total amount of money accumulated and the interest earned after two years.

Despite the weak performance, there were candidates who performed well on this question. They were able to apply the acquired competence in the basic concepts of sequences and series. In part (a), the candidates demonstrated adequate knowledge and skills by recalling and applying different formulae on sequences and series. They used the formulae, $A_5 = A_1 + 4d$, $A_{11} = A_1 + 10d$ and $S_n = \frac{n}{2}\left[2A_1 + (n-1)d\right]$. They solved the two equations for A_5 and A_{11} simultaneously to get $A_1 = 36$ and $d = -7$, where A_1 is the first term and d is the common difference. Finally, they substituted the values of A_1, d and n in the equation $S_n = \frac{n}{2}\left[2A_1 + (n-1)d\right]$ to get $S_{10} = 45$.

In part (b), the candidates demonstrated sufficient knowledge and skills since they were able to use the correct formula to calculate the total amount of money accumulated and interest earned after two years. That is, they applied correctly the formulae $A_n = P\left(1 + \frac{RT}{100}\right)^n$ and $\text{Interest} = \text{Amount} - \text{Principle}$ for the total amount of money accumulated and interest earned, respectively. They substituted the given values and calculated the total amount of money accumulated after two years, $A_2 = 100,000,000\left(1 + \frac{2\times 1}{100}\right)^2$ to obtain Tshs. 104,040,000. Lastly, they calculated the interest earned after two years, $I =$ Tshs. 104,040,000 $-$Tshs. 100,000,000 to obtain Tshs. 4,040,000. Extract 8.2 is a sample of a response of a candidate who answered the question correctly.

8.(a)	Soln:
	Given $A_5 = 8$
	$A_{11} = -34$
	$S_{10} = ?$
	from $A_5 = A_1 + 4d = 8$
	$A_{11} = A_1 + 10d = -34$
	$- \mid A_1 + 4d = 8$
	$A_1 + 10d = -34$
	$-6d = 42$
	$\frac{}{-6} \quad \frac{}{-6}$
	$d = -7$
	Also $A_1 + 4d = 8$
	$A_1 + (4 \times -7) = 8$
	$A_1 + -28 = 8$
	$A_1 = 8 + 28$
	$A_1 = 36$
	Then from $S_n = \frac{n}{2}(2A_1 + (n-1)d)$
	$S_{10} = \frac{10}{2}((2 \times 36) + ((10-1) \times -7))$
	$S_{10} = 5(72 + (9 \times -7))$
	$S_{10} = 5(72 - 63)$
	$S_{10} = 5 \times 9$
	$S_{10} = 45$
	$\therefore$ The sum of the first ten terms is 45.
8.(b)	Soln:
	Given Principal (P) = 100,000,000 Tsh. Number of years (n) = 2
	Rate (R) = 2% & T = 1
	(i) Then from $A_n = P\left(1 + \frac{RT}{100}\right)^n$
	$A_2 = 100,000,000 \text{Tsh} \left(1 + \frac{2 \times 1}{100}\right)^2$
	$A_2 = 100,000,000 \text{Tsh} (1 + 0.02)^2$
	$A_2 = 100,000,000 \text{Tsh} \times (1.02)^2$
	$A_2 = 100,000,000 \text{Tsh} \times 1.0404$

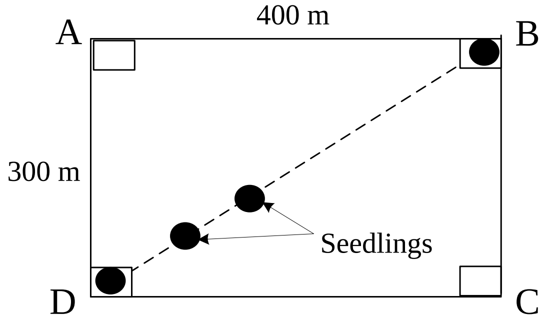
8 (b)	(i) $A_2 = 100;000,000$ Tsh $\times 1.0404$
	$A_2 = 104,040,000$ Tsh
	∴ The total amount of money that will be accumulated
	after two years is 104,040,000 Tsh.
	(ii) Soln:
	From Principal (P) + Interest (I) = A_n
B	$A_n = P + I$
	$I = A_n - P$
	$I = 104,040,000$ Tsh $- 100,000,000$ Tsh
	$I = 4,040,000$ Tsh
	∴ The interest after two years is 4,040,000 Tsh.

Extract 8.2: A sample of the candidate's correct responses to question 8

Extract 8.2 shows that, in part (a), the candidate formulated properly the equations for A_5 and A_{11} and managed to solve the equations simultaneously. The candidate was able to compute the sum of the first ten terms of the progression. In part (b), the candidate was able to recall the formula for compound interest and used it correctly to calculate the total amount of money accumulated and the interest earned after two years.

2.9 Question 9: Trigonometry and Pythagoras' Theorem

This question had two parts, (a) and (b). In part (a)(i), the candidates were required to find the value of $\frac{\sin 690°}{\cos 690°}$ without using mathematical table. In part (a)(ii), the candidates were examined to find the number of seedlings to be planted in a rectangular garden ABCD of the length of 400 m and the width of 300 m. The seedlings were to be planted along the diagonal $\overline{BD}$ at equal intervals of 1.25 m as shown in the following figure.



In part (b), the candidates were given the information that "From the top of a tower which is 50 m high, the angle of depression of a car parked on the ground is $30°$". They were required to find in surd form, the distance from the car to the base of the tower.

This question was attempted by 521,886 (100%) candidates, whereby only 41,876 (8.0%) candidates passed. This indicates the candidates' weak performance on this question. The analysis also indicates that, only 558 (0.1%) candidates scored all the allotted marks while a total of 422,269 (80.9%) scored zero. The summary of the candidates' performance on question 9 is portrayed in Figure 9.

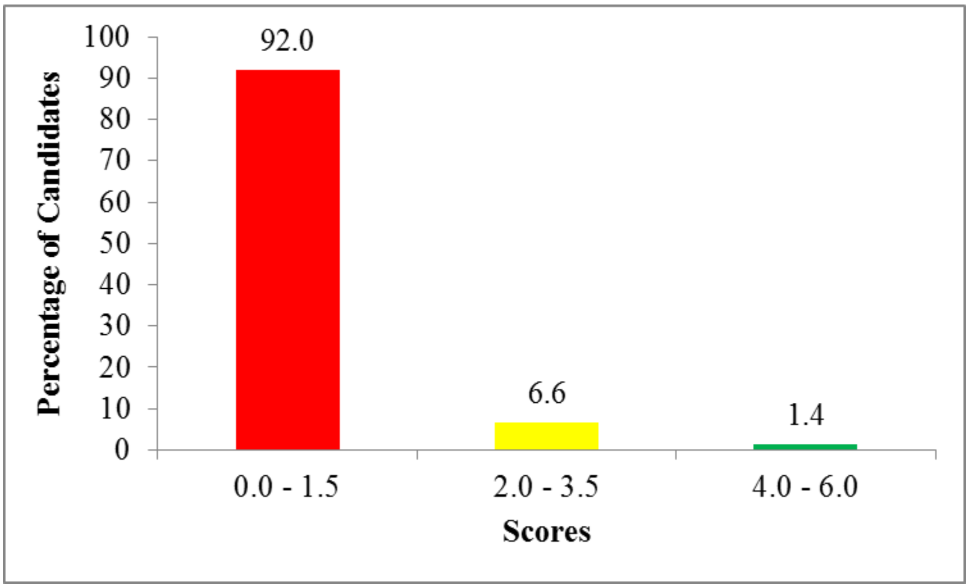


Figure 9: *Candidates' performance on question 9*

The response analysis shows that majority of the candidates performed poorly on this question. The candidates lacked the expected competence in the concepts of trigonometry and Pythagoras' theorem. In part (a)(i), the candidates failed to read the angle $690°$ in the interval $0° \le \theta \le 360°$. The analysis shows that some candidates used inappropriate factor formula such as $\cos x + \cos y = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x+y}{2}\right)$ and $\sin x + \sin y = 2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$. Further analysis shows that some of the candidates were able to read the angle $690°$ in the interval $0° \le \theta \le 360°$ and expressed $\frac{\sin 690°}{\cos 690°}$ as $\frac{\sin 330°}{\cos 330°}$ but failed to recognize that $\sin 330° = -\sin(360° - 330°)$ since $330°$ is in fourth quadrant where *sine* is negative. Furthermore, the analysis shows that some of the candidates used non-programmable scientific calculators to evaluate $\frac{\sin 690°}{\cos 690°}$ contrary to the requirement of the question.

In part (a)(ii), most of the candidates failed to find the length of the diagonal $\overline{\mathrm{BD}}$. The analysis shows that the candidates lacked knowledge and skills on applications of Pythagoras' theorem. They failed to get the correct length of the diagonal and the number of seedlings to be planted. For instance, the candidates used incorrect formulae such as $\overline{\mathrm{AB}}^2 - \overline{\mathrm{AD}}^2 = \overline{\mathrm{DB}}^2$. Moreover, there were candidates who managed to find the length of the diagonal but they failed to get the correct number of seedlings to be planted. They wrongly computed $n = \frac{500}{1.25}$ to obtain 400 seedlings instead of computing $n = \frac{500}{1.25} + 1$ to obtain 401 seedlings. Also, there were candidates who used inappropriate formulae such as, $\text{Area of triangle} = \frac{1}{2} \times \text{base} \times \text{height}$ and $\text{Area of rectangle} = \text{length} \times \text{width}$, which produced incorrect answers. In addition, some of the candidates failed to perform basic mathematical operations. For instance, they made wrong calculations such as $400^2 + 300^2 = \sqrt{90800}$, $400^2 + 300^2 = \sqrt{25000}$ instead of $400^2 + 300^2 = \sqrt{250000}$ while others were not able to evaluate $\sqrt{250000}$.

The analysis also shows that, the candidates failed to interpret the given word problem. For instance, the candidates multiplied the given dimensions of the rectangular garden to get the area of the rectangle. That is, $300\,\text{m}\times 400\,\text{m}$ to get $120000\,\text{m}^2$ which was incorrect.

In part (b), the candidates who performed poorly could not present the given word problem in relevant mathematical statements using relevant sketches. The response analysis shows that they failed to locate the angle of depression; hence they were not able to determine the distance of the car from the base of the tower. Some of the candidates used trigonometric ratios such as $\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$ and $\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$ which were not suitable to the given information. Extract 9.1 presents a sample of a response of a candidate who answered the question incorrectly.

9:(a)(i) Find sin 690° Without Using mathematical tables

cos 690°

Soln:

$= \sin 690^\circ + \cos 690^\circ$

$= 1380^\circ$

$= 13\cdot80^\circ$

$\cos^\circ \sin^\circ = 13\cdot80^\circ$

ii) Soln:

$= 400\text{m} + 1\cdot25\text{m}$

$= \dfrac{500\text{m}}{2}$

$= 0\cdot5\text{m}$

∴ The number of seedling that were planted is 0·5m.

b) Soln.

$50^\circ + 30^\circ = 180$

$= 80^\circ + 100^\circ$

$= 180^\circ$

∴ The area angle $= 180^\circ$

Extract 9.1: A sample of the candidate's incorrect responses to question 9

As Extract 9.1, part (a)(i) shows, the candidate failed to read the angle corresponding to $690°$ in the interval $0° \le \theta \le 360°$. Such a candidate just added $\sin 690°$ and $\cos 690°$ to get an incorrect answer. In part (a)(ii), the candidate lacked competence on the applications of Pythagoras' theorem, thus just added the length of a rectangular garden and interval of the seedlings. In part (b), the candidate performed irrelevant mathematical computations contrary to the requirement of the question.

On the other hand, the response analysis shows that the candidates who scored all the marks allotted to this question applied the competence developed through learning the concepts of trigonometry and Pythagoras' theorem. In part (a)(i), the candidates were able to determine the measure of an angle which is greater than $360°$ in the interval $0° \le \theta \le 360°$. They made the right substitution to get $\frac{\sin 330°}{\cos 330°}$. Since $330°$ is in the fourth quadrant, the candidates correctly expressed $\frac{\sin 330°}{\cos 330°}$ as $\frac{-\sin(360° - 330°)}{\cos(360° - 330°)}$ to obtain $-\frac{1}{\sqrt{3}}$ or $-\frac{\sqrt{3}}{3}$ which was the correct answer. Further analysis shows that some of the candidates expressed $\frac{\sin 690°}{\cos 690°}$ as $\tan 690°$. They were able to simplify $\tan 690°$ to get $-\tan 30° = -\frac{1}{\sqrt{3}}$. In part (a)(ii), the candidates were able to apply the Pythagoras' theorem $\overline{AB}^2 + \overline{AD}^2 = \overline{DB}^2$ to find the length of the diagonal $\overline{BD}$. That is, $\overline{BD}^2 = 400^2 + 300^2$ to obtain $\overline{BD} = 500$ m. Then, they calculated the number of seedlings by writing $n = \frac{500}{1.25} + 1$ to get 401 seedlings. Moreover, some candidates used the concept of arithmetic progression to obtain the number of seedlings to be planted. They applied the formula for the n^{th} term of an arithmetic progression, that is $A_n = A_1 + (n-1)d$, where $A_n = 500$, $A_1 = 0$, and $d = 1.25$.

In part (b), the candidates who managed to answer the question correctly sketched the diagram representing a right-angled triangle extracted from the given information. They were able to locate the angle of depression and height of the tower. From the right-angled triangle, they obtained $\tan 30° = \frac{50\,\text{m}}{x}$ such that $x = 50\sqrt{3}$ m, where x is the distance of the car from the base of the tower. The presented responses show that, the candidates had relevant knowledge and skills of the concepts of trigonometry and Pythagoras' theorem. Extract 9.2 is a sample of a response of a candidate who answered the question correctly.

9 a) $\frac{\sin 690°}{\cos 690°}$

$\frac{\sin\theta}{\cos\theta} = \tan\theta$.

$= \tan 690°$ IV Quadrant.

$= \tan 330°$.

$= \theta - 360$. $= 330 - 360$.

$\tan 690° = \tan 330° = \tan -30°$.

$\tan -30 =$.

Neglect negative.

$\tan 30° = \frac{\sin 30}{\cos 30}$.

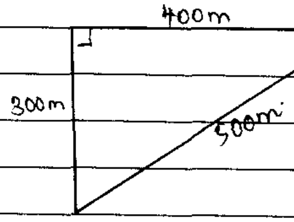
$= \frac{1}{\cancel{2}} \times \frac{\cancel{2}}{\sqrt{3}} = \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$\tan 30 = \frac{\sqrt{3}}{3}$.

$\tan -30 = \frac{-\sqrt{3}}{3}$.

$\therefore \frac{\sin 690°}{\cos 690°} = \frac{-\sqrt{3}}{3}$

a | 9a ii) From pythagoras theorem



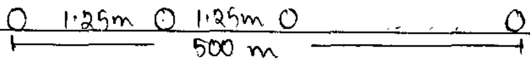
$$c^2 = a^2 + b^2$$

$$c^2 = (400)^2 + (300)^2$$

$$c^2 = 160000 + 90000$$

$$c^2 = 250000$$

$$c = 500m.$$

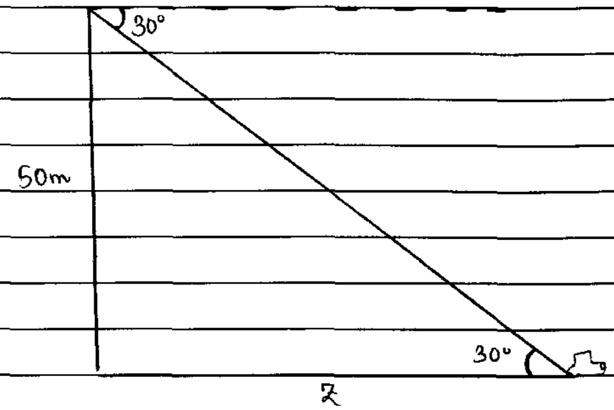


$$n = \frac{500m}{1.25} + 1.$$

$$= 400 + 1 = 401.$$

The number of seedlings are 401 seedlings

9 b)



~~Cos θ = op~~

$$\tan\theta = \frac{Opp}{Adj}$$

$$\tan 30 = \frac{50}{z}$$

$$\frac{\sqrt{3}}{3} = \frac{50}{z}$$

$$\frac{z\sqrt{3}}{\sqrt{3}} = \frac{150}{\sqrt{3}}$$

$$z = \frac{150}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$z = \frac{150\sqrt{3}}{3}$$

$$z = 50\sqrt{3}$$

The car is $50\sqrt{3}$ m away from the tower

Extract 9.2: A sample of the candidate's correct responses to question 9

Extract 9.2 shows that, in part (a), the candidate defined $\frac{\sin 690°}{\cos 690°}$ correctly as $\tan 690°$ and evaluated it without using mathematical table. Also, the candidate applied the Pythagoras' theorem to get the length of the diagonal and hence the required number of seedlings.

2.10 Question 10: Quadratic Equations

This question had parts (a) and (b). In part (a), the candidates were required to express the equation $2t^{-10} - 3t^{-5} + 1 = 0$ in terms of x, where $x = \frac{1}{t^5}$. In part (b), they were required to find the value(s) of x that satisfy the equation obtained in part (a) by using the quadratic formula.

The data analysis indicates that, a total of 521,886 (100%) candidates attempted this question and out of them, only 22,103 (4.2%) candidates passed. This indicates the candidates' weak performance on this question. Further analysis reveals that, 9,986 (1.9%) candidates scored all the allotted marks while 485,996 (93.1%) scored zero. Figure 10 depicts the candidates' performance on question 10.

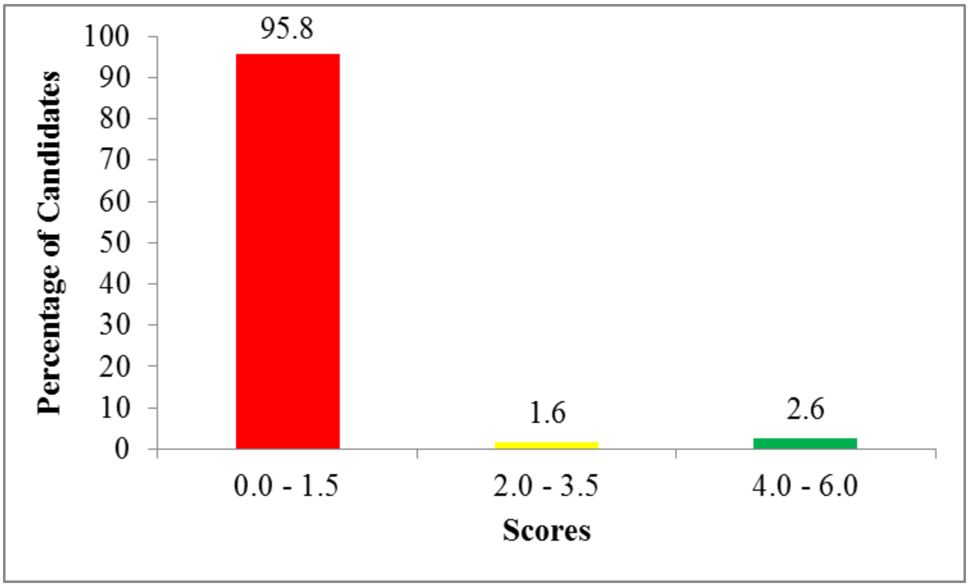


Figure 10: *Candidates' performance on question 10*

The response analysis shows that the weak performance on this question was a result of lack of competence by candidates in the basic concepts of quadratic equations. In part (a), most of the candidates failed to transform the given equation into positive exponents to obtain an equation corresponding to $2\frac{1}{t^{10}}-3\frac{1}{t^5}+1=0$. Other candidates failed to correctly substitute $x=\frac{1}{t^5}$ in the given equation to obtain the quadratic equation $2x^2-3x+1=0$. Also, there were candidates who transformed the given equation into positive exponent but failed to express the equation into $2\left(\frac{1}{t^5}\right)^2-3\left(\frac{1}{t^5}\right)+1=0$.

In part (b), majority of the candidates were unable to recall and apply the quadratic formula for solving quadratic equation. They used incorrect formulae such as $x=\frac{b\pm\sqrt{b^2-4ac}}{2a}$ and $x=-b\pm\frac{\sqrt{b^2-4ac}}{2a}$ instead of $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$. Also, some of the candidates were able to recall the quadratic formula but they failed to correctly substitute the values of a, b and c in the formula. Moreover, other candidates could not perform correct calculations since they lacked computation skills. Also, there were candidates who opted to use other methods such as splitting the middle term and factorization contrary to the requirement of the question. Extract 10.1 shows a sample of a response from one of the candidates who answered this question incorrectly.

10. a/ $2t^{-10} - 3t^{-5} + 1 = 0$ where $x = \frac{1}{t^5}$

$2t^{-10+-5} + 3t + 1 = 0 - x = \frac{1}{t^5}$

$5t^{-15} + 1 = 0 - x = \frac{1}{t^5}$

$6t^{-15} - x = \frac{1}{t^5}$

$\therefore \cancel{6}\, x = 6t^{-15} - x = \frac{1}{t^5}$

b/ Soln

$x = \frac{-b \pm \sqrt{b^2 \times 4ac}}{2a}$

$x = \frac{-3 \pm \sqrt{3^2 \times 4 + 2 \times 1}}{2 \times 2}$

$x = \frac{-3 \pm \sqrt{9 \times 6}}{4}$

$x = \frac{-3 \pm \sqrt{54^2}}{4\,\cancel{2}}$

$x = \frac{-3 + \sqrt{52}}{2}$ or $x = \frac{-3 - \sqrt{52}}{2}$

Extract 10.1: A sample of the candidate's incorrect responses to question 10

Extract 10.1 shows that, in part (a), the candidate was not able to write the given equation in terms of x. In part (b), the candidate solved the equation incorrectly due to lack of computation skills and inability to use the quadratic formula.

Further analysis shows that the candidates who scored all the marks allotted to this question had enough knowledge and skills of solving quadratic equations by using the quadratic formula. In part (a), the candidates were able to express the given equation in terms of x. They re-arranged the given equation in the form $2\left(\frac{1}{t^5}\right)^2 - 3\left(\frac{1}{t^5}\right) + 1 = 0$ and then substituted

$x = \frac{1}{t^5}$ to obtain the equation $2x^2 - 3x + 1 = 0$. In part (b), the candidates were able to solve the equation $2x^2 - 3x + 1 = 0$ by using the quadratic formula, that is $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. They performed proper substitution and calculations to obtain $x = 1$ or $x = \frac{1}{2}$. Extract 10.2 shows a sample of a response of a candidate who answered this question correctly.

10 a)	sln
	$2t^{-10} - 3t^{-5} + 1 = 0$
	$2(t^{2})^{-5} - 3t^{-5} + 1 = 0$
	$2(t^{-5})^{2} - 3t^{-5} + 1 = 0$
	$2\left(\frac{1}{t^5}\right)^2 - 3\left(\frac{1}{t^5}\right) + 1 = 0$
	But $\frac{1}{t^5} = x$
	$\therefore 2x^2 - 3x + 1 = 0$
	Expressed.
10 b)	sln
	from; $2x^2 - 3x + 1 = 0$
	By quadratic formulae.
	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
	$x = \frac{-(-3) \pm \sqrt{-3^2 - (4 \times 2 \times 1)}}{2 \times 2}$
	$x = \frac{3 \pm \sqrt{9 - 8}}{4}$

10(b) $x = \frac{3 \pm 1}{4}$

Either $x = \frac{3+1}{4}$

$x = \frac{4}{4}$

$x = 1$

or $x = \frac{3-1}{4}$

$x = \frac{2}{4}$

$x = 0.5$

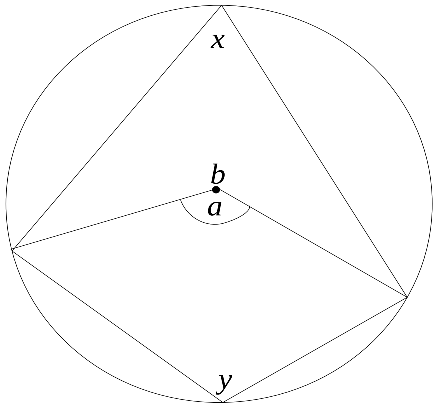
$\therefore$ Either $x = 0.5$ or $x = 1$

Extract 10.2: A sample of the candidate's correct responses to question 10

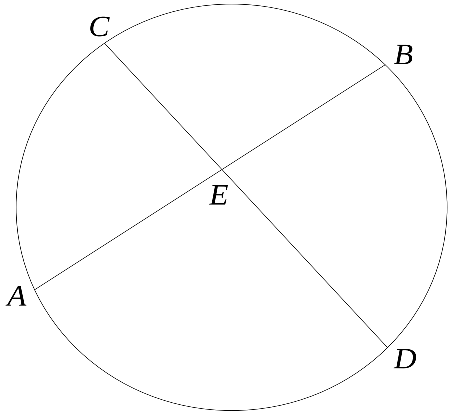
Extract 10.2 shows that, in part (a), the candidate was able to write the given equation in terms of x. In part (b), the candidate solved correctly the equation obtained in (a) by using the quadratic formula to get the correct values of x.

2.11 Question 11: Circles

This question consisted of three parts, (a), (b), and (c). Part (a) intended to examine the candidates' ability to calculate in degrees, the size of the central angle formed by an arc of the length of 22 cm in a circle whose radius is 63 cm. In part (b), the candidates were examined to prove that the angles x and y are supplementary given that a and b are the angles at the centre of the circle as shown in the following figure.



Part (c) examined if the candidates were able to apply the competence developed in the concepts of circles. They were required to use one of the chord properties to determine the length of the line segment $\overline{DE}$ from two intersecting chords $\overline{CD}$ and $\overline{AB}$ inside a circle, given that $\overline{AE}=8\text{cm}$, $\overline{BE}=3\text{cm}$ and $\overline{CE}=4\text{cm}$ as shown in the following figure.



The analysis indicates that, this question was attempted by a total of 521,886 (100%) candidates, out of whom 52,539 (10.1%) candidates passed. This shows that the candidates' performance on this question was weak. It was further revealed that, 7,596 (1.5%) candidates scored all the allotted marks whereas a total of 456,177 (87.4%) scored 0 marks. The candidates' performance on question 11 is summarised in Figure 11.

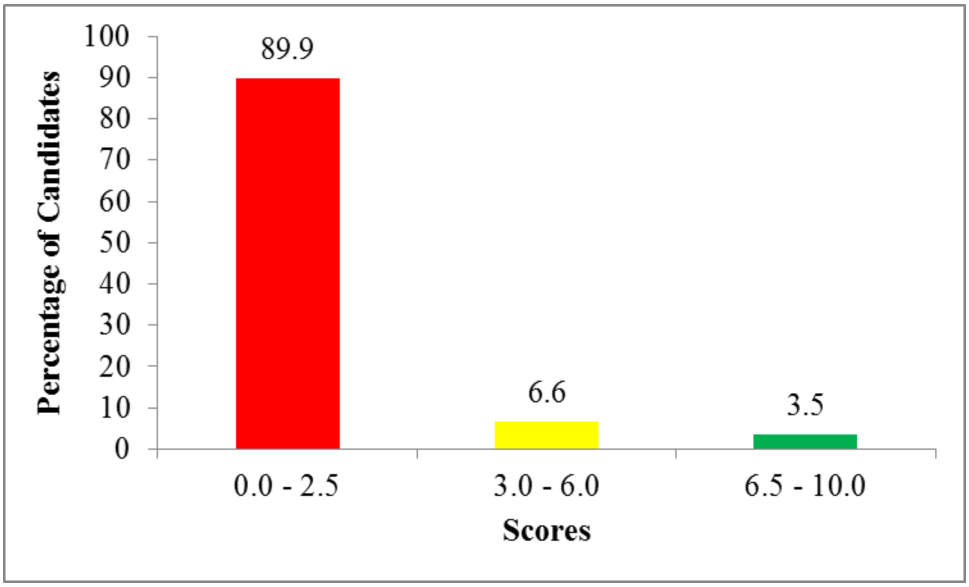
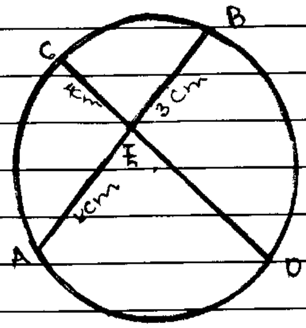


Figure 11: *Candidates' performance on question 11*

The analysis of responses shows that the candidates who scored zero on this question lacked competence in the concepts of circles. They failed to recall and apply the formula for arc length, that is $\text{arc length}, l = \frac{\pi r\theta}{180°}$. Other candidates failed to transpose the formula for finding the arc length to obtain $\theta = \frac{180° l}{\pi r}$. Further analysis shows that some of the candidates applied wrong formulae such as $central\,angle = \pi r^2$ or $central\ angle = \pi rl$ instead of $\text{arc length}, l = \frac{\pi r\theta}{180°}$. Moreover, most of the candidates were not able to verify that, opposite angles in a cyclic quadrilateral are supplementary. Also, they were not able to apply the theorem of intersecting chords to find the length of the line segment $\overline{\mathrm{DE}}$. Extract 11.1 shows a sample of the candidate's incorrect responses to this question.

	soln.
11.a	$L = \pi R \theta$
	$22cm = 22/7 \times 6370 \times \theta$
	$\frac{22}{20020} = \frac{20020\theta}{20020}$
	$\theta = 1.098901099 \times 10^{-3}$.
	soln.
11.b	$x+y = 90^\circ$
	$a = 2x$.
	$a - x = 180^\circ$
	$2x - x = 180^\circ$
	$x = 180^\circ$
	$180^\circ + y = 90^\circ$
	$y = 190^\circ - 90^\circ$
	$y = 90^\circ$
	$y = 180 - 2a$
	$90^\circ = 180^\circ - 2a$
	$180 - 90^\circ = -2a$
	$\frac{-90^\circ}{-2} = \frac{-2a}{-2}$
	$a = 45^\circ$.
	$x+y = 90^\circ$



soln.

11c

$\overline{AE} \times \overline{BE} = \overline{CE} \times \overline{CD}$

$8cm \times 3cm = 4cm \times (CE + ED)$

$8cm \times 3cm = 4cm \times (4 + x cm)$

$24cm = 4(4 + x)$

$24 = 16 + 4x$

$24 - 16 = 4x$

$\frac{8}{4} = \frac{4x}{4}$

$x = 2cm$

$\overline{DE} = 2cm.$

Extract 11.1: A sample of the candidate's incorrect responses to question 11

Extract 11.1 shows that, in part (a), the candidate applied inappropriate formula for finding the arc length and substituted the given values to get $\theta = 1.098901099 \times 10^{-3}$, which was an incorrect answer. In part (b), the candidate did not get the required condition for angles x and y to be supplementary. The candidate obtained $x + y = 90°$, which was not the correct condition. In part (c), the candidate applied incorrectly the theorem of intersecting chords, that is $\overline{CE} \times \overline{CD} = \overline{AE} \times \overline{BE}$ instead of $\overline{CE} \times \overline{DE} = \overline{AE} \times \overline{BE}$. The candidate substituted the given values and computed the length of $\overline{DE} = 2\,\text{cm}$, which was not the correct answer.

On the other hand, the analysis of responses shows that, the candidates who scored all the marks allotted to this question were able to apply the competence developed through learning the basic concepts of circles. In part (a), the candidates recognized the relationship among the arc length, central angle, and radius of a circle and applied the appropriate formula to determine the measure of the central angle, that is, arc length, $l = \frac{\pi r\theta}{180°}$. They transposed the formula to obtain $\theta = \frac{180° l}{\pi r}$. Finally, they substituted the given values and computed the value of θ to get $\theta = 20°$, which was the correct answer.

In part (b), the candidates who performed well had sufficient knowledge and skills to verify one of circle theorems which states that "the opposite angles in a cyclic quadrilateral are supplementary". They were able to recognize that, the angles a and b at the centre are twice the degree measures of angles x and y subtending on the remaining parts of the circumference, respectively. Similarly, they stated that $a + b = 360°$, since angles a and b complete the degree measure of a complete circle. The candidates performed proper substitutions to obtain $2x + 2y = 360°$. Simplifications were done to get $x + y = 180°$ which was the required condition for angles x and y to be supplementary. This highlights that the candidates had sufficient knowledge on circle theorems.

In part (c), the candidates applied chords' properties of a circle. They were able to recognize that, the chords $\overline{\text{CD}}$ and $\overline{\text{AB}}$ intersect internally at point E. They applied the theorem of intersecting chords such that $\overline{\text{CE}} \times \overline{\text{DE}} = \overline{\text{AE}} \times \overline{\text{BE}}$. The necessary calculations were performed to get $\overline{\text{DE}} = 6\,\text{cm}$. Alternatively, other candidates applied the concept of similar triangles by constructing new chords $\overline{\text{AC}}$ and $\overline{\text{BD}}$ to make two similar triangles, ΔACE and ΔBDE. They used similarity property to get $\frac{\overline{\text{AE}}}{\overline{\text{DE}}} = \frac{\overline{\text{CE}}}{\overline{\text{BE}}}$. Computations were done to get $\overline{\text{DE}} = 6\,\text{cm}$. Extract 11.2 is a sample of a response of a candidate who answered this question correctly.

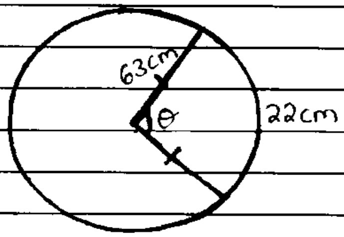
11a) Soln.

Length (L) = 22cm

Radius (r) = 63cm

$\pi = 22/7$

$L = \frac{\pi r \theta}{180^\circ}$



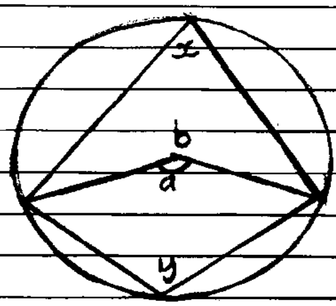
$22cm = \frac{\pi \times 63cm \times \theta}{180^\circ}$

$\theta = \frac{22cm \times 180^\circ}{\pi \times 63cm}$

$\theta = 20^\circ$

$\therefore$ The central angle $= 20^\circ$

11b) Soln.



$a = 2x$

$b = 2y$

$a + b = 360°$

when

$a = 2x$

$b = 2y$

$2x + 2y = 360°$

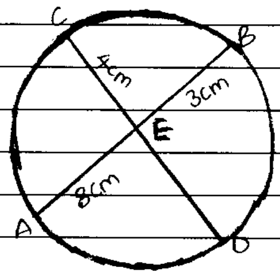
$\frac{2(x+y)}{2} = \frac{360°}{2}$

$x + y = 180°$

Supplementary angles add up to 180°

$\therefore$ Proven that the angles x and y are supplementary.

11c) Soln.



But

$\overline{AE} \times \overline{BE} = \overline{CE} \times \overline{DE}$

$\frac{8cm \times 3cm}{4cm} = \frac{4cm \times \overline{DE}}{4cm}$

$\overline{DE} = 6cm$

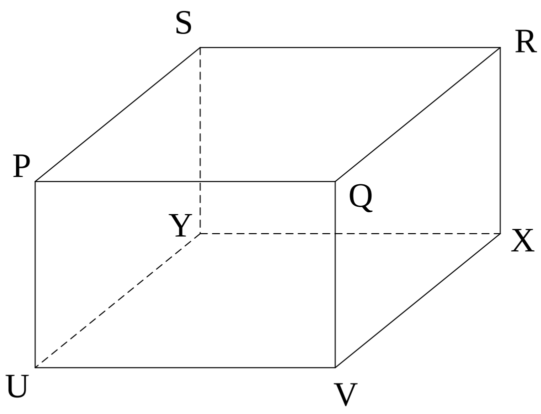
$\therefore$ The length $\overline{DE} = 6cm$.

Extract 11.2: A sample of the candidate's correct responses to question 11

Extract 11.2 shows that, the candidate applied the appropriate formula for finding the arc length and substituted the given values to get $\theta = 20°$, which was the correct answer. In part (b), the candidate stated that $a = 2x$ and $b = 2y$, then substituted into $a + b = 360°$ to get $x + y = 180°$ which was the required condition for angles x and y to be supplementary. In part (c), the candidate applied the theorem of intersecting chords such that $\overline{CE} \times \overline{DE} = \overline{AE} \times \overline{BE}$. The candidate substituted the values $\overline{AE} = 8\text{cm}$, $\overline{BE} = 3\text{cm}$ and $\overline{CE} = 4\text{cm}$ to get $\overline{DE} = 6\text{cm}$ which was the correct answer.

2.12 Question 12: The Earth as a Sphere and Three-Dimensional Figures

This question had two parts, (a) and (b). In part (a), the candidates were required to calculate the time taken by a bus which leaves town $A\,(3°S,\ 39°E)$ to $B\,(12°S,\ 39°E)$ at a constant speed of $40\,\text{km/h}$, given that $\pi = 3.14$ and radius of the earth, $R = 6,400\text{ km}$. In part (b), the candidates were given a rectangular box with base UVXY and plane PQRS being vertically above UVXY as shown in the following figure.



Given that $\overline{UV} = 4.2$ cm, $\overline{VX} = 2$ cm and $\overline{XR} = 2.5$ cm. The candidates were required to determine the lengths of $\overline{VR}$ and $\overline{UR}$, correct to one decimal place, and the angle between the diagonal $\overline{UR}$ and the base UVXY.

The analysis indicates that, the question was attempted by 521,886 (100%) candidates, whereby only 109,124 (20.9%) candidates passed. This indicates that the candidates' performance on this question was weak. Further analysis shows that 18,089 (3.5%) candidates scored all the allotted marks while 370,951 (71.1%) scored zero. Figure 12 shows the candidates' performance on question 12.

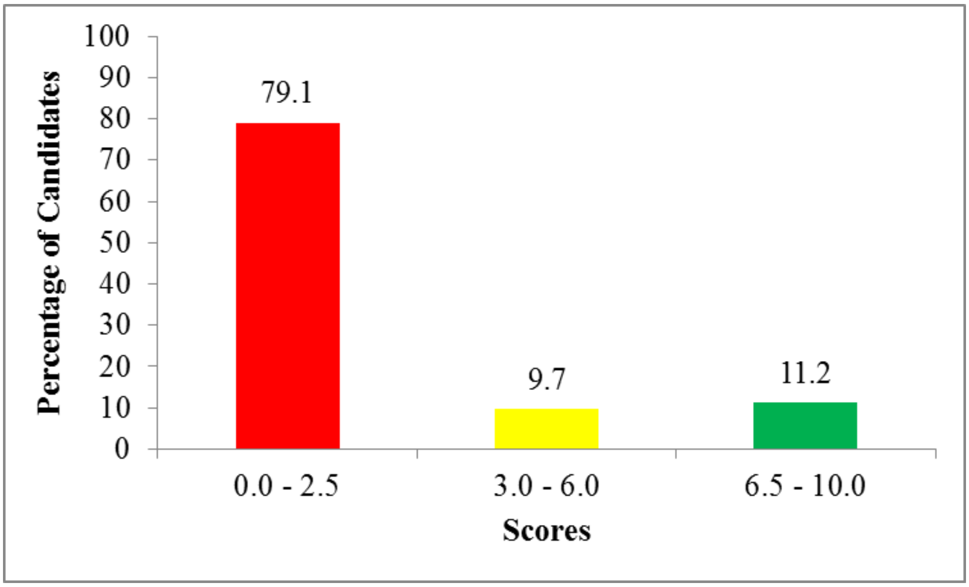


Figure 12: *Candidates' performance on question 12*

The analysis shows that majority of the candidates were not able to answer this question correctly. They demonstrated lack of competence on the basic concepts of the Earth as a sphere and three-dimensional figures. In part (a), the candidates could not recall and apply the appropriate formula to calculate the distance between two points lying on the same meridian. For instance, the candidates added the latitudes, $\theta = 12^\circ + 3^\circ = 15^\circ$ instead of subtracting $\theta = 12^\circ - 3^\circ = 9^\circ$. Also, other candidates applied wrong formulae for distance such as $d = \frac{\pi R\theta \cos\alpha}{180^\circ}$ instead of $d = \frac{\pi R\theta}{180^\circ}$. Moreover, some of the candidates could not recall the formula for calculating the time, that is $\text{Time} = \frac{\text{Distance}}{\text{Speed}}$. Thus, they used wrong formulae such as $\text{Time} = \text{Distance} - \text{Speed}$, $\text{Time} = \text{Distance} \times \text{Speed}$ or $\text{Time} = \text{Distance} + \text{Speed}$.

In part (b), the candidates were not able to extract and construct the required right-angled triangles from the rectangular box. Thus, they failed to apply the Pythagoras' theorem to calculate the lengths of $\overline{\text{VR}}$ and $\overline{\text{UR}}$. The analysis shows that some of the candidates performed irrelevant calculations for the intended lengths. For instance, they added the lengths of the sides of the rectangular box. That is, $\overline{\text{VR}} = 4.2 \text{ cm} + 2 \text{ cm} + 2.5 \text{ cm}$

to get $\overline{VR} = 8.7$ cm and $\overline{VR} = \overline{XR} + \overline{VX} = 4.5$ cm, which were incorrect answers. Further analysis shows that the candidates failed to locate the angle formed by the diagonal $\overline{UR}$ and the base $UVXY$, hence they failed to calculate its measure. Extract 12.1 is a sample of a response from a certain candidate who failed to answer the question correctly.

12. @ Given, Speed = 40Km/hr

$\theta = \cancel{\theta}^{\circ}\ 78^{\circ} - 15^{\circ} = 63^{\circ}$

Time = ?

1st; Recall distance on Earth.

$D = \left(\frac{\pi R}{180^{\circ}}\right)\theta$

12. @ $D = \left(\frac{3.14 \times 6,400}{180^{\circ}}\right) 63^{\circ}$

$= 111.6 \times 63$

Distance $= 70308$ Km

2nd; Recall Speed.

Speed $= \frac{\text{Distance}}{\text{Time}}$

$\longrightarrow 40\text{Km/hr} = \frac{7030.8 \text{ Km}}{\text{Time}}$

$\longrightarrow \text{Time} = \frac{7031}{40}$

$\text{Time} = \frac{176}{60}$

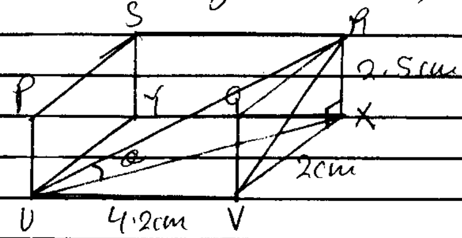
$= 2 : 93$

$- \ 60$

$3 \cdot 33$

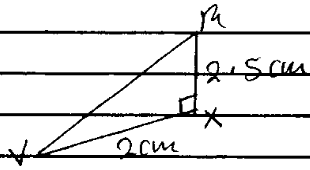
∴ Time taken was 3 hours 33 minutes.

12. b) Consider the figure below;



i) $\overline{VR} = ?$

Take ΔVXR, Apply pythagorus theorems.



$\longrightarrow \quad c^2 = a^2 + b^2$

$\overline{VR}^2 = (2.5cm)^2 + (2cm)^2$

$\overline{VR}^2 = 62.5cm^2 + 4cm^2$

$\sqrt{\overline{VR}^2} = \sqrt{66.5\ cm^2}$

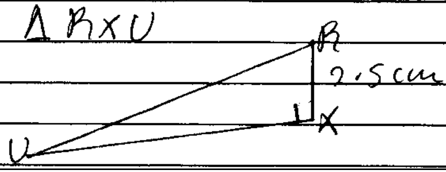
$\overline{VR} = \sqrt{66.5}\ cm$

$\overline{VR} = 8.155\ cm$

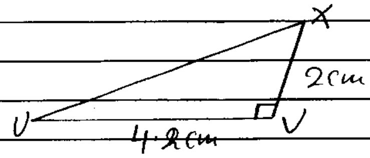
$\therefore \ \underline{\overline{VR} = 8.155\ cm} \simeq 8.2\ cm\ (1\ d.p)$

$\longrightarrow \ \overline{UR} = ?$

Take ΔRXU



12. b) Supporter figure; ΔXVU



Apply pythagorus theory.

$C^2 = a^2 + b^2$

$\overline{XU}^2 = (2cm)^2 + (4.2cm)^2$

$\overline{XU}^2 = 4cm^2 + 176.4\,cm^2$

$\sqrt{\overline{XU}^2} = \sqrt{180.4\ cm^2}$

$\overline{XU} = \sqrt{180.4}\ cm$

$\overline{XU} = \sqrt{18.04} \times \sqrt{100}$

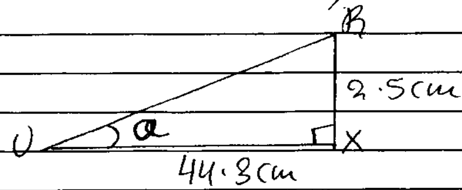
$\overline{XU} = \sqrt{18.04} \times 10\ cm$ 4.423

$\overline{XU} = 4.428 \times 10\ cm$

$= 44.28\ cm$

$\therefore \overline{XU} = 44.28\ cm \simeq 44.3\ cm\ (1d.p)$

ii) Take ΔRUX below;



Apply Trig ratios

$\text{Tan } a = \frac{\text{Opposite}}{\text{Adjacent}}$

12. b(ii) $\text{Tan } a = \frac{2.5}{44.3}$

$\text{Tan } a = 0.5643$ 0.5628 0.5542

$a = 29°$ 30

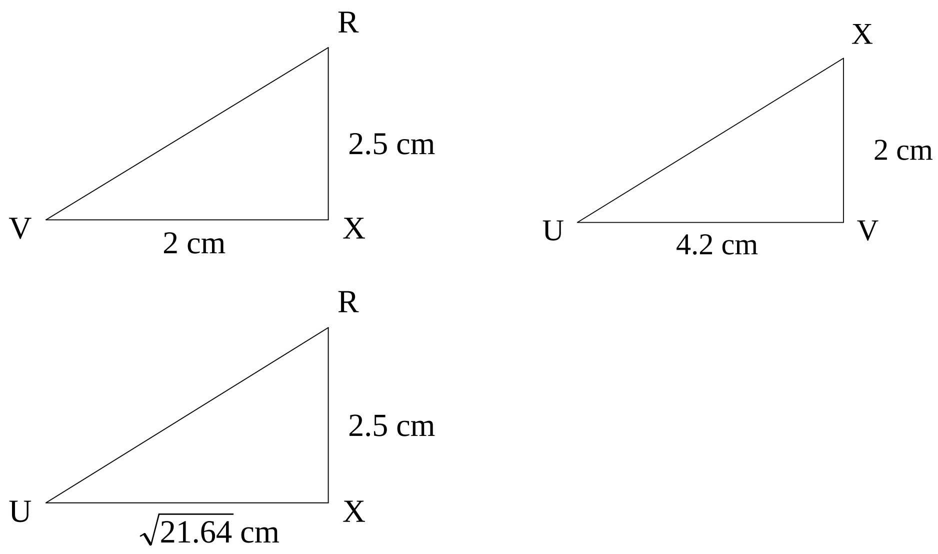
$\therefore$ Angle $R\hat{U}X$ is $29°$

Extract 12.1: A sample of the candidate's incorrect responses to question 12

Extract 12.1 shows that, in part (a), the candidate was not able to apply the correct formula for calculating distance, thus failed to obtain the time taken by the bus to travel from town A to town B. In part (b), the candidate failed to identify the sides which form the right-angled triangles that could help in calculating the lengths of $\overline{\mathrm{VR}}$ and $\overline{\mathrm{UR}}$. Also, the candidate failed to locate the angle U formed by the diagonal $\overline{\mathrm{UR}}$ and the base UVXY.

On the other hand, the analysis shows that the candidates who scored all the marks allotted to this question were able to apply their competence on the basic concepts of the Earth as a sphere and three-dimensional figures. In part (a), the candidates were able to recognize and apply appropriate formulae to arrive at the correct answer. First, they calculated the distance between the two towns, $\mathrm{A}\,(3°\mathrm{S},\ 39°\mathrm{E})$ and $\mathrm{B}\,(12°\mathrm{S},\ 39°\mathrm{E})$. Thus, they calculated the distance d using the formula $d = \dfrac{\pi R\theta}{180°}$, where $\theta = \alpha - \beta = 12° - 3° = 9°$ is the difference in latitudes. They substituted the values in the formula and made appropriate calculation to obtain $d = \dfrac{3.14 \times 6,400 \times 9°}{180°} = 1004.8\,\mathrm{km}$. Next, the candidates were able to recall and apply the formula for time, that is $\mathrm{Time} = \dfrac{\mathrm{Distance}}{\mathrm{Speed}}$, where $\mathrm{Speed} = 40\,\mathrm{km/h}$. They substituted the values in the formula to get $\mathrm{Time} = \dfrac{1004.8\,\mathrm{km}}{40\,\mathrm{km/h}} = 25.12\,\mathrm{hours}$, which was the correct time taken by the bus to travel from town A to town B.

In part (b), candidates were able to find the lengths of $\overline{\mathrm{VR}}$ and $\overline{\mathrm{UR}}$ by making suitable projections to construct the required right-angled triangles. The following right-angled triangles were constructed from the rectangular box to simplify the calculations.



The candidates calculated the length of $\overline{VR}$ by taking the right-angled triangle VXR and applied the Pythagoras' theorem, $\overline{VR}^2 = \overline{VX}^2 + \overline{RX}^2$ to obtain $\overline{VR} = 3.2\,\text{cm}$. Also, the length of $\overline{UR}$ was calculated from the right-angled triangle UXR such that, $\overline{UR}^2 = \overline{UX}^2 + \overline{XR}^2$, hence $\overline{UR} = 5.3\,\text{cm}$. The right-angled triangle UVX was constructed in order to get the length of $\overline{UX}$ which was used to obtain the length of $\overline{UR}$. The candidates were able to approximate the lengths of $\overline{VR}$ and $\overline{UR}$ in one decimal place.

Further analysis shows that the candidates were able to identify the angle U formed between the diagonal $\overline{UR}$ and the base UVXY. They correctly applied knowledge of trigonometric ratios of sine, cosine or tangent to calculate the measure of angle U. For instance, the candidates used $\tan U = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{2.5}{4.6519}$ to obtain $28.25°$ or $28°15'$. Extract 12.2 shows a sample of a response of a candidate who answered the question correctly.

12.	(a): Given Locations:
	A (3°S, 39°E) to B(12°S, 39°E):
	Speed = 40km/h ; θ = 12°-3° = 9°

12. (a): Recall the distance:

$\longrightarrow$ Distance $= \left(\frac{\pi R}{180^\circ}\right)\theta$

$$d = \left(\frac{3.14 \times 6400km}{180^\circ}\right) 9$$

$$= (111.64 km)(9)$$

$$= 1004.76km$$

But;

$\longrightarrow$ Speed $= \frac{distance}{time}$.

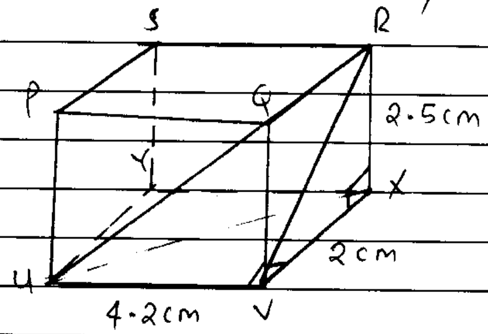
time $= \frac{distance}{speed} = \frac{1004.76km}{40km/h}$

$\approx 25.119 hrs$

$\approx 25.12 hrs$

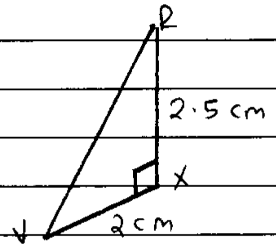
$\therefore$ The bus will take ≈ 25.119 hours to reach town B:

12. (b): Consider the box below;



(i): $\overline{VR} = ?$ and $\overline{UR} = ?$

12. (b) (i). Consider Δ RXV below



Apply:

$c^2 = a^2 + b^2$

$\overline{VR}^2 = (2cm)^2 + (2.5cm)^2$

$\overline{VR}^2 = 4cm + 6.25cm^2$

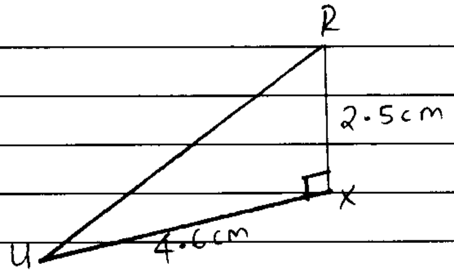
$VR^2 = 10.25cm^2$

$\sqrt{VR^2} = \sqrt{10.25cm^2}$

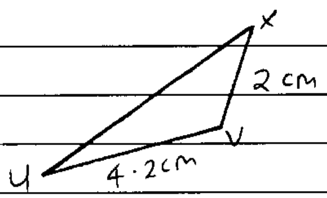
$\overline{VR} \approx 3.2$ cm. (1 d.p)

$\Longrightarrow \overline{UR} = ?$

from Δ RXU below



$\Longrightarrow$ Consider Δ UVX below



Apply!

$c^2 = a^2 + b^2$

$UX^2 = (4.2cm)^2 + (2cm)^2$

$\overline{UX}^2 = 17.64cm^2 + 4cm^2$

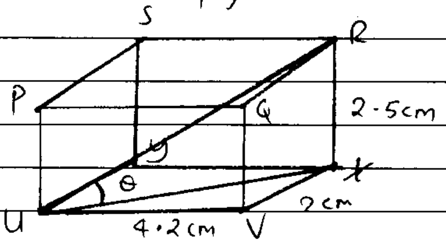
$UX^2 = 21.64cm^2$

12.	(b) (i); $\sqrt{\overline{UX}^2} = \sqrt{21.64\text{cm}^2}$	But; $\overline{UR} = \sqrt{(4.6\text{cm})^2 + (2.5\text{cm})^2}$
	$\overline{UX} \approx 4.65\text{ cm}$	$= \sqrt{27.41\text{cm}^2}$
	$\overline{UX} \approx 4.6\text{cm}$ (1 d.p)	$= 5.2\text{cm}$

$\therefore$ The value of $\overline{VR} \approx 3.2$ cm and $\overline{UR} \approx 5.2$ cm correct to 1 d.p.

(ii); < btn diagonal $\overline{UR}$ and UVXY.

Consider figure below;



→ From Δ RXU apply trig ratios.

$\text{Tan } \theta = \frac{\text{opp}}{\text{Adj}}$

$\text{Tan } \theta = \frac{2.5\text{cm}}{4.6\text{cm}}$

$\text{Tan } \theta = 0.5435$

$\theta = 28°31'$

$\therefore$ The angle $= 28°31'$

Extract 12.2: A sample of the candidate's correct responses to question 12

Extract 12.2 shows that, the candidate was able to apply a correct formula for distance between two points along the same meridian in part (a). The candidate used the results to calculate the time taken by the bus to travel from town A (3°S, 39°E) to town B (12°S, 39°E). In part (b), the candidate was able to apply the Pythagoras' theorem correctly to calculate the lengths of $\overline{VR}$ and $\overline{UR}$ as well the measure of angle U between the diagonal $\overline{UR}$ and the base UVXY by using appropriate trigonometric ratios.

2.13 Question 13: Matrices and Transformations

This question consisted of three parts, (a), (b) and (c). In part (a), the candidates were required to find the values of x, y, z and w from the matrix equation, $\begin{pmatrix} x & 4 \\ 4 & y \end{pmatrix}\begin{pmatrix} -5 & -7 \\ 2 & z \end{pmatrix} = \begin{pmatrix} 38 & 46 \\ -10 & w \end{pmatrix}$. In part (b), the candidates were required to find the image of the point $(3,-2)$ after a reflection on the line $y=-x$, followed by another reflection on the line $x=0$. In part (c), the candidates were given the information that, "A translation takes the point $(5, 5)$ to $(-7,-7)$". The candidates were required to find the values of x and y if the reflection takes the point (x, y) to $(-4,-4)$.

The data analysis shows that a total of 521,886 (100%) candidates attempted this question and among them, 56,052 (10.7%) candidates passed, indicating the candidates' weak performance on this question. The analysis further indicates that, only 5,841 (1.1%) candidates scored all the allotted marks while a total of 430,710 (82.5%) candidates scored zero. Figure 13 summarizes the candidates' performance on question 13.

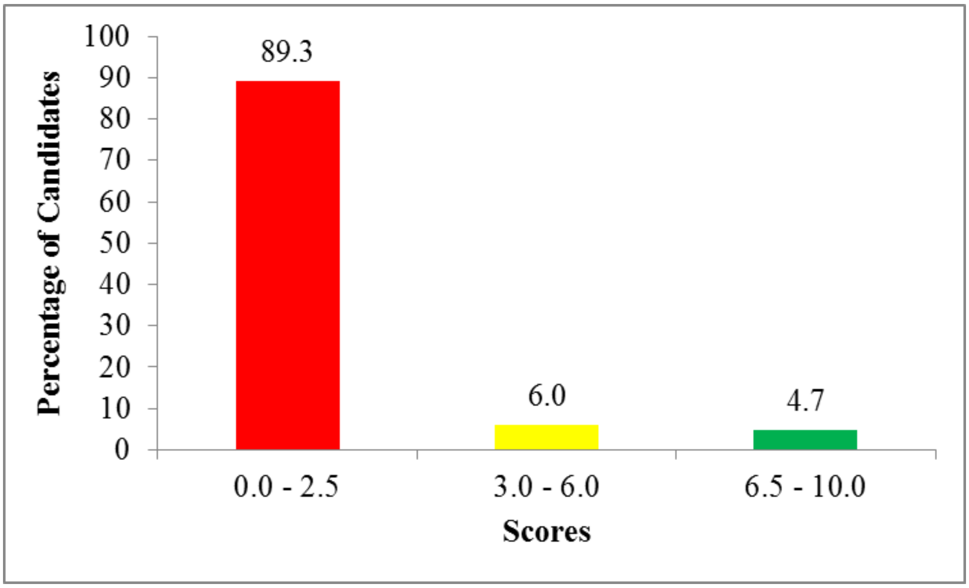


Figure 13: *Candidates' performance on question 13*

The analysis shows that the candidates who performed poorly on this question lacked knowledge and skills on the basic concepts of matrices and transformations. In part (a), the candidates failed to multiply 2×2 matrices in the given equation. They multiplied the entries instead of multiplying the matrices. Some of the candidates multiplied the diagonal entries while others added the two matrices. Further analysis shows that some of the candidates computed the determinant and inverse of the matrices. Also, the candidates pre-multiplied column entries by row entries instead of pre-multiplying row entries by column entries, that is $\begin{pmatrix} x & 4 \\ 4 & y \end{pmatrix}\begin{pmatrix} -5 & -7 \\ 2 & z \end{pmatrix}$ was wrongly multiplied to get $\begin{pmatrix} 5x-28 & 20-7y \\ -20-7y & 8+yz \end{pmatrix}$ instead of $\begin{pmatrix} -5x+8 & -7x+4z \\ -20+2y & -28+yz \end{pmatrix}$. There were candidates who multiplied the matrices and obtained the identity matrix $\mathrm{I}=\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ while others applied the inverse of the matrix to solve simultaneous equations which resulted to incorrect answers.

In part (b), the candidates failed to use a reflection matrix to determine the image of the point $(3,-2)$. They used graphical method contrary to the requirement of the question. They failed to recognize the angles of reflection on the lines $y=-x$ and $x=0$. Other candidates failed to identify the angle of reflection on the line $x=0$. They used $\alpha=0°$ instead of $\alpha=90°$. Generally, the candidates lacked knowledge on the properties of matrices. They were unable to recall that, matrix multiplication is not commutative. They multiplied the matrices such that,

$$M_{y=-x}M_{x=0}\begin{pmatrix} x'' \\ y'' \end{pmatrix}=\begin{pmatrix} \cos 270° & \sin 270° \\ \sin 270° & -\cos 270° \end{pmatrix}\begin{pmatrix} \cos 180° & \sin 180° \\ \sin 180° & -\cos 180° \end{pmatrix}\begin{pmatrix} 3 \\ -2 \end{pmatrix}$$

which is not equal to

$$M_{x=0}M_{y=-x}\begin{pmatrix} x'' \\ y'' \end{pmatrix}=\begin{pmatrix} \cos 180° & \sin 180° \\ \sin 180° & -\cos 180° \end{pmatrix}\begin{pmatrix} \cos 270° & \sin 270° \\ \sin 270° & -\cos 270° \end{pmatrix}\begin{pmatrix} 3 \\ -2 \end{pmatrix}.$$

Some of the candidates used the matrix formula for rotation instead of reflection, that is, $\begin{pmatrix} \cos 135^\circ & -\sin 135^\circ \\ \sin 135^\circ & -\cos 135^\circ \end{pmatrix}$ and $\begin{pmatrix} \cos 90^\circ & -\sin 90^\circ \\ \sin 90^\circ & \cos 90^\circ \end{pmatrix}$. Also, the candidates were able to get the image of the point after first reflection but failed to get the second image because they used the original point $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$ instead of the first image $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$. There were candidates who used radian angles instead of degree angles and they obtained incorrect answers while other candidates computed the slope and equation of the line as $m = -1$, $y = m(x - x_0) + y_0$ and obtained $y = -x + 1$.

In part (c), the candidates were not able to distinguish between the image and the translated point (x, y). Thus, they interchanged the image with the translated point. It was further noted that, some of the candidates treated the question as an enlargement problem. They applied the formula $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$ such that $\begin{pmatrix} -7 \\ -7 \end{pmatrix} = \begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix} \begin{pmatrix} 5 \\ 5 \end{pmatrix}$ which was contrary to the requirement of the question. Other candidates failed to recall the formula for translation, $\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} x' \\ y' \end{pmatrix}$ as they subtracted instead of adding the object point and the translation vector. Extract 13.1 shows a sample of a response by one of the candidates who failed to answer the question correctly.

13	a) $x + 4 = 38$
	$x = 38 - 4$
	$x = 34$
	$4 + y = 46$
	$y = 46 - 4$
	$y = 42$
	$-5 + -7 = -10$
	-12
	$-5 + -7 = w$
	$w = -5 + -7$
	$w = -12$
	$2 + z = -10$
	$z = -10 - 2$
	$z = -12$

13 b/ $(3, -2)$

$y = -x$

$\alpha = 45 \times 2 = 90$

from

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \cos 2\alpha & \sin 2\alpha \\ \sin 2\alpha & -\cos 2\alpha \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}'$$

$$= \begin{pmatrix} \cos 90 & \sin 90 \\ \sin 90 & -\cos 90 \end{pmatrix} \begin{pmatrix} 3 \\ -2 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 3 \\ -2 \end{pmatrix}$$

$$\begin{pmatrix} 0 + 3 \\ -2 + 0 \end{pmatrix}$$

$$\begin{pmatrix} 3 \\ -2 \end{pmatrix}$$

$x, y = {}^{+}3, {}^{-}2$

followed by another reflection

$$\begin{pmatrix} \cos 360 & \sin 360 \\ \sin 360 & -\cos 360 \end{pmatrix} \begin{pmatrix} 3 \\ -2 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 3 \\ -2 \end{pmatrix}$$

$$\begin{pmatrix} 3 + 0 \\ 0 + 2 \end{pmatrix}$$

$3, 2$

$\therefore$ The image of the point $= (3, 2)$

13(c) solution

Given that

$(5, 5)$ and $(-7, 7)$

If takes point (x, y) to $(-4, 4)$

$(5, 5) - (-7, 7) = (4, 4)$

$(5, 5) + (7, 7) - (4, 4)$

$$\begin{pmatrix} 5 \\ 7 \end{pmatrix} = \begin{pmatrix} 4 \\ 4 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 1 \end{pmatrix} \begin{pmatrix} 5 \\ 5 \end{pmatrix}$$

Extract 13.1: A sample of the candidate's incorrect responses to question 13

Extract 13.1 shows that, the candidate failed to multiply the matrices in part (a). In part (b), the candidate failed to get the image of point $(3,-2)$ after a reflection on the line $y=-x$ that was followed by the reflection on the line $x=0$. In part (c), the candidate failed to find the coordinates of the point (x, y) using the image of translation.

Despite the weak performance of the majority of candidates, the analysis shows that the candidates who scored all the marks allotted to this question were competent on the tested concepts of matrices and transformations. In part (a), the candidates multiplied the 2×2 matrices in the equation, $\begin{pmatrix} x & 4 \\ 4 & y \end{pmatrix}\begin{pmatrix} -5 & -7 \\ 2 & z \end{pmatrix}=\begin{pmatrix} 38 & 46 \\ -10 & w \end{pmatrix}$ to obtain $\begin{pmatrix} -5x+8 & -7x+4z \\ -20+2y & -28+yz \end{pmatrix}=\begin{pmatrix} 38 & 46 \\ -10 & w \end{pmatrix}$. The entries of the matrices were equated to form four equations. They solved for the values of the variables to obtain $x=-6,\ y=5,\ z=1$ and $w=-23$.

In part (b), the candidates were able to determine that the angles of reflection on the lines $y=-x$ and $x=0$ are $135°$ and $90°$, respectively. They applied correctly the formula for reflection, $\text{M}_{y=mx+c}\begin{pmatrix} x' \\ y' \end{pmatrix}=\begin{pmatrix} \cos 2\alpha & \sin 2\alpha \\ \sin 2\alpha & -\cos 2\alpha \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$, where α is the angle of reflection on the line $y=mx+c$. They substituted $\alpha=135°$ to get $\text{M}_{y=-x}\begin{pmatrix} x' \\ y' \end{pmatrix}=\begin{pmatrix} \cos 270° & \sin 270° \\ \sin 270° & -\cos 270° \end{pmatrix}\begin{pmatrix} 3 \\ -2 \end{pmatrix}$. Simplifications were done to obtain $\begin{pmatrix} x' \\ y' \end{pmatrix}=\begin{pmatrix} 2 \\ -3 \end{pmatrix}$. Next, the candidates determined the reflection of the point $(x', y')=(2,-3)$ on the line $x=0$, that is $\text{M}_{x=0}\begin{pmatrix} x'' \\ y'' \end{pmatrix}=\begin{pmatrix} \cos 180° & \sin 180° \\ \sin 180° & -\cos 180° \end{pmatrix}\begin{pmatrix} 3 \\ -2 \end{pmatrix}$ and obtained $\begin{pmatrix} x'' \\ y'' \end{pmatrix}=\begin{pmatrix} -2 \\ -3 \end{pmatrix}$.

In part (c), the candidates were able to get the translation vector of the point $(5,\ 5)$ to $(-7,-7)$ using the mapping $(x, y)\xrightarrow{\text{T}(a,b)}(x', y')$, where $(x, y)=(5,\ 5)$ and $(x', y')=(5+a,\ 5+b)=(-7,-7)$. They solved for the values of a and b to get $a = -12$ and $b = -12$. The translation vector $(a,\ b) = (-12,\ -12)$ was used to find the coordinates of the point (x, y) if it is taken to $(-4,\ -4)$. The candidates determined the point (x, y) by using the fact that, $(x, y) \xrightarrow{T(-12,-12)} (-4, -4)$. That is, $x - 12 = -4$ and $y - 12 = -4$. They determined the values of x and y to get $x = 8$ and $y = 8$, which was the correct answer. Extract 13.2 shows a sample of a response of a candidate who answered the question correctly.

13a) $\begin{pmatrix} x & 4 \\ 4 & y \end{pmatrix} \begin{pmatrix} -5 & -7 \\ 2 & z \end{pmatrix} = \begin{pmatrix} 38 & 46 \\ -10 & w \end{pmatrix}$

$\begin{pmatrix} x \times -5 + 4 \times 2 & x \times -7 + 4 \times z \\ 4 \times -5 + y \times 2 & 4 \times -7 + y \times z \end{pmatrix} = \begin{pmatrix} 38 & 46 \\ -10 & w \end{pmatrix}$

$\begin{pmatrix} -5x + 8 & -\overset{7}{\cancel{28}}x + 4z \\ -20 + 2y & -28 + yz \end{pmatrix} = \begin{pmatrix} 38 & 46 \\ -10 & w \end{pmatrix}$

$-5x + 8 = 38$

$-5x = 38 - 8$

$\frac{-5x}{-5} = \frac{30}{-5}$

$x = -6$

$-7(-6) + 4z = 46$

$+42 + 4z = 46$

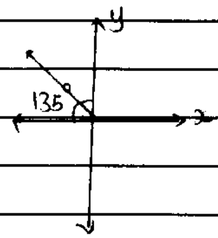
$4z = 46 - 42$

$4z = 4$

$z = 1$

13a	$-20+2y=-10$
	$2y=-10+20$
	$\frac{2y}{2}=\frac{10}{2}$
	$y=5$
	$-28+yz=w$
	$-28+5(1)=w$
	$-28+5=w$
	$w=-23.$
	$\therefore x=-6,\ y=5,\ z=1,\ w=-23.$

13 b	solution
	Given
	$P=(3,-2)$
	$R=y=-x$
	$x=0$
	For
	$y=-x$



From Reflection

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos 2\alpha & \sin 2\alpha \\ \sin 2\alpha & -\cos 2\alpha \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$= \begin{pmatrix} \cos 2\times135^\circ & \sin 2\times135^\circ \\ \sin 2\times135^\circ & -\cos 2\times135^\circ \end{pmatrix}$$

$$M = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$$

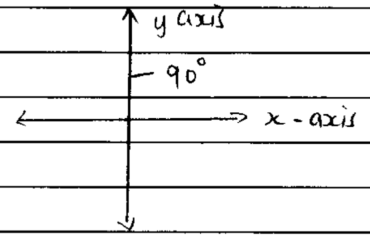
13 (b)

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 3 \\ -2 \end{pmatrix}$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$$

Then followed by another reflection in line

$x = 0$



From Reflection

$$\begin{pmatrix} x'' \\ y'' \end{pmatrix} = \begin{pmatrix} \cos 2\alpha & \sin 2\alpha \\ \sin 2\alpha & -\cos 2\alpha \end{pmatrix} \begin{pmatrix} x' \\ y' \end{pmatrix}$$

$$= \begin{pmatrix} \cos 2\times90^\circ & \sin 2\times90^\circ \\ \sin 2\times90^\circ & -\cos 2\times90^\circ \end{pmatrix}$$

$$= \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} x'' \\ y'' \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ -3 \end{pmatrix}$$

$$\begin{pmatrix} x'' \\ y'' \end{pmatrix} = \begin{pmatrix} -2 \\ -3 \end{pmatrix}$$

$$\therefore \begin{pmatrix} x'' \\ y'' \end{pmatrix} = \begin{pmatrix} -2 \\ -3 \end{pmatrix}$$

13 (c) solution

Translation

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = T \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} -7 \\ -7 \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix}\begin{pmatrix} 5 \\ 5 \end{pmatrix}$$

$$\begin{pmatrix} -7 \\ -7 \end{pmatrix} - \begin{pmatrix} 5 \\ 5 \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix}$$

$$\begin{pmatrix} -12 \\ -12 \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix}$$

$$\therefore T\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} -12 \\ -12 \end{pmatrix}$$

Again

$$\begin{pmatrix} -4 \\ -4 \end{pmatrix} = \begin{pmatrix} -12 \\ -12 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} -4 & + & 12 \\ -4 & & 12 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} 8 \\ 8 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\therefore (x, y) = (8, 8)$$

Extract 13.2: A sample of the candidate's correct responses to question 13

Extract 13.2 shows that, the candidate multiplied the 2×2 matrices in the question and compared the corresponding entries to get $x=-6$, $y=5$, $z=1$ and $w=-23$. The candidate determined the image of the point $(3,-2)$ under the reflection on the line $y=-x$, followed by a reflection on the line $x=0$ to obtain $(-2,-3)$. Also, the candidate used knowledge on the concept of translation to get the translation vector $(-12,-12)$ and the point $(8, 8)$.

2.14 Question 14: Functions and Linear Programming

This question had parts (a) and (b). In part (a), the candidates were required to determine the domain, range and inverse of $f(x)=\frac{1}{x-2}$ at $x=\frac{1}{3}$. In part (b), the candidates were given the information "Antony wishes to buy black shirts and white shirts. He intends to buy at most five black shirts. A black shirt costs $\text{Tsh. } 24,000$ while a white shirt costs $\text{Tsh. } 30,000$ and he is planning to spend up to $\text{Tsh. } 180,000$ for buying shirts". The candidates were required to find the maximum number of shirts for each kind and the greatest number of shirts to be bought.

The analysis shows that this question was attempted by a total of 521,886 (100%) candidates, out of whom, 49,865 (9.6%) candidates passed. This shows that the candidates' performance on this question was weak. Further analysis indicates that, only 1,221 (0.2%) candidates scored all the allotted marks while 413,040 (79.1%) scored zero. Figure 14 summarizes the candidates' performance on question 14.

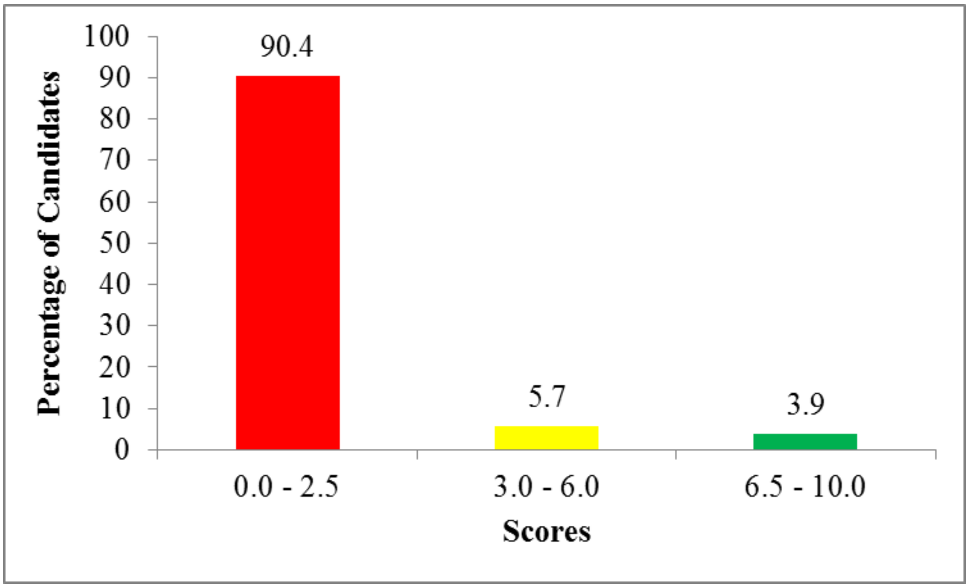


Figure 14: *Candidates' performance on question 14*

The response analysis reveals that, the candidates who performed poorly on this question lacked knowledge and skills on basic concepts of functions and linear programming. In part (a), the candidates failed to state the domain and range of the given function. They were not aware of the fact that the function $f(x)$ is not defined at $x=2$ and $f(x)\neq 0$ for all values of x in the domain. For instance, the candidates wrongly responded that $\text{Domain}=\{x: x\in\mathbb{R}\}$ and $\text{Range}=\{y: y\in\mathbb{R}\}$ instead of $\text{Domain}=\{x: x\in\mathbb{R},\ x\neq 2\}$ and $\text{Range}=\{y: y\in\mathbb{R},\ y\neq 0\}$. Moreover, other candidates sketched the graph of $f(x)=\dfrac{1}{x-2}$ wrongly, thus they failed to state the domain and range correctly. Further analysis shows that majority of the candidates failed to find the inverse of $f(x)$. For instance, some candidates made illogical calculations to get $f^{-1}(x)=1-2x$ while others solved inappropriately for the value of x. That is, $x-2=1$ to obtain $x=3$. Furthermore, some of the candidates substituted $x=\dfrac{1}{3}$ in $f(x)$ instead of $f^{-1}(x)$.

In part (b), the candidates failed to formulate the inequalities and objective function from the given linear programming problem. They formulated inequalities that are not suitable to the given maximization problem. For instance, some of the candidates formulated inequalities of the form $x\geq 5$, $x<5$, $x+y\leq 5$ and $24x+30y=180$ which were incorrect. Further analysis shows that some of the candidates could not identify the objective function, $f(x,y)=x+y$. They presented some functions that did not suit the problem such as $f(x,y)=4x+5y$ or $f(x,y)=5x+4y$. Also, most of the candidates failed to sketch the graph representing the linear programming problem as a result of inability to make correct formulation. Extract 14.1 shows a sample of a response of a candidate who failed to answer this question correctly.

14. a) $f(x) = \frac{1}{x-2}$

$y = \frac{1}{x-2}$

$y \times x - 2 = 1$

$y \frac{\cancel{x-2}}{\cancel{x-2}} = \frac{1}{x-2}$

$y = \frac{1}{x-2}$

i) $\therefore$ Domain $\{x : x \in \mathbb{R}\}$

Range $\{y : y \in \mathbb{R}\}$

ii) $f^{-1}\left(\frac{1}{3}\right)$

but $y = \frac{1}{x-2}$

$\left(\frac{1}{x-2}\right) \cdot \left(\frac{1}{3}\right)$

$\frac{1}{3x-2}$

$\frac{3x}{3} = \frac{3}{3}$

$x = 1$

$\therefore f^{-1}\left(\frac{1}{3}\right) = 1$

14(b) (i) 1 black shirt = 24000 Tsh.

5 black shirt $\cancel{=}$?

$$\frac{1\text{ black shirt} \times x}{1\text{ black shirt}} = \frac{5\text{ black shirt} \times 24000}{1\text{ black shirt}}$$

$x = 120,000$

Five black shirts = Tsh. 120,000

Then

1 white shirts = 30,000

5 white shirts $\cancel{=}$?

$$\frac{1\text{ white shirt} \times x}{1\text{ white shirt}} = \frac{5\text{ white shirts} \times 30,000}{1\text{ white shirt}}$$

$x = 150,000$

Five black shirt = Tsh. 150,000

Then

5 shirt = 120,000

? $\cancel{=}$ 180,000

$$\frac{5\text{ shirt} \times \text{Tsh } 180,000}{\text{Tsh}} = \frac{x \times 120,000\text{ Tsh.}}{\text{Tsh}}$$

$$\frac{900000}{120,000} = \frac{120,000}{120,000}$$

$x = 7.5 \approx 8$

∴ The black shirts will be 8

5 white shirt = 150,000

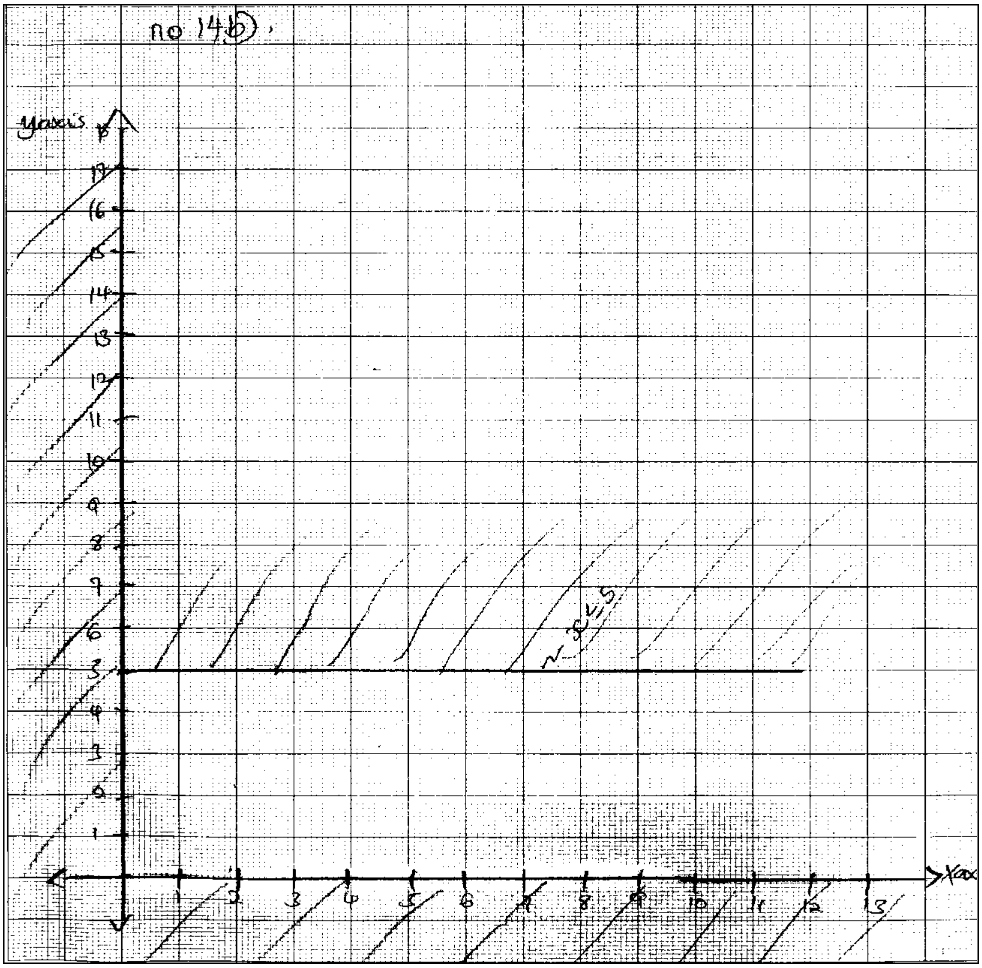
? = 180,000

$$\frac{5 \times 180,000}{150,000} = \frac{150,000x}{150,000}$$

$$x = \frac{900000}{150,000}$$

$x = 6$

∴ The white shirts will be 6



Extract 14.1: A sample of the candidate's incorrect responses to question 14

Extract 14.1 shows that, in part (a), the candidate failed to state the domain and range of the function $f(x)=\frac{1}{x-2}$. The candidate was unable to determine the inverse of the given function, hence failed to evaluate $f^{-1}\left(\frac{1}{3}\right)$. In part (b), the candidate was unable to formulate inequalities and objective function representing the given linear programming problem. The candidate used irrelevant approaches and ended up with incorrect answers.

On the other hand, the analysis shows that the candidates who responded to this question correctly were sufficiently competent in the basic concepts of functions and linear programming. In part (a), they stated correctly the domain and range of $f(x)=\frac{1}{x-2}$, that is, domain $=\{x: x\in\mathbb{R},\ x\neq 2\}$ and range $=\{\mathrm{y}:\mathrm{y}\in\mathbb{R},\ y\neq 0\}$. The candidates were able to find the inverse of the function by letting $y=f(x)$ and interchanging the variables x and y. They expressed y in terms of x to get $f^{-1}(x)=\frac{1}{x}+2$. Finally, they evaluated the value of $f^{-1}(x)$ at $x=\frac{1}{3}$ to get $f^{-1}\left(\frac{1}{3}\right)=5$. Further analysis shows that other candidates expressed x in terms of y and then interchanged the variables to obtain $f^{-1}(x)=\frac{1}{x}+2$.

In part (b), the candidates were able to formulate the intended inequalities (constraints) and objective function. First, they identified the decision variables by letting x and y be the number of black and white shirts, respectively. Next, they obtained the objective function and its constraints by maximizing $f(x,y)=x+y$ subject to $x\leq 5$, $4x+5y\leq 30$, $x\geq 0$ and $y\geq 0$. Finally, they solved the problem graphically to determine the corner points and substituted them in the objective function. The candidates correctly concluded that, 5 black and 2 white shirts could be bought and the greatest number of shirts was $5+2=7$. Extract 14.2 shows a sample of a response by a candidate who answered this question correctly.

14.	(a) $f(x) = \dfrac{1}{x-2}$
	(i) · For Domain.
	⟶ 1st : Make y subject
	$y = \dfrac{1}{x-2}$
	⟶ Let; $x - 2 \neq 0$
	$\underline{x \neq 2}$
	$\underline{\text{Domain} = \{x : x \neq 2\}}$
	For Range
	⟶ 1st : Make x subject
	$y = \dfrac{1}{x-2}$
	$(x-2)y = 1$
	$xy - 2y = 1$

14. (a)(i). $xy - 2y = 1$

$$xy = 1 + 2y$$

$$\frac{xy}{y} = \frac{1+2y}{y}$$

$$x = \frac{1+2y}{y}$$

$\Longrightarrow$ Let; $y \neq 0$

Range $= \{y : y \neq 0\}$.

$\therefore$ Domain $= \{x : x \neq 2\}$ and Range $= \{y : y \neq 0\}$

(ii): $f^{-1}(1/3) = ?$

$\Longrightarrow$ 1st : find $f^{-1}(x)$

$\Longrightarrow$ $f(x) = \dfrac{1}{x-2}$

$$y = \frac{1}{x-2}$$

1st : Interchange variables.

$$x = \frac{1}{y-2}$$

2nd : Make y the subject.

$$x(y-2) = 1$$

$$xy - 2x = 1$$

$$xy = 1 + 2x$$

$$\frac{xy}{x} = \frac{1+2x}{x}$$

$$y = \frac{1+2x}{x}$$

$$f^{-1}(x) = \frac{1+2x}{x}$$

But;

$\longrightarrow$ $f^{-1}(1/3) = ?$

14. (a) (ii). $f^{-1}(1/3) = \frac{1+2x}{x}, \quad x = 1/3$

$$= \frac{1 + 2(1/3)}{1/3}$$

$$= \frac{1 + 2/3}{1/3}$$

$$= 5/3 \div 1/1$$

$$= 5/3 \times 3/1$$

$$= 5$$

$\therefore f^{-1}(1/3) = 5$:

14. (b): Let; x be the number of black shirts to be bought

; y be the number of white t-shirts to be bought

⟶ The constraints are :-

- $x \leq 5$ ——— (i):
- $24000x + 30,000y \leq 180,000$

$24x + 30y \leq 180$

$4x + 5y \leq 30$ ——— (ii):

Avoid negative

$x \geq 0$ and $y \geq 0$:

⟶ The Objective function is :-

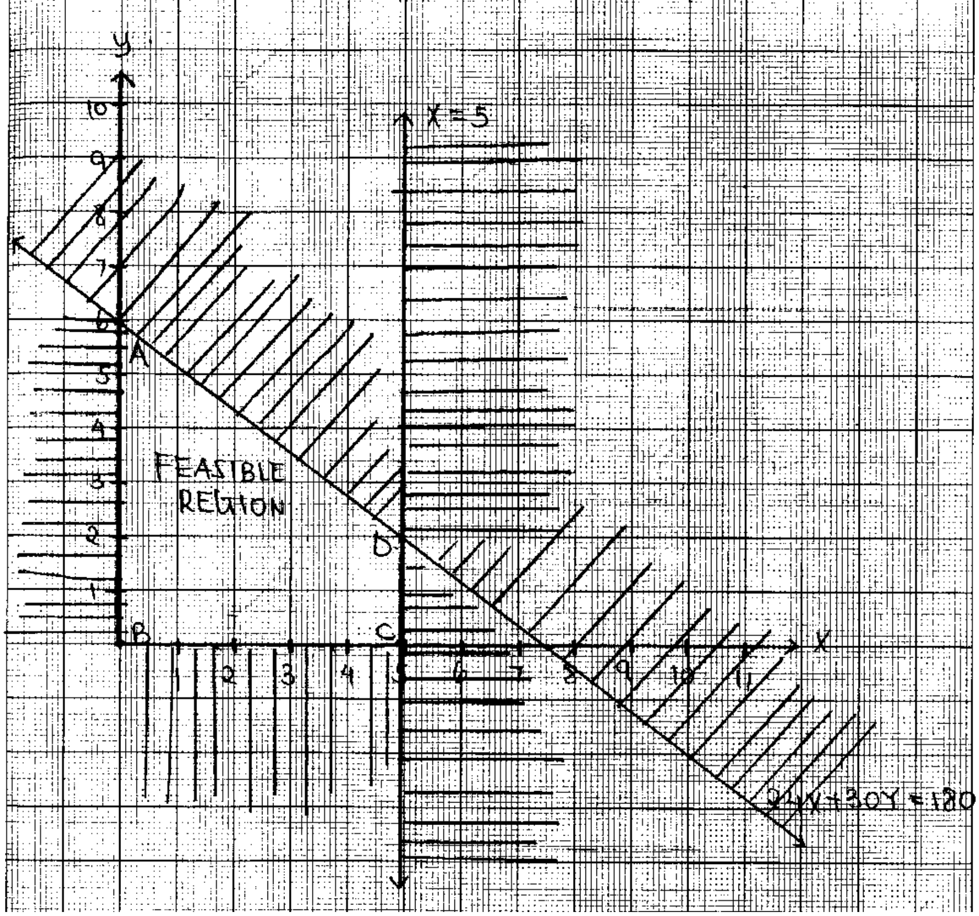
$f(x,y) = x + y$:

⟶ By table of values..

- $x = 5$
- $4x + 5y = 30$

x	0.	7.5
y	6	0.

14 b)



14. ~~(ii)~~ TABLE OF RESULTS SUMMARY.

Corner points	$f(x,y) = x+y$	Value.
A (0, 0)	$= 0 + 0$ $= 0$	0
B (0, 6)	$= 0 + 6$ $= 6$	6
C (5, 2)	$= 5 + 2$ $= 7$	7
D (5, 0):	$= 5 + 0$ $= 5$	5

$\Longrightarrow$ Optimal point = (5, 2)

~~(i)~~ (i):

$\therefore$ 5 black shirts and 2 white shirts should be bought so as to have the maximum number of 7 shirts.

(ii) $\therefore$ The greatest number of shirts to be bought is 7 shirts in order to obtain maximum number of shirts.

Extract 14.2: A sample of the candidate's correct responses to question 14

Extract 14.2 shows that, the candidate stated the domain and range correctly in part (a). The candidate made proper mathematical manipulation to find the inverse of the function $f(x)$ and evaluated $f^{-1}\left(\frac{1}{3}\right)$ correctly. In part (b), the candidate managed to formulate the objective function and the constraints and finally solved them graphically.

3.0 ANALYSIS OF CANDIDATES' PERFORMANCE ON EACH TOPIC

The data analysis reveals that, out of 26 topics which were examined in Basic Mathematics in CSEE 2022, only two (2) topics on *Units* and *Rates and variations* had an average performance of 32.7 per cent. The rest of the topics had weak performance. Those topics include: *Ratios, Profit and loss* and *Accounts (26.9%), Exponents and radicals* and *Logarithms (26.1%), Numbers, Fractions, Decimals and Percentages* and *Logarithms (22.4%), The Earth as a sphere* and *Three dimensional figures (20.9%), Vectors* and *Coordinate geometry (16.2%), Sets* and *Probability (12.6%), Similarity* and *Perimeters and areas (12.4%), Sequences and series (11.8%), Matrices and transformations (10.7%), Circles (10.1%), Functions* and *Linear programming (9.6%), Trigonometry* and *Pythagoras' theorem (8.0%)* and *Quadratic equations (4.2%)*. The summary of the candidates' performance per topic is indicated in the Appendix attached with this report.

4.0 CONCLUSION

The Basic Mathematics paper in CSEE 2022 had 14 questions and among them, only one (01) question had an average performance of 32.7 per cent. The question was set from the topics on *Units* and *Rates and variations*. Further analysis indicated that, the remaining 13 questions had weak performance ranging from 26.1 to 4.2 per cent. Those questions were set from the following topics: *Ratios, Profit and loss; Accounts; Exponents and radicals; Logarithms; Numbers; Fractions; Decimals and Percentages; Logarithms; The Earth as a sphere; Three dimensional figures; Vectors; Coordinate geometry; Sets; Probability; Similarity; Perimeter and areas; Sequences and series; Matrices and transformations; Circles; Functions; Linear programming; Trigonometry; Pythagoras' theorem and Quadratic equations.*

The response analysis reveals that, the main reasons that contributed to weak performance were the candidates' inability to recall and apply the correct formulae, rules, theorems, properties and postulates, formulate mathematical inequalities, expressions and equations from word problems as well as applying appropriate procedures when performing calculations. They also failed to present relevant sketches, figures, graphs and diagrams.

5.0 RECOMMENDATIONS

In order to improve the candidates' performance in Basic Mathematics examinations in future, the Council would like to make the following recommendations:

(a) In the topic on Quadratic equations, teachers should guide students in groups through applying the substitution of variable in order to formulate quadratic equation and solve it using various methods.

(b) Teachers should demonstrate on how to use sine, cosine and tangent of angles in solving problems about angles of elevation and depression. They should also demonstrate on how to calculate the heights and angles of real structures by employing knowledge of Trigonometry and Pythagoras' theorem.

(c) Teachers should use teaching and learning resources such as square cuttings, flat objects in shape of right-angled triangle, square root tables in guiding students on how to apply the Pythagoras theorem in solving related problems.

(d) Teachers should use teaching and learning resources available in their surroundings when explaining all the necessary procedures to lead students discussion on formulating the inequalities/equations and objective functions, drawing graphs and calculating maximum and minimum values using real examples related to linear programming.

(e) In the topic on Circles, teachers should demonstrate to students using diagrams, charts and circular objects to explain the angle and chord properties of a circle as well as applying theorems to prove and solve problems related to circle.

(f) Teachers should use participatory method and creativity techniques in teaching students on how to multiply matrices and show all the necessary steps in determining the application of matrices such as reflection, rotations, translations and enlargements in the topic on Matrices and transformations.

(g) Teachers should use techniques of planning cities and agricultural activities like planting seedlings to understand the concepts of sequences and series as well as using savings in bank to teach students about compound interest in Sequence and series.

(h) In the topic on Perimeters and areas, teachers should identify and describe geometrical figures and their properties and apply them in solving problems such as areas and perimeters, and determining the relations of two similar polygons.

(i) Teachers should guide students through discussion on how to apply knowledge of sets and probabilities in predicting, combining and making decision in real life situations.

(j) Teachers should demonstrate to students on how to apply knowledge of Vectors in finding distance, direction, size and location of objects and other properties.

(k) Teachers should demonstrate to students in small groups on how to locate a pair of points on xy-planes, read coordinate points and find slopes and equations of parallel and perpendicular lines. They should also show how to apply Coordinate geometry in real life situation like land survey, construction of buildings, including locations and distance of points.

(l) In teaching and learning Three dimensional figures, teachers should use flat figures, hall shapes in teaching three-dimensional objects and simple techniques for finding angles, side lengths and diagonals, angle formed between two planes, angle formed between a line segment and a plane, and procedures for finding areas and volumes of hall shapes including prism.

(m) In the topic on The Earth as a sphere, teachers should use variety of tools such as globe, orange, water melon, ball, atlas and graphs to guide students through using the latitudes and longitudes between two places on the earth's surface and find distance between two places.

(n) Teachers should use participatory method to teach students on how to analyze factors and multiples of numbers, identify order of numbers (ascending and descending) by using various methods like percentage, decimals and fractions in the topics on Numbers, Fractions, Decimals and Percentages.

(o) Teachers should use various teaching and learning resources like charts, number cards and multiplication tables to familiarize students with the laws and concepts on Exponents, Radicals and Logarithms. They should also lead students in groups to write numbers in standard notation and approximating numerals to a given number of significant figures and decimal places.

(p) In the topics on Ratios, profit and loss, and Accounts, teachers should conduct group discussions on the meaning, difference and applications of profit, loss, percentage profit, percentage loss as well as trial balance in relation to daily business activities in the society.

APPENDIX: Analysis of Candidates' Performance per Topic – CSEE 2022

S/N	Topics	Question Number	Percentage of Candidates who Passed	Remarks
1.	Units; Rates and variations	6	32.7	Average
2.	Ratios, Profit and loss; and Accounts	7	26.9	Weak
3.	Exponents and radicals; and Logarithms	2	26.1	Weak
4.	Numbers; Fractions; Decimals and Percentages; and Logarithms	1	22.4	Weak
5.	The Earth as a sphere; and Three dimensional figures	12	20.9	Weak
6.	Vectors; and Coordinate geometry	4	16.2	Weak
7.	Sets; and Probability	3	12.6	Weak
8.	Similarity; and Perimeters and areas	5	12.4	Weak
9.	Sequences and series	8	11.8	Weak
10.	Matrices and transformations	13	10.7	Weak
11.	Circles	11	10.1	Weak
12.	Functions; and Linear programming	14	9.6	Weak
13.	Trigonometry; and Pythagoras' theorem	9	8.0	Weak
14.	Quadratic equations	10	4.2	Weak