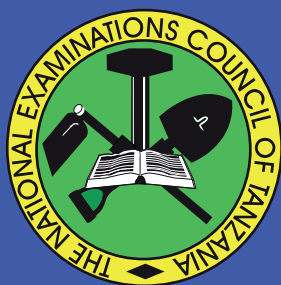


THE NATIONAL EXAMINATIONS COUNCIL OF TANZANIA



**CANDIDATES' ITEM RESPONSE ANALYSIS REPORT FOR
DIPLOMA IN SECONDARY EDUCATION EXAMINATION
(DSEE) 2019**

740 MATHEMATICS

THE NATIONAL EXAMINATIONS COUNCIL OF TANZANIA



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(DSEE) 2019**

740 MATHEMATICS

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TABLE OF CONTENTS

FOREWORD	iv
1.0 INTRODUCTION.....	1
2.0 ANALYSIS OF CANDIDATES' RESPONSES ON EACH QUESTION ...	2
2.1 Section A: Short Answer Questions.....	2
2.1.1 Question 1: Foundations of Mathematics	2
2.1.2 Question 2: Foundations of Mathematics	4
2.1.3 Question 3: Logic.....	6
2.1.4 Question 4: Similarity and Congruence	7
2.1.5 Question 5: Analysis of Mathematics Curriculum Materials	9
2.1.6 Question 6: Probability	10
2.1.7 Question 7: Coordinate Geometry II.....	12
2.1.8 Question 8: Coordinate Geometry II.....	14
2.1.9 Question 9: Hyperbolic Functions	17
2.1.10 Question 10: Vectors	19
2.2 Section B: Essay Answer Questions (Academic)	22
2.2.1 Question 11: Algebra	22
2.2.2 Question 12: Logic.....	25
2.2.3 Question 13: Similarity and Congruence	28
2.3 Section C: Essay Answer Questions (Pedagogy).....	31
2.3.1 Question 14: Assessment in Mathematics	31
2.3.2 Question 15: Planning and Preparation for Teaching Mathematics	35
2.3.3 Question 16: Assessment in Mathematics	36
3.0 THE ANALYSIS OF CANDIDATES PERFORMANCE PER TOPIC	39
4.0 CONCLUSION	39
5.0 RECOMMENDATIONS	40
APPENDIX.....	41

FOREWORD

The report on the Candidates' Item Responses Analysis for Diploma in Secondary Education Examination in Mathematics was prepared for the purpose of providing feedback to candidates, tutors, policy makers, curriculum developers and other education stakeholders on how the candidates answered the examination questions.

The analysis of responses shows that some of the reasons that made the candidate fail to answer the examination questions correctly include; making errors while performing mathematical operations, failure to recall some basic formulae, lack of knowledge and skills on various concepts which were examined, applying incorrect formulae and failure to identify the requirements of the questions.

The National Examinations Council of Tanzania believes that this analysis will provide feedback to various education stakeholders in order to take appropriate measures in improving mathematics teaching techniques as well as creating some more strategies in teaching and learning mathematics subject.

Finally, the Council would like to extend sincere appreciation to the Examination Officers and all others who participated in the preparation of this report.



Dkt. Charles E. Msonde
EXECUTIVE SECRETARY

1.0 INTRODUCTION

The Diploma in Secondary Education Examinations is done every year with the aim of assessing knowledge and skills that have been achieved throughout the two years of implementing the Diploma in education curriculum. A total of 420 candidates were registered in the 2019 DSEE in Mathematics subject of which 417 (99.3%) candidates sat for the Examination.

The paper had a total of sixteen (16) questions divided into three sections namely A, B and C. Section A consisted of 10 short answer questions from both academic and pedagogic contents. Candidates were required to answer all questions in this section. Each correct answer had 4 marks, making a total of 40 marks. Section B consisted of 3 essay questions from academic subject matter. Candidates were required to answer 2 questions where each question had 15 marks, making a total of 30 marks. Section C consisted of 3 essay questions from pedagogy subject matter. Candidates were required to answer 2 questions; the total marks for each question were 15, so the section had a total of 30 marks. Candidates' responses for every question were analysed in order to identify the reasons for either better performance or failure. The analysis involved worked out scripts for each question. Extracts for every question were placed as a sample to the general performance of the question.

The analysis involved all questions that were in the examination paper. It based on the percentage of candidates who performed well the assessment in every question. The analysis on the performance for each question in section A had three categories of marks as follows: 3 - 4 marks; indicating good performance, 2 - 2.5 marks; indicating average performance and 0 - 1.5 marks; indicating poor performance. In sections, B and C, the performance analysis for each question was also categorised into three groups of marks as follows: 10.5 - 15 marks; indicating good performance, 6 - 10 marks; indicating average performance and 0 - 5.5 marks; indicating poor performance.

2.0 ANALYSIS OF CANDIDATES' RESPONSES ON EACH QUESTION

The analysis of candidates' responses for each of the sixteen questions in mathematics was observed as follows:

2.1 Section A: Short Answer Questions

2.1.1 Question 1: Foundations of Mathematics

In this question, the candidates were required to identify four great Mathematicians in mathematics history and explain briefly the contribution of each one.

This question examined candidates' ability to remember the contributions of greater mathematicians on developments of various properties and ideals in mathematics subject.

A total of 352 candidates (83.8%) attempted this question. The analysis of responses in this question shows that 225 candidates (63.9%) scored from 0 to 1.5 marks, 79 candidates (22.4%) scored from 2 to 2.5 marks while 48 candidates (13.6%) scored from 3 to 4 marks. The analysis shows that out of 225 candidates who scored from 0 to 1.5 marks, 68 candidates (19.3%) got a 0 mark. Therefore the general performance for this question was poor because the majority of the candidates (63.9%) scored from 0 to 1.5 marks as indicated in figure 1.

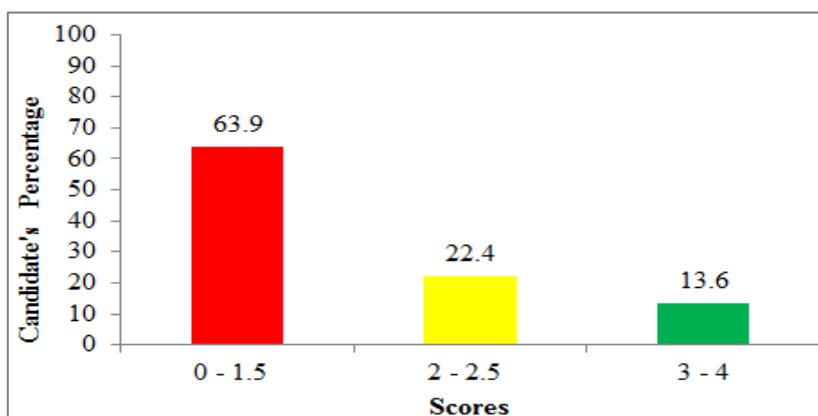


Figure 1: The performance of candidates in question 1.

Extract 1.1 shows the sample of answers from a candidate who failed to remember any of the contributions of the great mathematicians. And therefore he/she presented imaginary answers for each of the mathematician mentioned.

1.	o plato contributes and state children should
	be given conducive learning environment and he
	was greek.
	(2) Pascal said learner should learn by
	playing and he was French
	(3) Galileo contribute to the learning environment
	(4) Zoltan Dienes view learning is playing

Extract 1.1: Sample of incorrect candidate's response in question 1.

This implies that the candidates who failed in this question were not aware of the great mathematicians and their contribution on the subject. They focused on the contribution of education in general instead of mathematics subject.

On the contrary, a total of 48 candidates (13.6%) remembered the contributions of the great mathematicians and presented the correct responses as shown in extract 1.2.

1] Four great Mathematicians in Mathematics history and briefly explain the contribution on each.

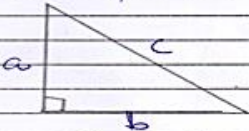
a. Pascal triangle.
 is the Mathematician, whose by the way to provide the expansion of function in form of triangle example.

```

      1
     1 1
    1 2 1
   1 3 3 1
  1 4 6 4 1
  
```

The structure was look like the triangle structure.

b. Pythagoras.
 is the Mathematician who proved the formulae of $a^2 + b^2 = c^2$ in the right angle triangle with 90° .
 Example.



where by $a = \text{opposite}$
 $b = \text{Adjacent}$
 $c = \text{hypotenuse}$
 $a^2 + b^2 = c^2$.

c. Machaulation
 is the Mathematician who discuss about the expansion of function (fth).
 is the theory for the expansion of the function.

d. Zoltan Dienes.
 is the Mathematician who concerned about the learning of Mathematics, it by doing practically solving.

Extract 1.2: Sample of correct candidate's response given in question 1.

2.1.2 Question 2: Foundations of Mathematics

In this question, the candidates were required to explain briefly Zoltan Dienes view in learning Mathematics.

This question assessed candidates' knowledge about the view of the mathematician Zoltan Dienes and his contributions in mathematics field. A total of 272 candidates (64.8%) attempted the question. The analysis of data for this question shows that 163 candidates (59.9%) scored from 0 to 1.5 marks, 32 candidates (11.8%) scored from 2 to 2.5 marks and 77 candidates (28.3%) scored from 3 to 4 marks. Out of 77 candidates who scored from 3 to 4 marks, 43 candidates (10.2%) scored full marks that were allocated to this question. The general performance for this question was poor because those who performed poorly were 59.9% compared to those who scored

from 2 to 4 marks who were only 109 (40.1%). Figure 2 shows the performance of the candidates in this question.

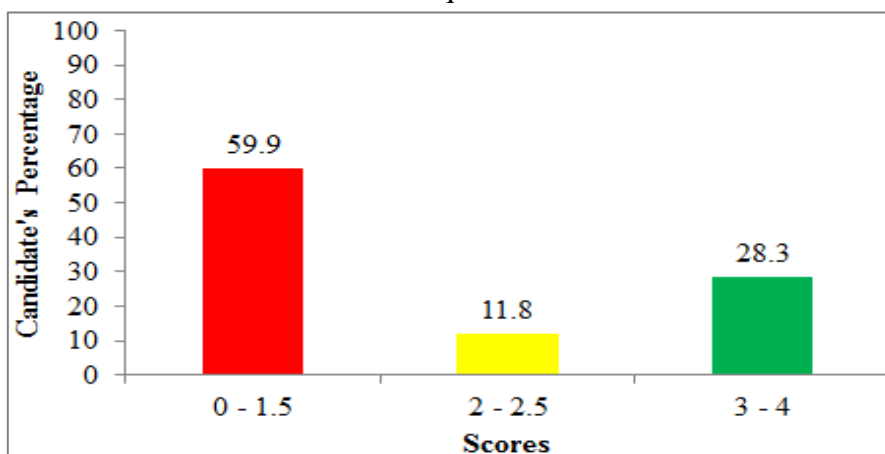


Figure 2: The performance of candidates in question 2.

There were candidates who managed to score full marks as can be seen in extract 2.1 below which is the sample of a candidate's response who managed to remember the Zoltan Dienes view in learning Mathematics.

2.	Zoltan Dienes view in learning mathematics
i/	Emphasize learning through practises
ii/	Emphasize critical thinking
iii/	Brings the rules and theories of learning mathematics.

Extract 2.1: Sample of a correct candidate's response in question 2.

However, 163 candidates (59.9%) of those who attempted this question failed to get it correctly. The reasons for failure were inability to remember and write the Zoltan Dienes view in learning Mathematics. For instance some of the candidates wrote about capacity, experience and age. Extract 2.2 is a sample of a candidate's response who failed to recall the view of the mathematician Zoltan Dienes.

2.	Zoltan Dienes he said the mathematics
	all about thinking capacity, age and
	experience.

Extract 2.2: Sample of incorrect response of a candidate in question 2.

2.1.3 Question 3: Logic

The candidates were required to write the Converse, Inverse and Contrapositive of the statement “If I am under nine years old, then I will go to school”.

This question examined candidates’ ability to apply logic for solving problems in their real life. The question was attempted by 417 (99.3%) out of 420 registered candidates. A total number of 146 candidates (35.0%) answered this question correctly; displayed a high level of interpretation of the given statements and presented the correct answers as shown in extract 3.1.

3	$P =$ I am Under nine years old $Q =$ I will go to School
	$P \rightarrow Q$
(i)	<u>Converse</u>
	$Q \rightarrow P$ I will go to school If I am Under nine years old
(ii)	<u>Inverse</u>
	$\sim P \rightarrow \sim Q$ I am not Under nine years old, I won't go to School.
(iii)	<u>Contrapositive</u>
	$\sim Q \rightarrow \sim P$ I won't go to School If I am not Under year old.

Extract 3.1: Sample of candidate’s correct response given in question 3.

Despite the fact that the question reflected a simple logical application of simple compound statement, 82 (19.7%) of the candidates who attempted

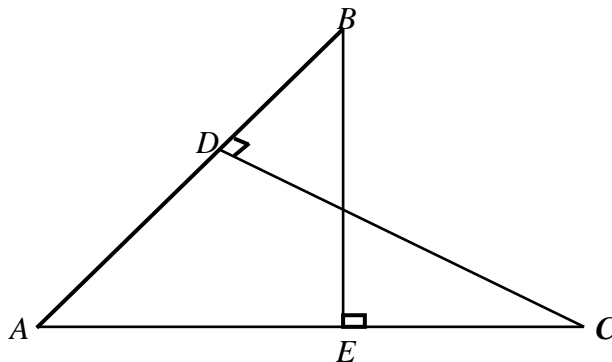
the question, failed to answer it correctly. The failure was due to lack of basic skills to interpret and use correctly the symbols used to represent converse, inverse and contrapositive of a statement. They wrote compound statements which had different meaning in logical statements. Extract 3.2 is the sample of the responses of a candidate who failed to make the correct interpretation that could lead him/her to attain the correct statements.

3.	<u>Soln:</u>
	let P be If I am Under nine years old
	q be I will go to school
	and , = \vee
	connection will be $P \vee q$
	$P \vee q \rightarrow$ Statement
	$q \vee P \rightarrow$ converse.
	$\sim q, \vee \sim P \rightarrow$ Contrapositive
	$\sim P \vee \sim q \rightarrow$ Inverse.

Extract 3.2: Sample of the incorrect candidate's response in question 3.

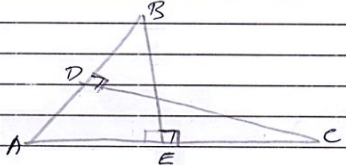
2.1.4 Question 4: Similarity and Congruence

The candidates were required to prove that triangle ABE and ACD are congruent by considering that, in the following figure, $\overline{AB} = \overline{AC}$, D and E are midpoints of \overline{AB} and \overline{AC} respectively.



This question aimed at measuring candidates' ability to identify the given figure and apply the similarity and congruence properties of figures to prove the congruence of triangles ABE and ACD .

A total of 389 candidates (92.6%) attempted this question. The performance of candidates for this question was good as 165 candidates (42.4%) scored from 3 to 4 marks and 98 candidates (25.2%) scored from 2 to 2.5 marks. Those candidates who responded to this question correctly were able to identify the similar triangles and the sides which were equal as well as applying their properties to conclude. See extract 4.1 which is a sample of the response of candidates who managed to answer this question correctly.

4.	
	<p>Since $AB = AC$ then $\triangle ABE$ and $\triangle ACD$ have equal hypotenuses.</p> <p>also, $\triangle ABE$ and $\triangle ACD$ both are right angled triangles</p> <p>if $AB = AC$ then $EB = DC$</p> <p>Therefore $\triangle ABE$ and $\triangle ACD$ are equal due to same hypotenuses, side and right angle (RHS)</p>

Extract 4.1: Illustrates a sample of correct candidate's response in question 4.

On the other hand, a total of 126 candidates (32.4%) failed to respond correctly to this question. The reasons for failure were due to lack of knowledge of identifying equal sides, equal triangles as well as failing to apply the properties to prove congruence of triangles. Extract 4.2 shows the situation where one of the candidates attempted the questions using incorrect procedures.

Qn 4	Consider $\triangle ABE$ and $\triangle ACD$.
	For a triangle to become congruency it must have same shape and size.
	Therefore the line or midpoints E and D

Extract 4.2: Sample of an incorrect response of the candidate in question 4.

2.1.5 Question 5: Analysis of Mathematics Curriculum Materials

The candidates were required to mention four curriculum materials applied in teaching and learning of mathematics subject.

This question assessed candidates' knowledge about curriculum materials relevant for teaching and learning mathematics. It required the candidates to remember and write the taught curriculum materials.

A total of 351 candidates (84.6%) of those who attempted this question responded it correctly; as a result the performance for this question was good. The candidates who performed well in this question were able to remember and write the curriculum materials required as indicated in extract 5.1.

5	Curriculum material applied in teaching and learning mathematics
	i) Reference book
	ii) Text book
	iii) Syllabus
	iv) Teacher guide

Extract 5.1: Illustrates a sample of candidate's correct response in question 5.

However, a total of 21 candidates (5.1%) failed to give correct answers to this question due to lack of knowledge about teaching and learning materials in mathematics. Some of them wrote about IT materials instead of writing the taught learning materials applied in mathematics as extract 5.2 reveals this analysis.

5	(i) Computer games
	(ii) Audio-visual Videos
	(iii) Projector
	(iv) Youtube Videos

Extract 5.2: Sample of the incorrect candidate's response in question 5.

2.1.6 Question 6: Probability

The candidates were required to calculate the probability that number 5 has appeared at least once as a die is thrown twice and the sum of the number appearing is observed to be 8.

The question assessed candidates' ability to apply the formulae $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$ where A is the event of getting number 5 appearing at least once and B is the event of getting the sum of numbers appeared equal to 8.

A total of 364 candidates (86.7%) attempted this question. The analysis of candidates' responses shows that 248 candidates (68.1%) scored from 0 to 1.5 marks, 55 candidates (15.1%) scored from 2 to 2.5 marks and 61 candidates (16.8%) scored from 3 to 4 marks. Among the 61 candidates who scored from 3 to 4 marks, 53 candidates (12.6%) scored full marks that were allocated to this question.

The general performance of candidates in this question was poor because those who scored from 0 to 1.5 marks were 68.1% compared to 31.9% of the candidates who scored between 2 to 4 marks as shown in figure 3.

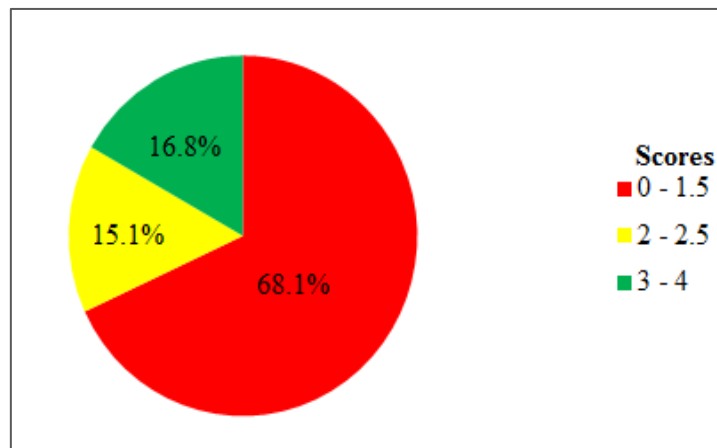


Figure 3: The performance of candidates in question 6.

Some of the candidates who responded wrongly to this question were using combination formula which was not needed in this case. Extract 6.1 shows an example of a response of the candidate who used incorrect formula.

6.	number of die is 6. the die is twice
	$6 \times 2 = 12$. total number appearing is
	observed to be 8. 5 has appeared at least
	once.
	Combination.
	${}^nC_r = \frac{n!}{(n-r)!r!}$
	$= \frac{8!}{(8-5)!5!}$
	$= \frac{40,320}{720}$
	$= 56.$
	The probability that number 5 has appeared
	at least once is 56.

Extract 6.1: Illustrates the incorrect candidate's response to question 6.

Some candidates who failed to respond correctly to this question used wrong formulae by using the combination formulae ${}^nC_r = \frac{n!}{r!(n-r)!}$ and replaced n by 8 and r by 5 which lead them to an incorrect answer because of the wrong procedure.

However, 61 candidates (16.8%) managed to get the correct answer. Some of the candidates used the method of listing the events to get the required probability as in extract 6.2.

Given that Die is thrown twice							
	1	2	3	4	5	6	
2	2,1	2,2	2,3	2,4	2,5	2,6	
3	3,1	3,2	3,3	3,4	3,5	3,6	
4	4,1	4,2	4,3	4,4	4,5	4,6	
5	5,1	5,2	5,3	5,4	5,5	5,6	
6	6,1	6,2	6,3	6,4	6,5	6,6	
The sum of appearing which is equal to 8 are							
2,6 .. 3,5 4,4 5,3 6,2							
$n(S) = 5$							
$n(E) =$ No 5 to appear atleast one							
3,5 and 5,3							
$n(E) = 2$							
$p(E) = \frac{n(E)}{n(S)}$							
$p(E) = \frac{2}{5} \approx 0.4$							
\therefore Probability of number five to appear atleast one = $\frac{2}{5}$ or 0.4							

Extract 6.2: Sample of a correct response of a candidate in question 6.

2.1.7 Question 7: Coordinate Geometry II

The candidates were required to find the equation of the tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at point $(a \cos \theta, b \sin \theta)$.

This question was intended to examine candidates' ability to apply chain rule in establishing the equation $\frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta} = \frac{-b \cos \theta}{a \sin \theta}$ and use it to get the required equation.

A total of 383 candidates (91.2%) attempted this question. The analysis shows that 232 candidates (60.6%) scored from 0 to 1.5 marks, 36 candidates (9.4%) scored from 2 to 2.5 marks and 115 candidates (30.0%) scored from 3 to 4 marks. Out of 115 candidates who scored from 3 to 4 marks, 101 candidates (24.0%) scored full marks which were allocated to this question.

The general performance of candidates to this question was poor because 232 candidates (60.6%) scored from 0 to 1.5 marks compared to those who

scored from 2 to 4 marks who were only 39.4% of those who attempted the question. The candidates who failed to solve this question assigned the given coordinates with the letters x and y and simplify instead of applying the differentiation methods with the chain rule as shown in extract 7.1.

7.	$a \cos \theta = x, b \sin \theta = y$
	$= \frac{(a \cos \theta)^2}{a^2} + \frac{(b \sin \theta)^2}{b^2} = 1$
	$= \frac{a^2 \cos^2 \theta}{a^2} + \frac{b^2 \sin^2 \theta}{b^2} = 1$
	$\cos^2 \theta + \sin^2 \theta = 1$
	$\therefore \text{The equation is } \cos^2 \theta + \sin^2 \theta = 1$

Extract 7.1: Sample of an incorrect response of a candidate.

On the other hand 115 candidates (30%) of those who attempted this question got it correct. This is due to the fact that they correctly used the method of differentiation and chain rule to find the equation as shown in extract 7.2.

7.	$\frac{dy}{dx} = \frac{-2xb^2}{2ya^2}$
	$\frac{dy}{dx} = \frac{-xb^2}{ya^2}$
	also given point = $(a\cos\theta, b\sin\theta)$ (x, y)
	gradient = $\frac{y - b\sin\theta}{x - a\cos\theta} = \frac{-xb^2}{ya^2}$
	$(y - b\sin\theta)ya^2 = (x - a\cos\theta) - xb^2$
	$y^2a^2 - ya^2b\sin\theta = -x^2b^2 + xb^2a\cos\theta$
	$y^2a^2 + x^2b^2 = xb^2a\cos\theta + ya^2b\sin\theta$
	Divide by a^2b^2 both side.
	$\frac{y^2a^2}{a^2b^2} + \frac{x^2b^2}{a^2b^2} = \frac{xb^2a\cos\theta}{a^2b^2} + \frac{ya^2b\sin\theta}{a^2b^2}$
	$\frac{y^2}{b^2} + \frac{x^2}{a^2} = \frac{x\cos\theta}{a} + \frac{y\sin\theta}{b}$
	but
	$\frac{b^2}{b^2} + \frac{x^2}{a^2} = 1$
	$\frac{x\cos\theta}{a} + \frac{y\sin\theta}{b} = 1$
	$\therefore \text{Equation} = \frac{x\cos\theta}{a} + \frac{y\sin\theta}{b} = 1$

Extract 7.2: Sample of the correct response of a candidate in question 7.

2.1.8 Question 8: Coordinate Geometry II

The candidates were required to find the coordinates of point P by considering a point P which divides internally the line joining point (4, 2, 2) and (10, 6, 4) in the ratio 1:1.

This question assessed candidates' knowledge of finding the coordinates of a point by using the concept of ratios and internal division of lines. It was intended to evaluate the candidates' ability to remember and use the

formulae $\underline{P} = \left(\frac{n}{n+m} \right) \underline{a} + \left(\frac{m}{n+m} \right) \underline{b}$ in finding the required coordinates.

Where n and m are the given ratio i.e., $n = 1$ and $m = 1$.

The question was attempted by 341 candidates (81.2%). The analysis of data shows that 116 candidates (34.0%) scored from 0 to 1.5 marks, 18 candidates (5.3%) scored from 2 to 2.5 marks and 177 candidates (60.7%) scored from 3 to 4 marks. This means that the performance of the candidates for this question was average because those who scored from 2 to 4 marks were 195 (66.0%). Figure 4 shows the performance of the candidates for this question.

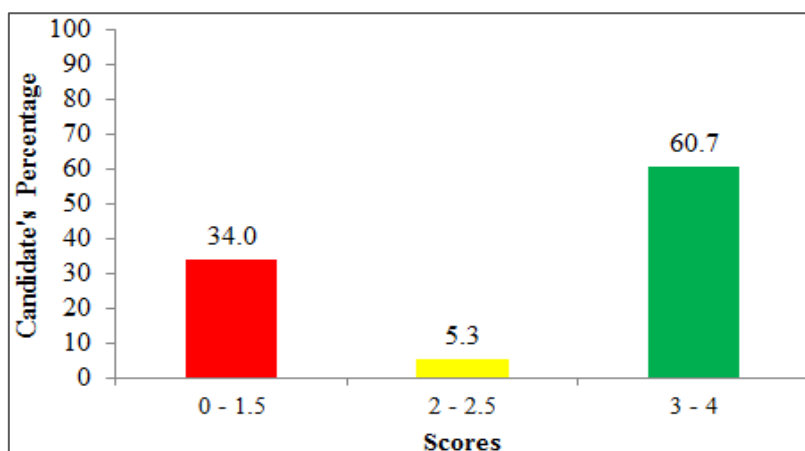


Figure 4: The performance of the candidates in question 8.

The candidates who managed to get the correct answer were able to remember the formulae and apply it correctly to find the coordinates of a point in a line which is $\underline{P} = \left(\frac{n}{n+m} \right) \underline{a} + \left(\frac{m}{n+m} \right) \underline{b}$ and attain the correct answer as indicated in extract 8.1.

Q8 Given that
 points $(4, 2, 2)$ and $(10, 6, 4)$
 such that $P:Q = \lambda:M$
 let $(a_1, b_1, c_1) = (4, 2, 2)$
 $(a_2, b_2, c_2) = (10, 6, 4)$
 Recall from internal division

$$\left(\frac{a_2}{b_2} \right) \lambda + \left(\frac{a_1}{b_1} \right) \lambda$$

$$\frac{\left(\frac{10}{6} \right) \lambda + \left(\frac{4}{2} \right) \lambda}{\lambda + \lambda}$$

$$\frac{\left(\frac{10}{6} \right) + \left(\frac{4}{2} \right)}{2}$$

$$\frac{(10+4) + (6+2) + (4+2)}{2}$$

$$\frac{14 + 8 + 6}{2}$$

$$\frac{28}{2} = 14$$

For the coordinate
 $\left(\frac{a_2 \lambda + a_1 \lambda}{\lambda + \lambda} \right), \left(\frac{b_2 \lambda + b_1 \lambda}{\lambda + \lambda} \right), \left(\frac{c_2 \lambda + c_1 \lambda}{\lambda + \lambda} \right)$
 $\left(\frac{10 \times 1 + 4 \times 1}{1+1} \right), \left(\frac{6 \times 1 + 2 \times 1}{1+1} \right), \left(\frac{4 \times 1 + 2 \times 1}{1+1} \right)$
 $\left(\frac{10+4}{2} \right), \left(\frac{6+2}{2} \right), \left(\frac{4+2}{2} \right)$
 $\frac{14}{2}, \left(\frac{8}{2} \right), \left(\frac{6}{2} \right)$
 $(7, 4, 3)$
 $\therefore \text{coordinate of } P = (7, 4, 3)$

Extract 8.1: A sample of candidate's correct response given in question 8.

However, 116 candidates (34.0%) of those who attempted this question failed to get it correctly due to incapability of remembering and using the relevant formula. Some of them were just adding the given coordinates to get the midpoint coordinates as indicated in extract 8.2 where candidate

failed to recall and apply the formula to get the coordinates required because he/she added the given coordinates, i.e.; $4+10=14$, $2+6=8$ and $2+4=6$ instead of using the formula.

8.	<u>Soln.</u>
	Given points $P(4, 2, 2)$ and $Q(10, 6, 4)$.
	Point $R = (4, 2, 2) + (10, 6, 4)$
	$= 14, 8, 6$
	<u>\therefore Point $R = (14, 8, 6)$</u>

In Extract 8.2: Illustrates an incorrect response of a candidate in question 8.

2.1.9 Question 9: Hyperbolic Functions

The candidates were required to differentiate $\cosh^{-1}(\sqrt{x^2+1})$ with respect to x .

This question evaluated the candidates' ability to differentiate hyperbolic functions by using the substitution method.

A total of 398 candidates (94.8%) attempted this question. The analysis of data shows that 153 candidates (38.4%) scored from 0 to 1.5 marks, 62 candidates (15.6%) scored from 2 to 2.5 marks and 183 candidates (46.0%) scored from 3 to 4 marks. Out of 183 candidates who scored from 3 to 4 marks, 121 candidates (28.8%) scored all marks that were allocated for this question. So the general performance of candidates for this question was average since 61.6% of the candidates who attempted the question scored from 2 to 4 marks. Extract 9.1 illustrate this analysis.

9.	$\frac{dy}{du} = \frac{1}{\sqrt{\cosh^2 u - 1}}$
	$\frac{dy}{du} = \frac{1}{\sqrt{u^2 - 1}}$
	but $u = \sqrt{x^2 + 1}$
	$\frac{dy}{du} = \frac{1}{\sqrt{(\sqrt{x^2 + 1})^2 - 1}}$
	$\frac{dy}{du} = \frac{1}{\sqrt{x^2 + 1 - 1}}$
	$\frac{dy}{du} = \frac{1}{\sqrt{x^2}}$
	$\frac{dy}{du} = \frac{1}{x}$
	so $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$
	$\frac{dy}{dx} = \frac{1}{x} \cdot \frac{x}{\sqrt{x^2 + 1}}$
	$\frac{dy}{dx} = \frac{1}{\sqrt{x^2 + 1}}$
	$\therefore \frac{d}{dx} (\cosh^{-1} \sqrt{x^2 + 1}) = \frac{1}{\sqrt{x^2 + 1}}$

Extract 9.1: Sample of a correct candidate's response given in question 9.

On the other hand, 38.4% of the candidates who attempted this question failed to get it correctly due to inability to make the substitution over the given expression so that it becomes simple before starting to differentiate. They were required to let $u = \cosh^{-1}(\sqrt{x^2 + 1})$ so that $\cosh u = \sqrt{x^2 + 1}$ then proceed with the process of differentiating. Extract 9.2 is a sample of a response from a candidate who failed to differentiate properly the given expression.

9	<u>Solution</u>
	Given that
	$\cosh^{-1}(\sqrt{x^2+1})$
	Let $y = \cosh^{-1}(\sqrt{x^2+1})$
	now
	$\cosh y = \sqrt{x^2+1}$
	Differentiate with respect to x .
	$\sinh y \frac{dy}{dx} = -\frac{1}{2}(x^2+1)^{-\frac{1}{2}} \cdot 2x$
	$\sinh y \frac{dy}{dx} = -x(x^2+1)^{-\frac{1}{2}}$
	$\frac{dy}{dx} = \frac{x^3+x}{\sinh y}$
	now from $-\sinh^2 y + \cosh^2 y = 1$
	$\sinh^2 y = 1 - \cosh^2 y$
	$\sinh y = \sqrt{1 - \cosh^2 y}$
	now $\frac{dy}{dx} = \frac{x^3+x}{\sqrt{1 - \cosh^2 y}}$
	$\frac{dy}{dx} = \frac{x^3+x}{\sqrt{1 - \sqrt{x^2+1}}}$
	$\therefore \frac{dy}{dx} = \frac{x^3+x}{\sqrt{1 - \sqrt{x^2+1}}}$

Extract 9.2: Illustrates the incorrect response of a candidate in question 9.

2.1.10 Question 10: Vectors

The candidates were required to find the angle between $\underline{a} = 2\underline{i} + \underline{j} - 2\underline{k}$ and $\underline{b} = \underline{i} - 2\underline{j} + \underline{k}$ by using the cross product of vectors.

The question examined candidates' skills on how to use the cross product formula which is $|\underline{a} \times \underline{b}| = |\underline{a}||\underline{b}|\sin \theta$ where \underline{a} and \underline{b} are two different vectors, in order to get the angle between the two vectors.

The question was attempted by 409 candidates (97.4%). The analysis of data for this question shows that 167 candidates (40.8%) scored from 0 to

1.5 marks, 48 candidates (11.7%) scored from 2 to 2.5 marks and 194 candidates (47.5%) scored from 3 to 4 marks. Out of 194 candidates who scored from 3 to 4 marks, 164 (39.0%) scored full marks that were allocated for this question. Figure 5 shows the performance of the question in question 10.

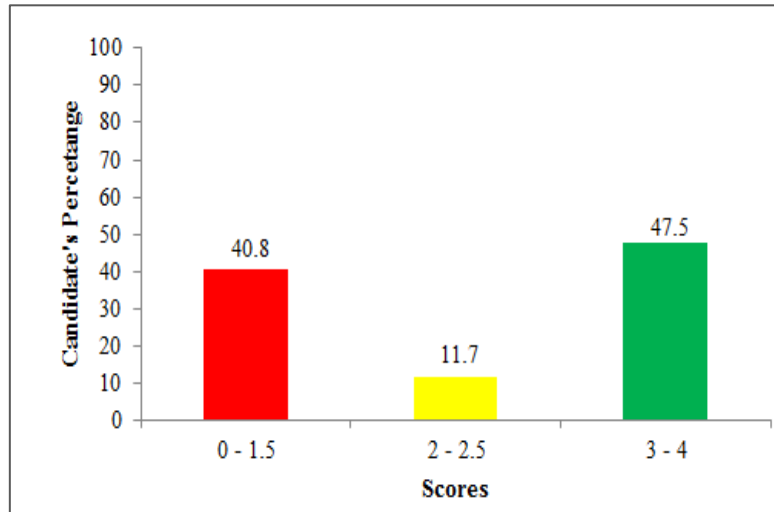


Figure 5: The performance of the candidate question 10.

The candidates who got the correct answer had enough skills on how to use cross product for calculating the angle between two different vectors. They managed to use the formula $|\underline{a} \times \underline{b}| = |\underline{a}| |\underline{b}| \sin \theta$ and proceed by finding $|\underline{a}| = \sqrt{4+1+4} = 3$ and $|\underline{b}| = \sqrt{1+4+1} = \sqrt{6}$ to attain the final answer as shown in extract 10.1.

10 AB /

Cross product = $A \times B$

$|A \times B| = |A||B| \sin \theta$

$a = -2\hat{i} + \hat{j} - 2\hat{k}$

$b = \hat{i} - 2\hat{j} + \hat{k}$

$a \times b = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 1 & -2 \\ 1 & -2 & 1 \end{vmatrix}$

$\hat{i}[(1)(1) - (-2)] - \hat{j}[2(1) + 2] + \hat{k}[2(-2) - (1)(1)]$

$\hat{i}(1-4) - \hat{j}(4) + \hat{k}(-4-1)$

$-3\hat{i} - 4\hat{j} - 5\hat{k}$

$|a \times b| = |-3\hat{i} - 4\hat{j} - 5\hat{k}|$ $|a \times b| = \sqrt{x^2 + y^2 + z^2}$

$|a \times b| = \sqrt{(-3)^2 + (-4)^2 + (-5)^2}$

$\sqrt{9 + 16 + 25}$

$\sqrt{50}$

$|a \times b| = \sqrt{50}$

$|a \times b| = |A||B| \sin \theta$

$|a| = \sqrt{2^2 + 1^2 + (-2)^2}$

$|a| = \sqrt{4 + 1 + 4}$ $|a| = \sqrt{9}$ $|a| = 3$

$|b| = \sqrt{1^2 + (-2)^2 + 1^2} = \sqrt{1 + 4 + 1} = \sqrt{6}$

$\sqrt{50} = 3 \times \sqrt{6} \sin \theta$

$\frac{\sqrt{50}}{3 \times \sqrt{6}} = \sin \theta$

$\frac{7.07}{3 \times 2.449} = \sin \theta$

$\frac{7.07}{7.348} = \sin \theta$

$\sin \theta = 0.9623$

$\theta = 74.2^\circ$

\therefore The Angle b/w is 74.2°

Extract 10.1: Is a sample of candidate's correct response in question 10.

Meanwhile, there were 167 candidates (40.8%) who attempted this question and performed poorly due to inability to remember and improper use of vector cross product techniques so as to get the required angle. Some of them just calculated the magnitude of both vectors and subtract before equating to $\sin \theta$ and find the angle θ resulting into incorrect answer as revealed in extract 10.2.

10 From cosine product formula

$$A \cdot B = |A||B| \cos \theta, \quad a = 2i + j - 2k$$

$$b = i - 2j + k$$

$$\begin{pmatrix} i & j & k \\ 2 & 1 & -2 \\ 1 & -2 & 1 \end{pmatrix}$$

$$2(1 + 4) - 1(2 + 2) + -2(-4 - 1)$$

$$= -6 - 4 + 10$$

$$-6 - 4 + 10 = \sqrt{2^2 + 1^2 + (-2)^2} \times \sqrt{1^2 + (-2)^2 + 1^2} \times \cos \theta$$

$$-6 - 4 + 10 = \sqrt{9} \times \sqrt{6} \cos \theta$$

$$\sqrt{6^2 + 4^2 + 10^2} = \sqrt{56} \cos \theta$$

$$\frac{\sqrt{152}}{\sqrt{56}}$$

$$0 = \sqrt{56} \cos \theta$$

$$\cos \theta = \left(\frac{0}{\sqrt{56}} \right)$$

$$\theta = \cos^{-1} \left(\frac{0}{\sqrt{56}} \right) = 0$$

Therefore the intended angle will be 0°

Extract 10.2: Is a sample of candidate's incorrect response in question 10.

2.2 Section B: Essay Answer Questions (Academic)

2.2.1 Question 11: Algebra

This question had three parts where by the candidates were to:

- (a) Evaluate $\sum_{n=4}^{10} n^2 + 3n$, given that $\sum r^2 = \frac{n}{6}(n+1)(2n+1)$ and

$$\sum r = \frac{n}{2}(n+1).$$

- (b) Show that $ac^3 = db^3$, if the roots of polynomial equation $ax^3 + bx^2 + cx + d = 0$ are in geometric progression.
- (c) Find an equation whose roots are $\alpha^2\beta$ and $\alpha\beta^2$, when the roots of quadratic equation $2x^2 - 7 = 0$ are α and β .

This question had three parts, in part (a) the question assessed candidates' ability to expand the expressions and apply the given information to substitute the values in order to get the required answer; in part (b) the question assessed the candidates' knowledge about roots of polynomials and part (c) assessed the candidates' ability to factorise and apply the factors obtained to form an equation.

The question was attempted by 132 candidates (31.4%) out of 420 registered candidates. Among these candidates; 95 (72.0%) scored from 0 to 5.5 marks, 35 candidates (26.4%) scored from 6 to 10 marks and 2 candidates (1.6%) scored from 10.5 to 15 marks. The analysis indicates that the general performance for this question was poor because the candidates who passed were only 28%. Figure 6 shows the performance of candidates for question 11.

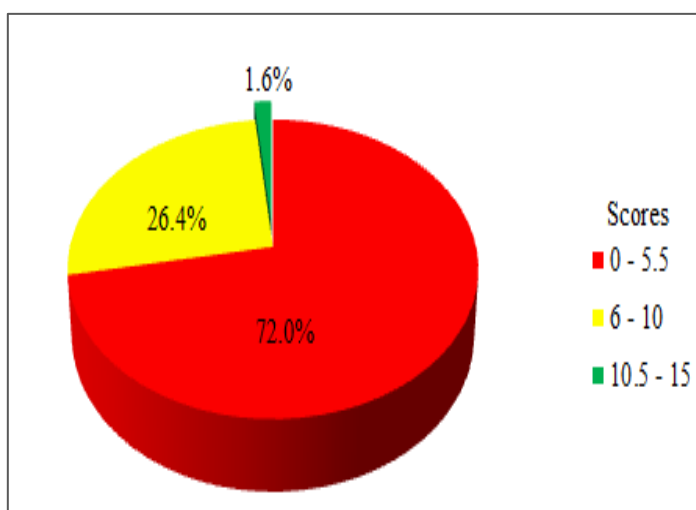


Figure 6: The general performance of candidates in question 11.

The candidates who failed to answer correctly this question lacked knowledge about the application of the given information; some of them substituted the given limits direct into the given expression and obtained incorrect answer as shown in extract 11.1.

11a)	$\sum r^2 = n/6 (n+1)(2n+1)$ and $\sum r = n/2 (n+1)$
	$\sum_{n=4}^{10} n^2 + 3n$
	<u>Solution:</u>
	$\sum_{n=4}^{10} n$
	$\sum_{n=4}^{10} n/6 (n+1)(2n+1)$ where $n = 4, 5, 6, 7, 8, 9, 10$
	$\sum_{n=4}^{10} 4/6 (4+1)(2 \times 4+1) + 5/6 (5+1)(2 \times 5+1) + 6/6 (6+1)(2 \times 6+1)$
	$+ 7/6 (7+1)(2 \times 7+1) + 8/6 (8+1)(2 \times 8+1) + 9/6 (9+1)(2 \times 9+1)$
	$+ 10/6 (10+1)(2 \times 10+1) + 3 \left[\frac{n}{2} (n+1) \right] \left[\frac{1}{2} (4+1) + \right.$
	$\left. \frac{5}{2} (5+1) + \frac{6}{2} (6+1) + \frac{7}{2} (7+1) + \frac{8}{2} (8+1) + \frac{9}{2} (9+1) \right.$
	$\left. + \frac{10}{2} (10+1) \right]$

Extract 11.1: Is the sample of incorrect response of a candidate in part (a).

Despite the failure of many candidates in this question, there were 37 candidates (28.0%) who managed to score from 6 to 13.5 marks. They managed to apply the given formula and able to factorise it as well as interpreting the values before making the substitution to attain the correct answer in part (a) as shown in extract 11.2.

11.	a). Given
	$\sum r^2 = n/6(n+1)(2n+1)$ and
	$\sum r = n/2(n+1)$ but required.
	$\sum_{4}^{10} n^2 + 3n$ where $n = 4, 5, 6, 7, 8, 9$ and 10 .
	$\sum_{4}^{10} n^2 + 3n = (4^2 + 3 \times 4) + (5^2 + 3 \times 5) + (6^2 + 3 \times 6) + (7^2 + 3 \times 7)$
	$+ (8^2 + 3 \times 8) + (9^2 + 3 \times 9) + (10^2 + 3 \times 10)$
	$= 28 + 40 + 54 + 70 + 88 + 108 + 130$.
	$= 518$.
	$\therefore \sum_{4}^{10} n^2 + 3n = 518$.

Extract 11.2: Illustrates the sample of a correct response of a candidate.

2.2.2 Question 12: Logic

In this question, the candidates were required to:

- Draw an electrical network of the simplified form of the compound statement; $\sim [(P \wedge \sim Q) \vee (\sim P \wedge Q) \vee (\sim P \wedge \sim Q)]$.
- Determine the validity of the argument "If it rains, the seedlings will survive. If seedlings survive well, animals will not die. But animals are dying. Therefore it is not raining", by using the laws of algebra of propositions.

This question assessed candidates' knowledge about logical properties and its application in simplification of compound statements. It was attempted by 355 candidates (84.5%) whereby 134 candidates (37.7%) scored from 0 to 5.5 marks, 134 (37.7%) scored from 6 to 10 marks and 87 (24.6%) scored from 10.5 to 15 marks. Out of 87 candidates who scored from 10.5 to 15 marks, 24 (5.7%) scored all marks that were allocated to this question.

The analysis indicates that the general performance of candidates in this question was average because those who scored from 6 to 15 marks were 221 (62.3%). Figure 7 presents the performance of candidates for question 12.

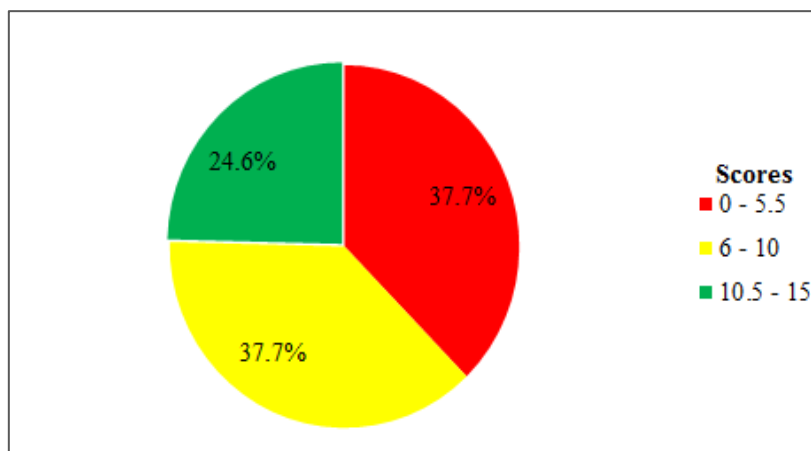
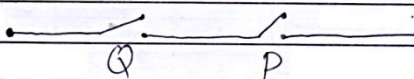


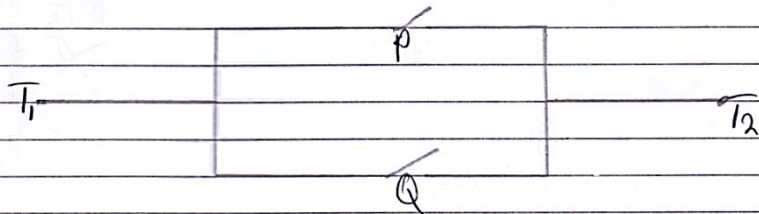
Figure 7: The performance of candidates in question 12.

Some of the candidates had adequate knowledge about formulating the compound statements and applying the laws of algebra of propositions to simplify as indicated in extract 12.1 which is the sample of the answers that were given by one of the candidates.

12a.	$\sim[(P \wedge \sim Q) \vee (\sim P \wedge Q) \vee (\sim P \wedge \sim Q)]$
	Let's simplify by using algebraic laws of propositions
	$\sim[(P \wedge \sim Q) \vee \sim P \wedge (Q \vee \sim Q)]$ - distributive law
	$\sim[(P \wedge \sim Q) \vee \sim P \wedge T]$ - negation law
	$\sim[(P \wedge \sim Q) \vee \sim P]$ - identity law.
	$\sim[(P \vee \sim P) \wedge (\sim Q \vee \sim P)]$ - distributive law
	$\sim[T \wedge (\sim Q \vee \sim P)]$ - negation
	$\sim(\sim Q \vee \sim P)$ - identity law
	$(Q \wedge P)$ - double negation.
	The electric net work will be in series.
	
12b.	let, p be It rains
	q be the seedling survive
	r be animals are dying
	In logic form the argument will be as follows.
	$[(P \rightarrow q) \wedge (q \rightarrow \sim r) \wedge r] \rightarrow \sim p$
	then $[(P \rightarrow \sim r) \wedge r] \rightarrow \sim p$ by definition: $(P \rightarrow Q) \wedge Q \rightarrow P$
	$[\sim(P \vee r) \wedge r] \rightarrow \sim p$ by definition
	$[\sim P \wedge \sim r] \wedge r \rightarrow \sim p$ negation
	$[\sim P \wedge (\sim r \wedge r)] \rightarrow \sim p$ Associative law
	$[\sim P \wedge F] \rightarrow \sim p$ Negation law
	$F \rightarrow \sim p$ Identity law
	$\sim F \vee \sim p$ by definition
	$T \vee \sim p$ Negation law
	T identity law
	Therefore the argument is valid.

Extract 12.1: Illustrates the response of a candidate statement correctly.

On the other hand, 134 candidates (37.7%) failed to get the correct answer to this question due to inability to remember the logical properties like; Distributive law, Negation law, Identity law and Double negation law. The analysis shows that some of them were using the laws which are not used in logic like Absorption law and Demogan's law. Therefore, they failed to simplify, ended up with the wrong statement as shown in extract 12.2.

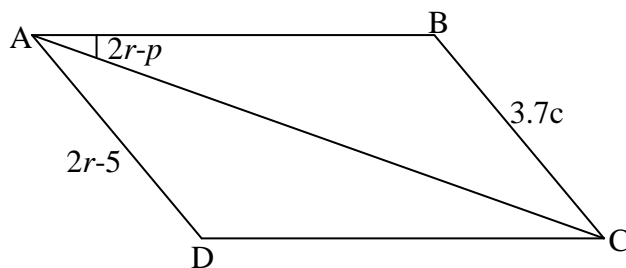
12. a)	Soln.
	Given.
	$\sim [P \wedge \sim Q] \vee (\sim P \wedge Q) \vee (\sim P \wedge \sim Q)$ - Given.
	$= \sim [P \vee \sim P \wedge \sim Q]$
	$\sim [T \vee (\sim P \wedge \sim Q)]$ - Absorption law.
	$\sim (\sim P \wedge \sim Q)$ - Identity law.
	$P \vee Q$ - Demogorgans law.
	\therefore electrical network of $P \vee Q$.
	
b).	Soln.
	Let P = rains
	Q = Survived
	r = animal
	$\therefore (P \rightarrow Q) \wedge (Q \rightarrow r) \rightarrow P \sim P$.
	Truth Table.
	$\neg [(P \rightarrow Q) \wedge (Q \rightarrow r) \wedge r] \rightarrow \sim P$.

Extract 12.2: is the response of a candidate who failed to simplify the statements in question 12(a), (b).

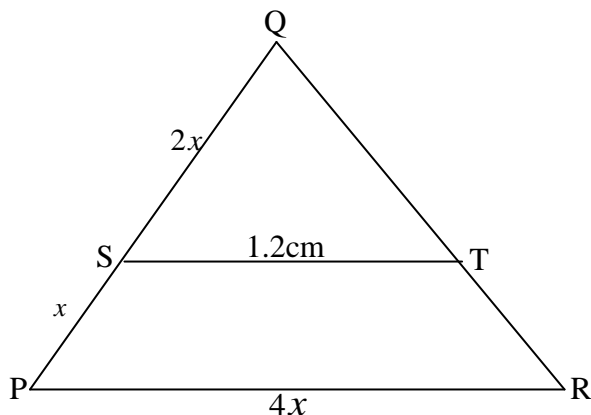
2.2.3 Question 13: Similarity and Congruence

In this question, the candidates were required to:

- (a) Find the value of r and p in the following figure, where $\triangle ABC$ is congruent to $\triangle CDA$.



- (b) Find the value of x in the following figure if $\triangle PQR \sim \triangle SQT$.



This question assessed candidates' ability to identify different figures and relate them in order to solve for values of r and p in part (a) and x in part (b).

A total of 349 candidates (83.1%) attempted this question. The analysis shows that 38 candidates (10.9%) scored from 0 to 5.5 marks, 118 (33.8%) scored from 6 to 10 marks and 193 candidates (55.3%) scored from 10.5 to 15 marks. Figure 8 presents the performance of the candidates in this question.

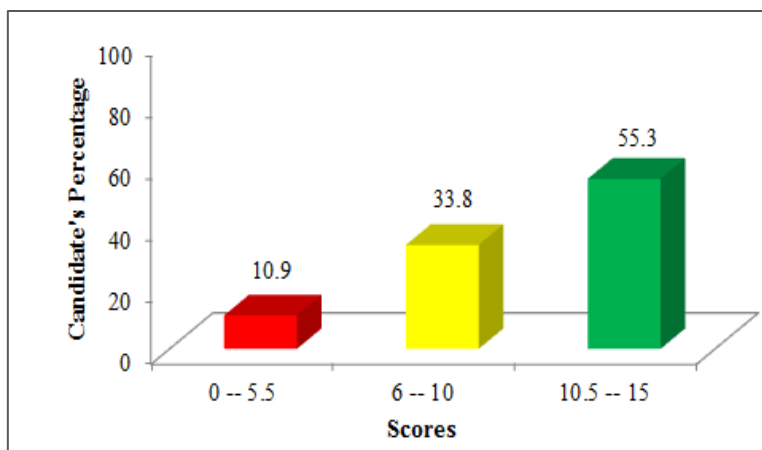


Figure 8: The performance of candidates for question 13.

The candidates who scored 10.5 marks and above, were able to solve for values of r and p in part (a) and determine the value of x in part (b) after detecting the angles which are equal and the corresponding sides. Extract 13.1 is a sample of a candidate's correct responses given in question 13 (a) and (b).

B@Required	r and P.
$\overline{AD} = \overline{BC}$.	
$2r - 5 = 3.7\text{cm}.$	
$2r = 3.7 + 5.$	
$\frac{2r}{2} = \frac{8.7}{2}.$	
$r = 4.35\text{cm}.$	
\therefore The value of $r = 4.35\text{cm}.$	
$2r - P = 23$	(Because of Alternate angles. Alternating angles.)
But $r = 4.35.$	
$8.7 - P = 23.$	
$P = -14.3.$	
\therefore Value of $P = -14.3.$	
b) from.	$\frac{PQ}{QS} = \frac{PR}{ST}.$
$3X = 4X.$	
$2X \quad 1.2\text{cm}.$	
By crossing multiplication	
$3X (1.2\text{cm}) = 4X (2X)$	
$3.6X\text{cm} = 8X^2.$	
$X \quad X.$	
$3.6\text{cm} = 8X.$	
$8 \quad 8.$	
\therefore The value of $X = 0.45\text{cm}.$	

Extract 13.1: Illustrates the correct response of a candidate in question 13.

Among 38 candidates (10.9%) who scored from 0 to 5.5 marks, some of them got incorrect answers to this question because they did not know the corresponding sides and angles in part (a) which are $\overline{AD} = \overline{BC}$ and $\angle BAC = \angle ACD$ also the corresponding sides in part (b) which are $\frac{PQ}{QS} = \frac{PR}{ST}.$

Therefore, they were unable to solve for values for r , p and x as shown in extract 13.2.

13. a/ $\triangle ABC$ is congruent to $\triangle CDA$. Find the value of r and P .

$2r - p = 23^\circ$
 $\therefore r = \frac{23^\circ}{2} = 11.5^\circ$
 \therefore The value of $r = 11.5^\circ$ and $p = 11.5^\circ$

b/ $\triangle PQR \sim \triangle STU$. Find the value of x .
 from $\sin = \frac{\text{opposite}}{\text{Hypotenuse}}$
 where opposite = $8x$
 Hypotenuse = 1.2 cm

Extract 13.2: Is a sample of candidate's incorrect response given in question 13(a) and (b).

2.3 Section C: Essay Answer Questions (Pedagogy)

2.3.1 Question 14: Assessment in Mathematics

The candidates were required to justify the statement that "Mathematics is a mother of various disciplines" by giving five points.

This question examined candidates' ability to express the application of Mathematics in various fields of life. It was attempted by 224 candidates (53.3%). The analysis of data shows that 22 candidates (9.8%) scored from 0 to 5.5 marks, 92 (41.1%) scored from 6 to 10 marks while 110 candidates (49.1%) scored from 10.5 to 15 marks. The general performance in this question was good because 90.2 percent of the candidates scored from 6 to 15 marks. This performance is indicated in figure 9.

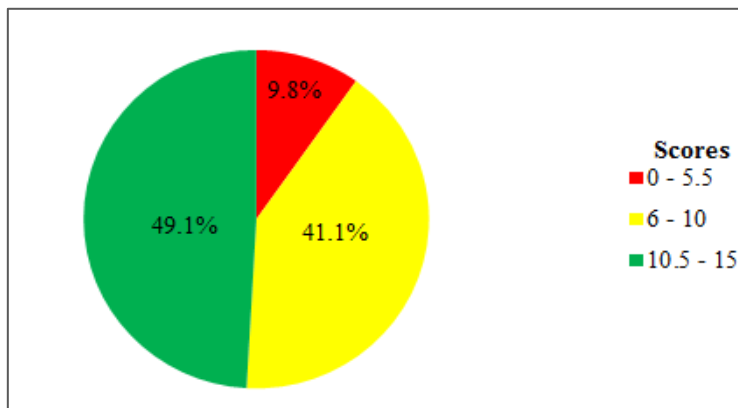


Figure 9: The performance of candidates in question 14.

The candidates who answered this question correctly had wide knowledge on the application of mathematics in their daily life. This implies that they were able to explain the applications of mathematics in their daily life. An example of a correct response is shown in extract 14.1.

14. Mathematic is the subject which deal with abstracting idea, critical thinking, and reasoning at given a particular decision making. The mathematic is also the subject deal with development of mental ability of the people and making correct decision making of a particular phenomena.

The mathematic is a mother of various disciplines as shown below:

Science subject like chemistry, physics, and other subject of art like geography it need depend to some extent the student to have the knowledge of mathematics. Such as knowledge of mathematical operation like addition, subtraction, multiplication, and division it more needed to be applicable for the field of various subject like physics, chemistry and geography.

Surveying activities, is another field or discipline which need the knowledge of mathematics application, the collection of data and performing different measurement during survey time, the mathematic calculation is needed in order to ensure the accuracy.

of collection of data.
Business and trade, is the field or discipline which need the knowledge of mathematics. This is because all transactions are operated through trade. It is based on buying and selling the goods and the media which is used to conduct is using the money, hence the trader needs to have the skill of mathematics in order to simplify their transaction.
Engineering activities, such as construction of road, railway, settlement plan, and other it needs the the constructor to and after it flow work to having the knowledge and idea of mathematics operations in order to enable them to perform some calculations during the measurement and ensuring the correct collection of the data.
Accountancy activities, this is a field or discipline which is based on exchange the money, saving money, and other such activities conducted through bank it needs the accountant to have the knowledge of the mathematics and the understand how to integrate the concept of mathematics with the activities based on bank systems.
Generally, the mathematics is another master of various disciplines because it's subject which guide the every people through daily activities and all every environment.

In Extract 14.1: Is a sample of candidate's correct response in question 14.

However, 9.8% of the candidates who attempted this question failed to do it correctly. The reasons for the failure were lack of knowledge and ability to explain the application of mathematics in different fields in their daily life. Extract 14.2 shows a poor response from one of the candidate who failed to answer correctly this question.

14	<p>Mathematics this is an art which deals with the study of logic, number, theory, principles in various aspects. Mathematics is a mother of various disciplines. The following are the points in order to justify this statement</p> <p>Science: Mathematics also is a science which used systematically to solve different problems. Many science subject depend on Mathematics for example chemistry and Physics</p> <p>Theory: Mathematics comprises different theories concern about different concepts which explain, predict the truth of a certain concept. For example the sum of two interior angles of a triangle is equal to the one opposite exterior angle of a triangle.</p> <p>Number: Also Mathematics always show or explain various concept by using numbers. This can be natural numbers, whole numbers, real numbers and integers</p> <p>Logic: This subject is a mother of other disciplines this is because of using reasoning, it means also Mathematics deals with the study of reasoning that is how, why, if $a=0$ then $a>-1$</p> <p>Systematic: Mathematics solve different problems systematically by following the procedure of solving a certain problem</p> <p>The above are the point which justify the statement that Mathematics is a mother of various discipline. Also Mathematics have got many application in our daily life. For example</p>
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Extract 14.2: Sample of incorrect response of a candidate in question 14.

2.3.2 Question 15: Planning and Preparation for Teaching Mathematics

The candidates were required to describe five criteria for selection of teaching and learning techniques in the teaching and learning of Mathematics.

This question assessed candidates' knowledge about Planning and Preparation for Teaching Mathematics. It required the candidates to recall and describe the criteria for selection of teaching and learning techniques.

A total of 251 candidates (65.6%) scored from 10.5 to 15 marks, 128 (33.5%) scored from 6 to 10 marks while only 3 candidates (0.9%) scored from 0 to 5.5 marks. Therefore the performance for this question was generally good.

The candidates who got the correct answer were able to remember and describe clearly the criteria for selection of teaching and learning techniques in the teaching and learning of Mathematics as indicated in extract 15.1.

15	Criteria for selection of teaching and learning technique in teaching and learning of mathematics
	Teaching is the process of facilitating learning but learning is the relative permanent change of behaviour which results from experiences. Therefore teaching and learning technique implies or means devices or plans used in the process of teaching and learning. When facilitating teaching and learning mathematics there are criteria to consider. The following below are criteria for selection of teaching and learning techniques in the teaching and learning of mathematics.
	Firstly, objective of the lesson or subject matter. Before you plan to teach a professional teacher must consider the objective of the topic to be taught. A teacher teach that subject for which reason. Here the term why is considered. A teacher teaches what intended to be taught and to be covered in a lesson.
	Secondly, cognitive ability of the learner. The learner or student are able to respond the lesson. This is because you can teach a lesson which is above the cognitive level or ability of the learner or student.
	Thirdly, class level or developmental level of the student. The student can master what you intend or need to teach. If the lesson taught is above his or her class level he or she can not master it effectively. The learning result will be difficult.

Extract 15.1: Is a sample of a correct answer given in question 15.

However, 3 candidates (0.9%) answered this question incorrectly due to lack of knowledge about criteria used for selection of teaching and learning techniques in the teaching and learning of Mathematics. Some of them just provided the definition of a teacher and described the terms like audience, behaviour condition and degree as extract 15.2 reveals.

15	Teaching, refers to the transmission of knowledge from one people to another, while teaching techniques are methods used by a teacher to facilitates a learning process effectively, also Learning techniques are methods used by the students given by their teacher to enhance them to participate well in teaching and learning process, The following are some of the criteria for selection of teaching and learning techniques in the teaching and learning of mathematics. Audience, Means that the population of the participant in a particular learning Behavioral, means the characteristics of the participants in a particular learning of the course. Condition, means an environment that the learning take place. Degree, Means ^{to} what the facilitator expected to implement to the learners Time, means of what facilitator prepare to implement to the learners.
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Extract 15.2: A sample of a candidate's incorrect response in question 15.

2.3.3 Question16: Assessment in Mathematics

In this question, the candidates were required to briefly describe three functions of an effective mathematics teacher.

This question was attempted by 229 candidates (54.5%). Out of 229 candidates who attempted this question, 196 (85.7%) failed by scoring from

0 to 5.5 marks. This is the question which was poorly performed in this paper as can be seen in figure 10.

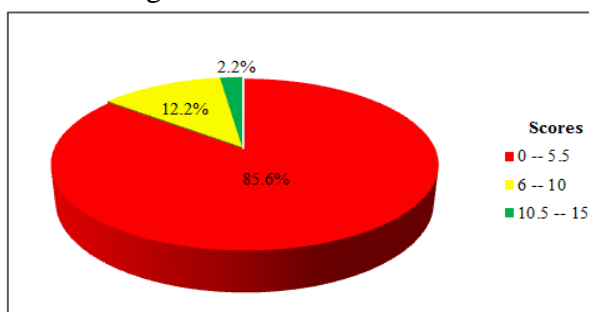


Figure 10: The performance of candidates in question 16.

The candidates who answered incorrectly this question failed to distinguish between the function of a mathematics teacher and the qualifications of a teacher. They wrote the definitions of a mathematics teacher and other requirements of a teacher when teaching in the classroom as shown in Extract 16.1.

16.	<p>Mathematics Teacher</p> <p>are few one that facilitate</p> <p>the Mathematics Concepts</p> <p>to learners the following</p> <p>are three functions of</p> <p>Mathematics Teacher, as</p> <p>Effective Teacher.</p> <p>To be good solver</p> <p>of problem of the students in</p> <p>Mathematics in which he</p> <p>she want to be with</p> <p>Enough knowledges to</p> <p>Facilitate Mathematics.</p> <p>to organise learn</p> <p>methods and Technique</p> <p>in facilitator learning process</p>
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Extract 16.1: illustrates the candidate's incorrect response in question 16.

On the other hand, there were only 5 candidates (2.2%) who managed to score from 10.5 to 14 marks. This shows that there were very few candidates who had adequate knowledge on the topic of assessment in

mathematics. Extract 16.2 shows the response of a candidate who managed to answer this question correctly.

16.	Effective mathematics teacher - is the one who is academically and professionally well prepared teacher in mathematics Subject.
	The following are the three functions of an effective mathematics teacher.
	Executive Functions - an effective mathematics teacher have a leadership role or function in a class, he/she has a duty to lead, facilitate for effective teaching and learning.
	Interactive function - an effective mathematics teacher has a function to employ various methods, technique and strategies for effective interaction between student and materials and student to teaching and learning situation (teacher).
	Organization Function - also an effective mathematics teacher has to make sure he/she work and cooperate with other teacher and students and the community to bring better environment for teaching and learning mathematics.
	Generally apart from those functions the government should also ^{support} an effective mathematics teacher such as with incentives so that he/she can accomplish his/her tasks.

Extract 16.2: Is a sample of the candidate's correct response in question 16.

Generally the performance of the candidates in this question was poor, which means that this topic was not understood by most of the candidates before they sat for the examination.

3.0 THE ANALYSIS OF CANDIDATES PERFORMANCE PER TOPIC

The topics that were examined in the 2019 mathematics examination paper were; *Foundations of Mathematics*, *Logic*, *Similarity and Congruence*, *Analysis of Mathematics Curriculum Materials*, *Probability*, *Coordinate Geometry II*, *Hyperbolic Functions*, *Vectors*, *Algebra*, *Assessment in Mathematics* and *Planning and Preparations for Teaching Mathematics*.

The analysis of candidates' performance per topic shows that, four topics had good performance, namely; *Planning and Preparation for Teaching Mathematics* (99.1%), *Analysis of Mathematics Curriculum Materials* (94.9%), *Similarity and Congruence* (78.4%) and *Logic* (71.3%).

In addition to that, there were four topics with average performance, namely; *Hyperbolic Functions* (61.6%), *Vectors* (59.2%), *Coordinate Geometry II* (53.1%) and *Assessment in Mathematics* (52.3%).

However, the data shows that the candidate had poor performance on three topics which are; *Foundations of Mathematics* (38.1%), *Probability* (31.9%) and *Algebra* (28%). This poor performance was due to lack of skills and competence of the candidates on these topics.

Further analysis was done on each topic by finding the average of the percentages of the candidates who correctly answered the questions in a particular topic. The analysis of the performance in each topic was categorised in three groups according to the average of the percentages of the candidates who correctly answered the questions. These groups were 70–100, 40–69 and 0–39 percent for good, average and poor performance respectively.

4.0 CONCLUSION

The general performance for 740-Mathematics subject, was average by an overall average of 55.7%. The analysis of performance per topics reveals that the reasons that contributed to the topics having an average and weak performances include; making errors while performing mathematical operations, lack of skills on various concepts which were examined in the questions, applying incorrect formulae and failure to identify the requirements of the questions by the candidates.

5.0 RECOMMENDATIONS

In order to improve the performance of mathematics subject for Diploma in Secondary Education in future, the following opinions are recommended:

- (a) Tutors should make sure that all topics in the syllabus are clearly taught and covered so as to equip the candidates with wide skills in answering the examination questions.
- (b) Candidates should be given enough exercise in order to improve their capacity in answering examination questions.
- (c) Candidates should be encouraged to read various materials such as books and pamphlets of mathematics in order to increase their knowledge and ability on different mathematics concepts particularly in the topics with poor performance.
- (d) Tutors should provide monthly tests especially in the poorly performed topics in order to improve the performance.

APPENDIX

ANALYSIS OF PERFORMANCE OF CANDIDATES IN EACH TOPIC

S/N	Topic	Number of question	Performance of Candidates in Percentage		Remarks
			Percentage for Each Question	Average	
1	Planning and Preparation for Teaching Mathematics	15	99.1	99.1	Good
2	Analysis of Mathematics Curriculum Materials	5	94.9	94.9	Good
3	Similarity and Congruence	4	67.6	78.4	Good
		13	89.1		
4	Logic	3	80.3	71.3	Good
		12	62.3		
5	Hyperbolic Functions	9	61.6	61.6	Average
6	Vectors	10	59.2	59.2	Average
7	Coordinate Geometry II	7	39.4	52.7	Average
		8	66.0		
8	Assessment in Mathematics	14	90.2	52.3	Average
		16	14.4		
9	Foundations of Mathematics	1	36.1	38.1	Poor
		2	40.1		
10	Probability	6	31.9	31.9	Poor
11	Algebra	11	28	28	Poor

THE NATIONAL EXAMINATIONS COUNCIL OF TANZANIA



**CANDIDATES' ITEM RESPONSE ANALYSIS REPORT FOR
DIPLOMA IN SECONDARY EDUCATION EXAMINATION
(DSEE) 2019**

740 MATHEMATICS

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TABLE OF CONTENTS

FOREWORD	iv
1.0 INTRODUCTION.....	1
2.0 ANALYSIS OF CANDIDATES' RESPONSES ON EACH QUESTION ...	2
2.1 Section A: Short Answer Questions.....	2
2.1.1 Question 1: Foundations of Mathematics	2
2.1.2 Question 2: Foundations of Mathematics	4
2.1.3 Question 3: Logic.....	6
2.1.4 Question 4: Similarity and Congruence	7
2.1.5 Question 5: Analysis of Mathematics Curriculum Materials	9
2.1.6 Question 6: Probability	10
2.1.7 Question 7: Coordinate Geometry II.....	12
2.1.8 Question 8: Coordinate Geometry II.....	14
2.1.9 Question 9: Hyperbolic Functions	17
2.1.10 Question 10: Vectors	19
2.2 Section B: Essay Answer Questions (Academic)	22
2.2.1 Question 11: Algebra	22
2.2.2 Question 12: Logic.....	25
2.2.3 Question 13: Similarity and Congruence	28
2.3 Section C: Essay Answer Questions (Pedagogy).....	31
2.3.1 Question 14: Assessment in Mathematics	31
2.3.2 Question 15: Planning and Preparation for Teaching Mathematics	35
2.3.3 Question 16: Assessment in Mathematics	36
3.0 THE ANALYSIS OF CANDIDATES PERFORMANCE PER TOPIC	39
4.0 CONCLUSION	39
5.0 RECOMMENDATIONS	40
APPENDIX.....	41

FOREWORD

The report on the Candidates' Item Responses Analysis for Diploma in Secondary Education Examination in Mathematics was prepared for the purpose of providing feedback to candidates, tutors, policy makers, curriculum developers and other education stakeholders on how the candidates answered the examination questions.

The analysis of responses shows that some of the reasons that made the candidate fail to answer the examination questions correctly include; making errors while performing mathematical operations, failure to recall some basic formulae, lack of knowledge and skills on various concepts which were examined, applying incorrect formulae and failure to identify the requirements of the questions.

The National Examinations Council of Tanzania believes that this analysis will provide feedback to various education stakeholders in order to take appropriate measures in improving mathematics teaching techniques as well as creating some more strategies in teaching and learning mathematics subject.

Finally, the Council would like to extend sincere appreciation to the Examination Officers and all others who participated in the preparation of this report.



Dkt. Charles E. Msonde
EXECUTIVE SECRETARY

1.0 INTRODUCTION

The Diploma in Secondary Education Examinations is done every year with the aim of assessing knowledge and skills that have been achieved throughout the two years of implementing the Diploma in education curriculum. A total of 420 candidates were registered in the 2019 DSEE in Mathematics subject of which 417 (99.3%) candidates sat for the Examination.

The paper had a total of sixteen (16) questions divided into three sections namely A, B and C. Section A consisted of 10 short answer questions from both academic and pedagogic contents. Candidates were required to answer all questions in this section. Each correct answer had 4 marks, making a total of 40 marks. Section B consisted of 3 essay questions from academic subject matter. Candidates were required to answer 2 questions where each question had 15 marks, making a total of 30 marks. Section C consisted of 3 essay questions from pedagogy subject matter. Candidates were required to answer 2 questions; the total marks for each question were 15, so the section had a total of 30 marks. Candidates' responses for every question were analysed in order to identify the reasons for either better performance or failure. The analysis involved worked out scripts for each question. Extracts for every question were placed as a sample to the general performance of the question.

The analysis involved all questions that were in the examination paper. It based on the percentage of candidates who performed well the assessment in every question. The analysis on the performance for each question in section A had three categories of marks as follows: 3 - 4 marks; indicating good performance, 2 - 2.5 marks; indicating average performance and 0 - 1.5 marks; indicating poor performance. In sections, B and C, the performance analysis for each question was also categorised into three groups of marks as follows: 10.5 - 15 marks; indicating good performance, 6 - 10 marks; indicating average performance and 0 - 5.5 marks; indicating poor performance.

2.0 ANALYSIS OF CANDIDATES' RESPONSES ON EACH QUESTION

The analysis of candidates' responses for each of the sixteen questions in mathematics was observed as follows:

2.1 Section A: Short Answer Questions

2.1.1 Question 1: Foundations of Mathematics

In this question, the candidates were required to identify four great Mathematicians in mathematics history and explain briefly the contribution of each one.

This question examined candidates' ability to remember the contributions of greater mathematicians on developments of various properties and ideals in mathematics subject.

A total of 352 candidates (83.8%) attempted this question. The analysis of responses in this question shows that 225 candidates (63.9%) scored from 0 to 1.5 marks, 79 candidates (22.4%) scored from 2 to 2.5 marks while 48 candidates (13.6%) scored from 3 to 4 marks. The analysis shows that out of 225 candidates who scored from 0 to 1.5 marks, 68 candidates (19.3%) got a 0 mark. Therefore the general performance for this question was poor because the majority of the candidates (63.9%) scored from 0 to 1.5 marks as indicated in figure 1.

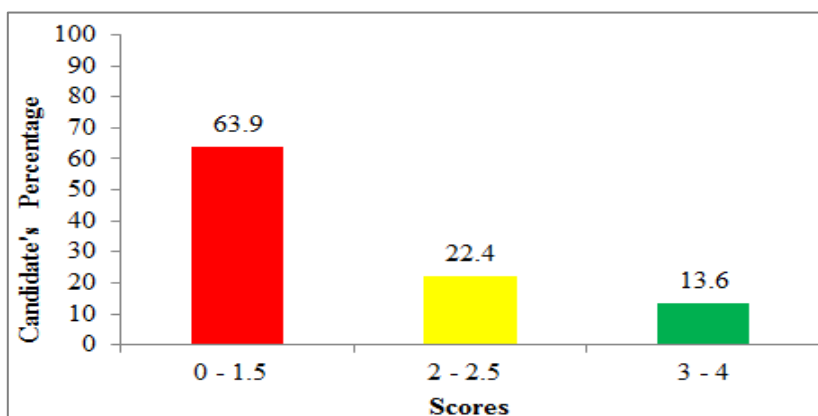


Figure 1: The performance of candidates in question 1.

Extract 1.1 shows the sample of answers from a candidate who failed to remember any of the contributions of the great mathematicians. And therefore he/she presented imaginary answers for each of the mathematician mentioned.

1.	o plato contributes and state children should
	be given conducive learning environment and he
	was greek.
	(2) Pascal said learner should learn by
	playing and he was French
	(3) Galileo contribute to the learning environment
	(4) Zoltan Dienes view learning is playing

Extract 1.1: Sample of incorrect candidate's response in question 1.

This implies that the candidates who failed in this question were not aware of the great mathematicians and their contribution on the subject. They focused on the contribution of education in general instead of mathematics subject.

On the contrary, a total of 48 candidates (13.6%) remembered the contributions of the great mathematicians and presented the correct responses as shown in extract 1.2.

1] Four great Mathematicians in Mathematics history and briefly explain the contribution on each.

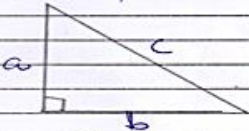
a. Pascal triangle.
 is the Mathematician, whose by the try to provide the expansion of function in form of triangle example.

```

      1
     1 1
    1 2 1
   1 3 3 1
  1 4 6 4 1
  
```

The structure was look like the triangle structure.

b. Pythagoras.
 is the Mathematician who proved the formulae of $a^2 + b^2 = c^2$ in the right angle triangle with 90° .
 Example.



where by a = opposite
 b = Adjacent.
 c = hypotenuse.
 $a^2 + b^2 = c^2$.

c. Machaulation
 is the Mathematician who discuss about the expansion of function (fth).
 is the theory for the expansion of the function.

d. Zoltan Dienes.
 is the Mathematician who concerned about the learning of Mathematics, it by doing practically solving.

Extract 1.2: Sample of correct candidate's response given in question 1.

2.1.2 Question 2: Foundations of Mathematics

In this question, the candidates were required to explain briefly Zoltan Dienes view in learning Mathematics.

This question assessed candidates' knowledge about the view of the mathematician Zoltan Dienes and his contributions in mathematics field. A total of 272 candidates (64.8%) attempted the question. The analysis of data for this question shows that 163 candidates (59.9%) scored from 0 to 1.5 marks, 32 candidates (11.8%) scored from 2 to 2.5 marks and 77 candidates (28.3%) scored from 3 to 4 marks. Out of 77 candidates who scored from 3 to 4 marks, 43 candidates (10.2%) scored full marks that were allocated to this question. The general performance for this question was poor because those who performed poorly were 59.9% compared to those who scored

from 2 to 4 marks who were only 109 (40.1%). Figure 2 shows the performance of the candidates in this question.

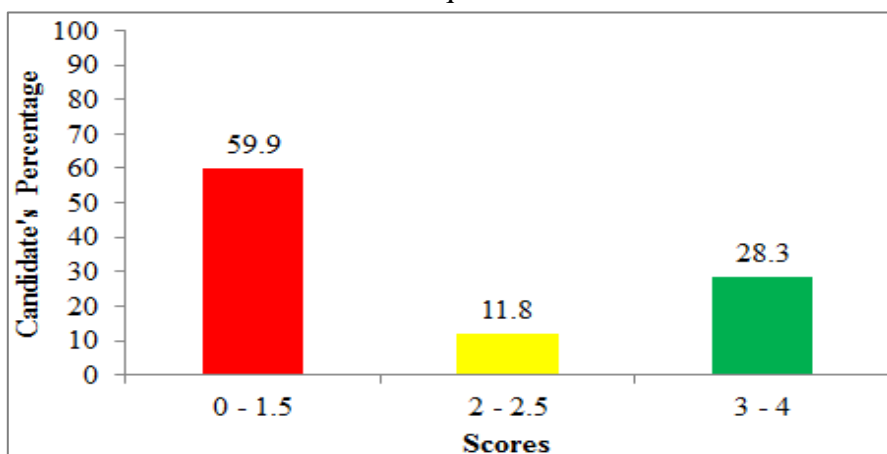


Figure 2: The performance of candidates in question 2.

There were candidates who managed to score full marks as can be seen in extract 2.1 below which is the sample of a candidate's response who managed to remember the Zoltan Dienes view in learning Mathematics.

2.	Zoltan Dienes view in learning mathematics
i/	Emphasize learning through practises
ii/	Emphasize critical thinking
iii/	Brings the rules and theories of learning mathematics.

Extract 2.1: Sample of a correct candidate's response in question 2.

However, 163 candidates (59.9%) of those who attempted this question failed to get it correctly. The reasons for failure were inability to remember and write the Zoltan Dienes view in learning Mathematics. For instance some of the candidates wrote about capacity, experience and age. Extract 2.2 is a sample of a candidate's response who failed to recall the view of the mathematician Zoltan Dienes.

2.	Zoltan Dienes he said the mathematics
	all about thinking capacity, age and
	experience.

Extract 2.2: Sample of incorrect response of a candidate in question 2.

2.1.3 Question 3: Logic

The candidates were required to write the Converse, Inverse and Contrapositive of the statement “If I am under nine years old, then I will go to school”.

This question examined candidates’ ability to apply logic for solving problems in their real life. The question was attempted by 417 (99.3%) out of 420 registered candidates. A total number of 146 candidates (35.0%) answered this question correctly; displayed a high level of interpretation of the given statements and presented the correct answers as shown in extract 3.1.

3	$P =$ I am Under nine years old $Q =$ I will go to School
	$P \rightarrow Q$
(i)	<u>Converse</u>
	$Q \rightarrow P$ I will go to school If I am Under nine years old
(ii)	<u>Inverse</u>
	$\sim P \rightarrow \sim Q$ I am not Under nine years old, I won't go to School.
(iii)	<u>Contrapositive</u>
	$\sim Q \rightarrow \sim P$ I won't go to School If I am not Under year old.

Extract 3.1: Sample of candidate’s correct response given in question 3.

Despite the fact that the question reflected a simple logical application of simple compound statement, 82 (19.7%) of the candidates who attempted

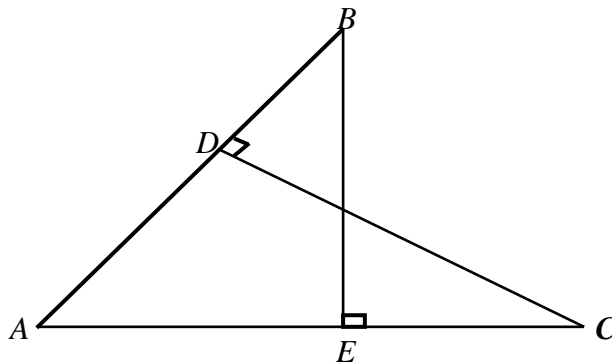
the question, failed to answer it correctly. The failure was due to lack of basic skills to interpret and use correctly the symbols used to represent converse, inverse and contrapositive of a statement. They wrote compound statements which had different meaning in logical statements. Extract 3.2 is the sample of the responses of a candidate who failed to make the correct interpretation that could lead him/her to attain the correct statements.

3.	<u>Soln:</u>
	let P be If I am Under nine years old
	q be I will go to school
	and , = \vee
	connection will be $P \vee q$
	$P \vee q \rightarrow$ Statement
	$q \vee P \rightarrow$ converse.
	$\sim q, \vee \sim P \rightarrow$ Contrapositive
	$\sim P \vee \sim q \rightarrow$ Inverse.

Extract 3.2: Sample of the incorrect candidate's response in question 3.

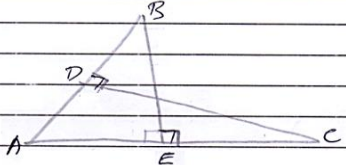
2.1.4 Question 4: Similarity and Congruence

The candidates were required to prove that triangle ABE and ACD are congruent by considering that, in the following figure, $\overline{AB} = \overline{AC}$, D and E are midpoints of \overline{AB} and \overline{AC} respectively.



This question aimed at measuring candidates' ability to identify the given figure and apply the similarity and congruence properties of figures to prove the congruence of triangles ABE and ACD .

A total of 389 candidates (92.6%) attempted this question. The performance of candidates for this question was good as 165 candidates (42.4%) scored from 3 to 4 marks and 98 candidates (25.2%) scored from 2 to 2.5 marks. Those candidates who responded to this question correctly were able to identify the similar triangles and the sides which were equal as well as applying their properties to conclude. See extract 4.1 which is a sample of the response of candidates who managed to answer this question correctly.

4.	
	<p>Since $AB = AC$ then $\triangle ABE$ and $\triangle ACD$ have equal hypotenuses.</p> <p>also, $\triangle ABE$ and $\triangle ACD$ both are right angled triangles</p> <p>if $AB = AC$ then $EB = DC$</p> <p>Therefore $\triangle ABE$ and $\triangle ACD$ are equal due to same hypotenous, side and right angle (RHS)</p>

Extract 4.1: Illustrates a sample of correct candidate's response in question 4.

On the other hand, a total of 126 candidates (32.4%) failed to respond correctly to this question. The reasons for failure were due to lack of knowledge of identifying equal sides, equal triangles as well as failing to apply the properties to prove congruence of triangles. Extract 4.2 shows the situation where one of the candidates attempted the questions using incorrect procedures.

Qn 4	Consider $\triangle ABE$ and $\triangle ACD$.
	For a triangle to become congruency it must have same shape and size.
	Therefore the line or midpoints E and D

Extract 4.2: Sample of an incorrect response of the candidate in question 4.

2.1.5 Question 5: Analysis of Mathematics Curriculum Materials

The candidates were required to mention four curriculum materials applied in teaching and learning of mathematics subject.

This question assessed candidates' knowledge about curriculum materials relevant for teaching and learning mathematics. It required the candidates to remember and write the taught curriculum materials.

A total of 351 candidates (84.6%) of those who attempted this question responded it correctly; as a result the performance for this question was good. The candidates who performed well in this question were able to remember and write the curriculum materials required as indicated in extract 5.1.

5	Curriculum material applied in teaching and learning mathematics
	i) Reference book
	ii) Text book
	iii) Syllabus
	iv) Teacher guide

Extract 5.1: Illustrates a sample of candidate's correct response in question 5.

However, a total of 21 candidates (5.1%) failed to give correct answers to this question due to lack of knowledge about teaching and learning materials in mathematics. Some of them wrote about IT materials instead of writing the taught learning materials applied in mathematics as extract 5.2 reveals this analysis.

5	(i) Computer games
	(ii) Audio-visual Videos
	(iii) Projector
	(iv) Youtube Videos

Extract 5.2: Sample of the incorrect candidate's response in question 5.

2.1.6 Question 6: Probability

The candidates were required to calculate the probability that number 5 has appeared at least once as a die is thrown twice and the sum of the number appearing is observed to be 8.

The question assessed candidates' ability to apply the formulae $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$ where A is the event of getting number 5 appearing at least once and B is the event of getting the sum of numbers appeared equal to 8.

A total of 364 candidates (86.7%) attempted this question. The analysis of candidates' responses shows that 248 candidates (68.1%) scored from 0 to 1.5 marks, 55 candidates (15.1%) scored from 2 to 2.5 marks and 61 candidates (16.8%) scored from 3 to 4 marks. Among the 61 candidates who scored from 3 to 4 marks, 53 candidates (12.6%) scored full marks that were allocated to this question.

The general performance of candidates in this question was poor because those who scored from 0 to 1.5 marks were 68.1% compared to 31.9% of the candidates who scored between 2 to 4 marks as shown in figure 3.

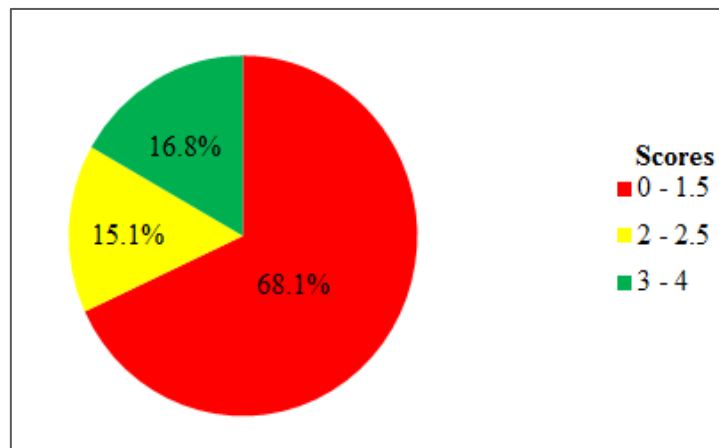


Figure 3: The performance of candidates in question 6.

Some of the candidates who responded wrongly to this question were using combination formula which was not needed in this case. Extract 6.1 shows an example of a response of the candidate who used incorrect formula.

6.	number of die is 6. the die is twice
	$6 \times 2 = 12$. total number appearing is
	observed to be 8. 5 has appeared at least
	once.
	Combination.
	${}^nC_r = \frac{n!}{(n-r)!r!}$
	$= \frac{8!}{(8-5)!5!}$
	$= \frac{40,320}{720}$
	$= 56.$
	The probability that number 5 has appeared
	at least once is 56.

Extract 6.1: Illustrates the incorrect candidate's response to question 6.

Some candidates who failed to respond correctly to this question used wrong formulae by using the combination formulae ${}^nC_r = \frac{n!}{r!(n-r)!}$ and replaced n by 8 and r by 5 which lead them to an incorrect answer because of the wrong procedure.

However, 61 candidates (16.8%) managed to get the correct answer. Some of the candidates used the method of listing the events to get the required probability as in extract 6.2.

Given that Die is thrown twice							
	1	2	3	4	5	6	
2	2,1	2,2	2,3	2,4	2,5	2,6	
3	3,1	3,2	3,3	3,4	3,5	3,6	
4	4,1	4,2	4,3	4,4	4,5	4,6	
5	5,1	5,2	5,3	5,4	5,5	5,6	
6	6,1	6,2	6,3	6,4	6,5	6,6	
The sum of appearing which is equal to 8 are							
2,6 .. 3,5 4,4 5,3 6,2							
$n(S) = 5$							
$n(E) =$ No 5 to appear atleast one							
3,5 and 5,3							
$n(E) = 2$							
$p(E) = \frac{n(E)}{n(S)}$							
$p(E) = \frac{2}{5} \approx 0.4$							
\therefore Probability of number five to appear atleast one = $\frac{2}{5}$ or 0.4							

Extract 6.2: Sample of a correct response of a candidate in question 6.

2.1.7 Question 7: Coordinate Geometry II

The candidates were required to find the equation of the tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at point $(a \cos \theta, b \sin \theta)$.

This question was intended to examine candidates' ability to apply chain rule in establishing the equation $\frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta} = \frac{-b \cos \theta}{a \sin \theta}$ and use it to get the required equation.

A total of 383 candidates (91.2%) attempted this question. The analysis shows that 232 candidates (60.6%) scored from 0 to 1.5 marks, 36 candidates (9.4%) scored from 2 to 2.5 marks and 115 candidates (30.0%) scored from 3 to 4 marks. Out of 115 candidates who scored from 3 to 4 marks, 101 candidates (24.0%) scored full marks which were allocated to this question.

The general performance of candidates to this question was poor because 232 candidates (60.6%) scored from 0 to 1.5 marks compared to those who

scored from 2 to 4 marks who were only 39.4% of those who attempted the question. The candidates who failed to solve this question assigned the given coordinates with the letters x and y and simplify instead of applying the differentiation methods with the chain rule as shown in extract 7.1.

7.	$a \cos \theta = x, b \sin \theta = y$
	$= \frac{(a \cos \theta)^2}{a^2} + \frac{(b \sin \theta)^2}{b^2} = 1$
	$= \frac{a^2 \cos^2 \theta}{a^2} + \frac{b^2 \sin^2 \theta}{b^2} = 1$
	$\cos^2 \theta + \sin^2 \theta = 1$
	$\therefore \text{The equation is } \cos^2 \theta + \sin^2 \theta = 1$

Extract 7.1: Sample of an incorrect response of a candidate.

On the other hand 115 candidates (30%) of those who attempted this question got it correct. This is due to the fact that they correctly used the method of differentiation and chain rule to find the equation as shown in extract 7.2.

7.	$\frac{dy}{dx} = \frac{-2xb^2}{2ya^2}$
	$\frac{dy}{dx} = -\frac{xb^2}{ya^2}$
	also given point = $(a\cos\theta, b\sin\theta)$ (x, y)
	gradient = $\frac{y - b\sin\theta}{x - a\cos\theta} = -\frac{xb^2}{ya^2}$
	$(y - b\sin\theta)ya^2 = (x - a\cos\theta) - xb^2$
	$y^2a^2 - ya^2b\sin\theta = -x^2b^2 + xb^2a\cos\theta$
	$y^2a^2 + x^2b^2 = xb^2a\cos\theta + ya^2b\sin\theta$
	Divide by a^2b^2 both side.
	$\frac{y^2a^2}{a^2b^2} + \frac{x^2b^2}{a^2b^2} = \frac{xb^2a\cos\theta}{a^2b^2} + \frac{ya^2b\sin\theta}{a^2b^2}$
	$\frac{y^2}{b^2} + \frac{x^2}{a^2} = \frac{x\cos\theta}{a} + \frac{y\sin\theta}{b}$
	but
	$\frac{b^2}{b^2} + \frac{x^2}{a^2} = 1$
	$\frac{x\cos\theta}{a} + \frac{y\sin\theta}{b} = 1$
	$\therefore \text{Equation} = \frac{x\cos\theta}{a} + \frac{y\sin\theta}{b} = 1$

Extract 7.2: Sample of the correct response of a candidate in question 7.

2.1.8 Question 8: Coordinate Geometry II

The candidates were required to find the coordinates of point P by considering a point P which divides internally the line joining point (4, 2, 2) and (10, 6, 4) in the ratio 1:1.

This question assessed candidates' knowledge of finding the coordinates of a point by using the concept of ratios and internal division of lines. It was intended to evaluate the candidates' ability to remember and use the

formulae $\underline{P} = \left(\frac{n}{n+m} \right) \underline{a} + \left(\frac{m}{n+m} \right) \underline{b}$ in finding the required coordinates.

Where n and m are the given ratio i.e., $n = 1$ and $m = 1$.

The question was attempted by 341 candidates (81.2%). The analysis of data shows that 116 candidates (34.0%) scored from 0 to 1.5 marks, 18 candidates (5.3%) scored from 2 to 2.5 marks and 177 candidates (60.7%) scored from 3 to 4 marks. This means that the performance of the candidates for this question was average because those who scored from 2 to 4 marks were 195 (66.0%). Figure 4 shows the performance of the candidates for this question.

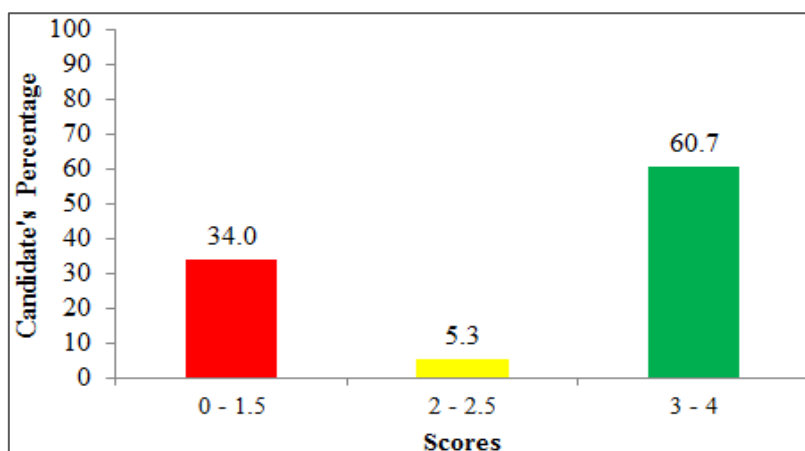


Figure 4: The performance of the candidates in question 8.

The candidates who managed to get the correct answer were able to remember the formulae and apply it correctly to find the coordinates of a point in a line which is $\underline{P} = \left(\frac{n}{n+m} \right) \underline{a} + \left(\frac{m}{n+m} \right) \underline{b}$ and attain the correct answer as indicated in extract 8.1.

Q8 Given that
 points $(4, 2, 2)$ and $(10, 6, 4)$
 such that $P:Q = \lambda:M$
 let $(a_1, b_1, c_1) = (4, 2, 2)$
 $(a_2, b_2, c_2) = (10, 6, 4)$
 Recall from internal division

$$\left(\frac{a_2}{b_2} \right) \lambda + \left(\frac{a_1}{b_1} \right) \lambda$$

$$\frac{\left(\frac{10}{6} \right) \lambda + \left(\frac{4}{2} \right) \lambda}{\lambda + \lambda}$$

$$\frac{\left(\frac{10}{6} \right) + \left(\frac{4}{2} \right)}{2}$$

$$\frac{(10+4) + (6+2) + (4+2)}{2}$$

$$\frac{14 + 8 + 6}{2}$$

$$\frac{28}{2} = 14$$

For the coordinate
 $\left(\frac{a_2 \lambda + a_1 \lambda}{\lambda + \lambda} \right), \left(\frac{b_2 \lambda + b_1 \lambda}{\lambda + \lambda} \right), \left(\frac{c_2 \lambda + c_1 \lambda}{\lambda + \lambda} \right)$
 $\left(\frac{10 \times 1 + 4 \times 1}{1+1} \right), \left(\frac{6 \times 1 + 2 \times 1}{1+1} \right), \left(\frac{4 \times 1 + 2 \times 1}{1+1} \right)$
 $\left(\frac{10+4}{2} \right), \left(\frac{6+2}{2} \right), \left(\frac{4+2}{2} \right)$
 $\frac{14}{2}, \left(\frac{8}{2} \right), \left(\frac{6}{2} \right)$
 $(7, 4, 3)$
 $\therefore \text{coordinate of } P = (7, 4, 3)$

Extract 8.1: A sample of candidate's correct response given in question 8.

However, 116 candidates (34.0%) of those who attempted this question failed to get it correctly due to incapability of remembering and using the relevant formula. Some of them were just adding the given coordinates to get the midpoint coordinates as indicated in extract 8.2 where candidate

failed to recall and apply the formula to get the coordinates required because he/she added the given coordinates, i.e.; $4+10=14$, $2+6=8$ and $2+4=6$ instead of using the formula.

8.	<u>Soln.</u>
	Given points $P(4, 2, 2)$ and $Q(10, 6, 4)$.
	Point $R = (4, 2, 2) + (10, 6, 4)$
	$= 14, 8, 6$
	<u>\therefore Point $R = (14, 8, 6)$</u>

In Extract 8.2: Illustrates an incorrect response of a candidate in question 8.

2.1.9 Question 9: Hyperbolic Functions

The candidates were required to differentiate $\cosh^{-1}(\sqrt{x^2+1})$ with respect to x .

This question evaluated the candidates' ability to differentiate hyperbolic functions by using the substitution method.

A total of 398 candidates (94.8%) attempted this question. The analysis of data shows that 153 candidates (38.4%) scored from 0 to 1.5 marks, 62 candidates (15.6%) scored from 2 to 2.5 marks and 183 candidates (46.0%) scored from 3 to 4 marks. Out of 183 candidates who scored from 3 to 4 marks, 121 candidates (28.8%) scored all marks that were allocated for this question. So the general performance of candidates for this question was average since 61.6% of the candidates who attempted the question scored from 2 to 4 marks. Extract 9.1 illustrate this analysis.

9.	$\frac{dy}{du} = \frac{1}{\sqrt{\cosh^2 u - 1}}$
	$\frac{dy}{du} = \frac{1}{\sqrt{u^2 - 1}}$
	but $u = \sqrt{x^2 + 1}$
	$\frac{dy}{du} = \frac{1}{\sqrt{(\sqrt{x^2 + 1})^2 - 1}}$
	$\frac{dy}{du} = \frac{1}{\sqrt{x^2 + 1 - 1}}$
	$\frac{dy}{du} = \frac{1}{\sqrt{x^2}}$
	$\frac{dy}{du} = \frac{1}{x}$
	so $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$
	$\frac{dy}{dx} = \frac{1}{x} \cdot \frac{x}{\sqrt{x^2 + 1}}$
	$\frac{dy}{dx} = \frac{1}{\sqrt{x^2 + 1}}$
	$\therefore \frac{d}{dx} (\cosh^{-1} \sqrt{x^2 + 1}) = \frac{1}{\sqrt{x^2 + 1}}$

Extract 9.1: Sample of a correct candidate's response given in question 9.

On the other hand, 38.4% of the candidates who attempted this question failed to get it correctly due to inability to make the substitution over the given expression so that it becomes simple before starting to differentiate. They were required to let $u = \cosh^{-1}(\sqrt{x^2 + 1})$ so that $\cosh u = \sqrt{x^2 + 1}$ then proceed with the process of differentiating. Extract 9.2 is a sample of a response from a candidate who failed to differentiate properly the given expression.

9	<u>Solution</u>
	Given that
	$\cosh^{-1}(\sqrt{x^2+1})$
	Let $y = \cosh^{-1} \sqrt{x^2+1}$
	now
	$\cosh y = \sqrt{x^2+1}$
	Differentiate with respect to x .
	$+ \sinh y \frac{dy}{dx} = -\frac{1}{2} (x^2+1)^{-\frac{1}{2}} \cdot 2x$
	$\sinh y \frac{dy}{dx} = -x(x^2+1)^{-\frac{1}{2}}$
	$\frac{dy}{dx} = \frac{x^3+x}{\sinh y}$
	now from $-\sinh^2 y + \cosh^2 y = 1$
	$\sinh^2 y = 1 - \cosh^2 y$
	$\sinh y = \sqrt{1 - \cosh^2 y}$
	now $\frac{dy}{dx} = \frac{x^3+x}{\sqrt{1 - \cosh^2 y}}$
	$\frac{dy}{dx} = \frac{x^3+x}{\sqrt{1 - \sqrt{x^2+1}}}$
	$\therefore \frac{dy}{dx} = \frac{x^3+x}{\sqrt{1 - \sqrt{x^2+1}}}$

Extract 9.2: Illustrates the incorrect response of a candidate in question 9.

2.1.10 Question 10: Vectors

The candidates were required to find the angle between $\underline{a} = 2\underline{i} + \underline{j} - 2\underline{k}$ and $\underline{b} = \underline{i} - 2\underline{j} + \underline{k}$ by using the cross product of vectors.

The question examined candidates' skills on how to use the cross product formula which is $|\underline{a} \times \underline{b}| = |\underline{a}| |\underline{b}| \sin \theta$ where \underline{a} and \underline{b} are two different vectors, in order to get the angle between the two vectors.

The question was attempted by 409 candidates (97.4%). The analysis of data for this question shows that 167 candidates (40.8%) scored from 0 to

1.5 marks, 48 candidates (11.7%) scored from 2 to 2.5 marks and 194 candidates (47.5%) scored from 3 to 4 marks. Out of 194 candidates who scored from 3 to 4 marks, 164 (39.0%) scored full marks that were allocated for this question. Figure 5 shows the performance of the question in question 10.

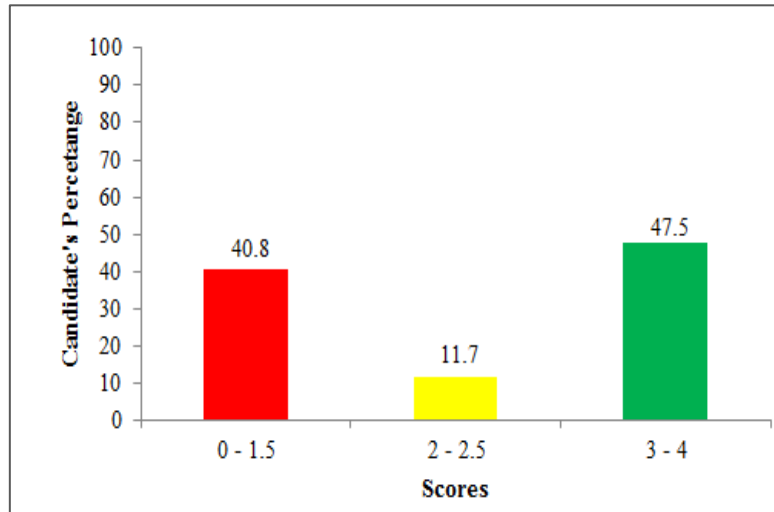


Figure 5: The performance of the candidate question 10.

The candidates who got the correct answer had enough skills on how to use cross product for calculating the angle between two different vectors. They managed to use the formula $|\underline{a} \times \underline{b}| = |\underline{a}| |\underline{b}| \sin \theta$ and proceed by finding $|\underline{a}| = \sqrt{4+1+4} = 3$ and $|\underline{b}| = \sqrt{1+4+1} = \sqrt{6}$ to attain the final answer as shown in extract 10.1.

10 AB /

Cross product = $A \times B$

$A \times B = |A||B|\sin\theta$

$a = -2i + j - 2k$

$b = i - 2j + k$

$a \times b = \begin{vmatrix} i & j & k \\ -2 & 1 & -2 \\ 1 & -2 & 1 \end{vmatrix}$

$i[(1)(1) - (-2)(-2)] - j[(-2)(1) + (-2)(1)] + k[(-2)(-2) - (1)(1)]$

$i(1 - 4) - j(-2 - 2) + k(4 - 1)$

$i(-3) - j(-4) + k(3)$

$|a \times b| = |-3i - 4j + 3k|$ $|a \times b| = \sqrt{x^2 + y^2 + z^2}$

$|a \times b| = \sqrt{(-3)^2 + (-4)^2 + (3)^2}$

$\sqrt{9 + 16 + 25}$

$\sqrt{50}$

$|a \times b| = \sqrt{50}$

$|a \times b| = |A||B|\sin\theta$

$|a| = \sqrt{2^2 + 1^2 + (-2)^2}$

$|a| = \sqrt{4 + 1 + 4}$ $|a| = \sqrt{9}$ $|a| = 3$

$|b| = \sqrt{1^2 + (-2)^2 + 1^2} = \sqrt{1 + 4 + 1} = \sqrt{6}$

$\sqrt{50} = 3 \times \sqrt{6} \sin\theta$

$\frac{\sqrt{50}}{3 \times \sqrt{6}} = \sin\theta$

$\frac{7.07}{3 \times 2.449} = \sin\theta$

$\frac{7.07}{7.348} = \sin\theta$

$\sin\theta = 0.9623$

$\theta = 74.2^\circ$

\therefore The Angle b/w is 74.2°

Extract 10.1: Is a sample of candidate's correct response in question 10.

Meanwhile, there were 167 candidates (40.8%) who attempted this question and performed poorly due to inability to remember and improper use of vector cross product techniques so as to get the required angle. Some of them just calculated the magnitude of both vectors and subtract before equating to $\sin\theta$ and find the angle θ resulting into incorrect answer as revealed in extract 10.2.

10 From Cos product formula

$$A \cdot B = |A||B| \cos \theta, \quad a = 2i + j - 2k$$

$$b = i - 2j + k$$

$$\begin{pmatrix} i & j & k \\ 2 & 1 & -2 \\ 1 & -2 & 1 \end{pmatrix}$$

$$2(1 + 4) - 1(2 + 2) + -2(-4 - 1)$$

$$= -6 - 4 + 10$$

$$-6 - 4 + 10 = \sqrt{2^2 + 1^2 + (-2)^2} \times \sqrt{1^2 + (-2)^2 + 1^2} \times \cos \theta$$

$$-6 - 4 + 10 = \sqrt{9} \times \sqrt{6} \cos \theta$$

$$\sqrt{6^2 + 4^2 + 10^2} = \sqrt{56} \cos \theta$$

$$\frac{\sqrt{152}}{\sqrt{56}}$$

$$0 = \sqrt{56} \cos \theta$$

$$\cos \theta = \left(\frac{0}{\sqrt{56}} \right)$$

$$\theta = \cos^{-1} \left(\frac{0}{\sqrt{56}} \right) = 0$$

Therefore the intended angle will be 0°

Extract 10.2: Is a sample of candidate's incorrect response in question 10.

2.2 Section B: Essay Answer Questions (Academic)

2.2.1 Question 11: Algebra

This question had three parts where by the candidates were to:

- (a) Evaluate $\sum_{n=4}^{10} n^2 + 3n$, given that $\sum r^2 = \frac{n}{6}(n+1)(2n+1)$ and

$$\sum r = \frac{n}{2}(n+1).$$

- (b) Show that $ac^3 = db^3$, if the roots of polynomial equation $ax^3 + bx^2 + cx + d = 0$ are in geometric progression.
- (c) Find an equation whose roots are $\alpha^2\beta$ and $\alpha\beta^2$, when the roots of quadratic equation $2x^2 - 7 = 0$ are α and β .

This question had three parts, in part (a) the question assessed candidates' ability to expand the expressions and apply the given information to substitute the values in order to get the required answer; in part (b) the question assessed the candidates' knowledge about roots of polynomials and part (c) assessed the candidates' ability to factorise and apply the factors obtained to form an equation.

The question was attempted by 132 candidates (31.4%) out of 420 registered candidates. Among these candidates; 95 (72.0%) scored from 0 to 5.5 marks, 35 candidates (26.4%) scored from 6 to 10 marks and 2 candidates (1.6%) scored from 10.5 to 15 marks. The analysis indicates that the general performance for this question was poor because the candidates who passed were only 28%. Figure 6 shows the performance of candidates for question 11.

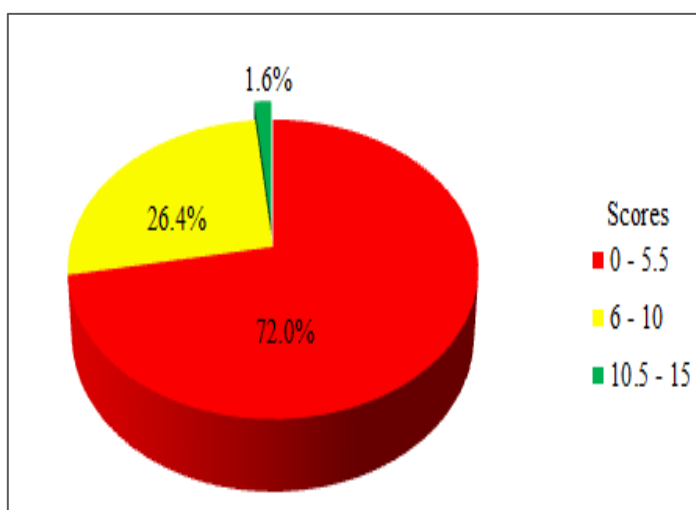


Figure 6: The general performance of candidates in question 11.

The candidates who failed to answer correctly this question lacked knowledge about the application of the given information; some of them substituted the given limits direct into the given expression and obtained incorrect answer as shown in extract 11.1.

11a)	$\sum r^2 = n/6 (n+1)(2n+1)$ and $\sum r = n/2 (n+1)$
	$\sum_{n=4}^{10} n^2 + 3n$
	<u>Solution:</u>
	$\sum_{n=4}^{10} n$
	$\sum_{n=4}^{10} n/6 (n+1)(2n+1)$ where $n = 4, 5, 6, 7, 8, 9, 10$
	$\sum_{n=4}^{10} 4/6 (4+1)(2 \times 4+1) + 5/6 (5+1)(2 \times 5+1) + 6/6 (6+1)(2 \times 6+1)$
	$+ 7/6 (7+1)(2 \times 7+1) + 8/6 (8+1)(2 \times 8+1) + 9/6 (9+1)(2 \times 9+1)$
	$+ 10/6 (10+1)(2 \times 10+1) + 3 \left[\frac{n}{2} (n+1) \right] \left[\frac{1}{2} (4+1) + \right.$
	$\left. \frac{5}{2} (5+1) + \frac{6}{2} (6+1) + \frac{7}{2} (7+1) + \frac{8}{2} (8+1) + \frac{9}{2} (9+1) \right.$
	$\left. + \frac{10}{2} (10+1) \right]$

Extract 11.1: Is the sample of incorrect response of a candidate in part (a).

Despite the failure of many candidates in this question, there were 37 candidates (28.0%) who managed to score from 6 to 13.5 marks. They managed to apply the given formula and able to factorise it as well as interpreting the values before making the substitution to attain the correct answer in part (a) as shown in extract 11.2.

11.	a). Given
	$\sum r^2 = n/6(n+1)(2n+1)$ and
	$\sum r = n/2(n+1)$ but required.
	$\sum_{4}^{10} n^2 + 3n$ where $n = 4, 5, 6, 7, 8, 9$ and 10 .
	$\sum_{4}^{10} n^2 + 3n = (4^2 + 3 \times 4) + (5^2 + 3 \times 5) + (6^2 + 3 \times 6) + (7^2 + 3 \times 7)$
	$+ (8^2 + 3 \times 8) + (9^2 + 3 \times 9) + (10^2 + 3 \times 10)$
	$= 28 + 40 + 54 + 70 + 88 + 108 + 130$.
	$= 518$.
	$\therefore \sum_{4}^{10} n^2 + 3n = 518$.

Extract 11.2: Illustrates the sample of a correct response of a candidate.

2.2.2 Question 12: Logic

In this question, the candidates were required to:

- Draw an electrical network of the simplified form of the compound statement; $\sim [(P \wedge \sim Q) \vee (\sim P \wedge Q) \vee (\sim P \wedge \sim Q)]$.
- Determine the validity of the argument "If it rains, the seedlings will survive. If seedlings survive well, animals will not die. But animals are dying. Therefore it is not raining", by using the laws of algebra of propositions.

This question assessed candidates' knowledge about logical properties and its application in simplification of compound statements. It was attempted by 355 candidates (84.5%) whereby 134 candidates (37.7%) scored from 0 to 5.5 marks, 134 (37.7%) scored from 6 to 10 marks and 87 (24.6%) scored from 10.5 to 15 marks. Out of 87 candidates who scored from 10.5 to 15 marks, 24 (5.7%) scored all marks that were allocated to this question.

The analysis indicates that the general performance of candidates in this question was average because those who scored from 6 to 15 marks were 221 (62.3%). Figure 7 presents the performance of candidates for question 12.

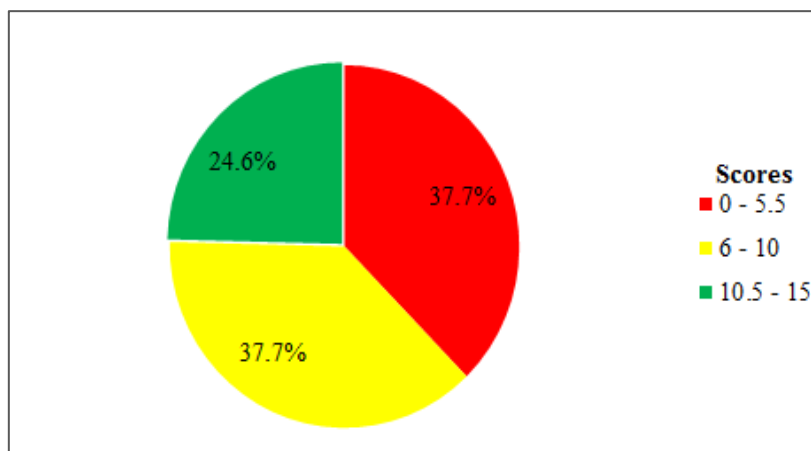
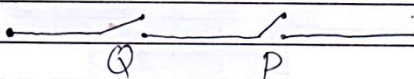


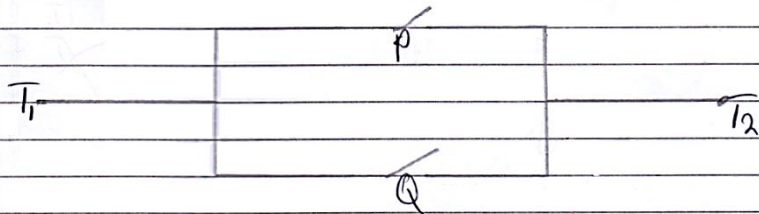
Figure 7: The performance of candidates in question 12.

Some of the candidates had adequate knowledge about formulating the compound statements and applying the laws of algebra of propositions to simplify as indicated in extract 12.1 which is the sample of the answers that were given by one of the candidates.

12a.	$\sim[(P \wedge \sim Q) \vee (\sim P \wedge Q) \vee (\sim P \wedge \sim Q)]$
	Let's simplify by using algebraic laws of propositions
	$\sim[(P \wedge \sim Q) \vee \sim P \wedge (Q \vee \sim Q)]$ - distributive law
	$\sim[(P \wedge \sim Q) \vee \sim P \wedge T]$ - negation law
	$\sim[(P \wedge \sim Q) \vee \sim P]$ - identity law.
	$\sim[(P \vee \sim P) \wedge (\sim Q \vee \sim P)]$ - distributive law
	$\sim[T \wedge (\sim Q \vee \sim P)]$ - negation
	$\sim(\sim Q \vee \sim P)$ - identity law
	$(Q \wedge P)$ - double negation.
	The electric net work will be in series.
	
12b.	let, p be It rains
	q be the seedling survive
	r be animals are dying
	In logic form the argument will be as follows.
	$[(P \rightarrow q) \wedge (q \rightarrow \sim r) \wedge r] \rightarrow \sim P$
	then $[(P \rightarrow \sim r) \wedge r] \rightarrow \sim P$ by definition: $(P \rightarrow Q) \wedge Q \rightarrow P$
	$[\sim(P \vee r) \wedge r] \rightarrow \sim P$ by definition
	$[\sim P \wedge \sim r] \wedge r \rightarrow \sim P$ negation
	$[\sim P \wedge (\sim r \wedge r)] \rightarrow \sim P$ Associative law
	$[\sim P \wedge F] \rightarrow \sim P$ Negation law
	$F \rightarrow \sim P$ Identity law
	$\sim F \vee P$ by definition
	$T \vee P$ Negation law
	T identity law
	Therefore the argument is valid.

Extract 12.1: Illustrates the response of a candidate statement correctly.

On the other hand, 134 candidates (37.7%) failed to get the correct answer to this question due to inability to remember the logical properties like; Distributive law, Negation law, Identity law and Double negation law. The analysis shows that some of them were using the laws which are not used in logic like Absorption law and Demorgan's law. Therefore, they failed to simplify, ended up with the wrong statement as shown in extract 12.2.

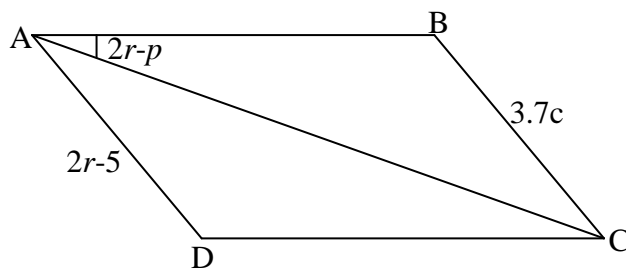
12. a)	Soln.
	Given.
	$\sim [P \wedge \sim Q] \vee (\sim P \wedge Q) \vee (\sim P \wedge \sim Q)$ - Given.
	$= \sim [P \vee \sim P \wedge \sim Q]$
	$\sim [T \vee (\sim P \wedge \sim Q)]$ - Absorption law.
	$\sim (\sim P \wedge \sim Q)$ - Identity law.
	$P \vee Q$ - Demogorgans law.
	\therefore electrical network of $P \vee Q$.
	
b).	Soln.
	Let P = rains
	q = survived
	r = animal
	$\therefore (P \rightarrow q) \wedge (q \rightarrow r) \rightarrow P \sim p$.
	Truth table.
	$\therefore [(P \rightarrow q \wedge q \rightarrow r) \wedge \sim r] \rightarrow \sim p$.

Extract 12.2: is the response of a candidate who failed to simplify the statements in question 12(a), (b).

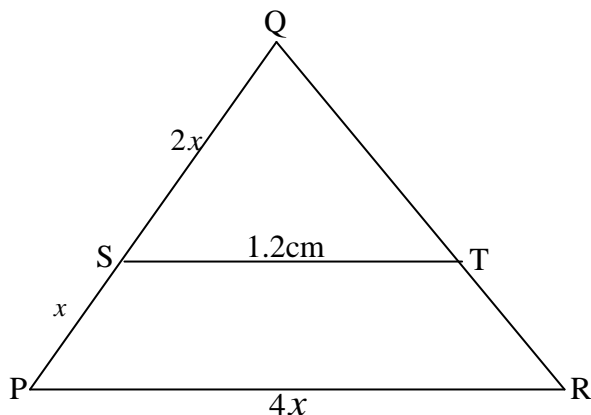
2.2.3 Question 13: Similarity and Congruence

In this question, the candidates were required to:

- (a) Find the value of r and p in the following figure, where $\triangle ABC$ is congruent to $\triangle CDA$.



- (b) Find the value of x in the following figure if $\triangle PQR \sim \triangle SQT$.



This question assessed candidates' ability to identify different figures and relate them in order to solve for values of r and p in part (a) and x in part (b).

A total of 349 candidates (83.1%) attempted this question. The analysis shows that 38 candidates (10.9%) scored from 0 to 5.5 marks, 118 (33.8%) scored from 6 to 10 marks and 193 candidates (55.3%) scored from 10.5 to 15 marks. Figure 8 presents the performance of the candidates in this question.

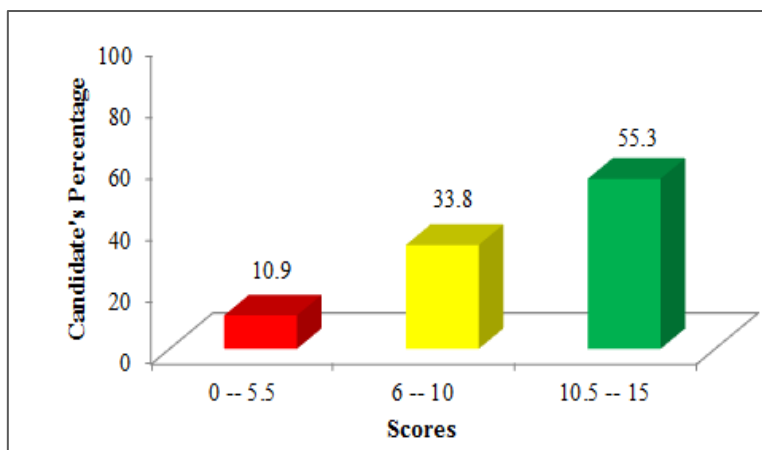


Figure 8: The performance of candidates for question 13.

The candidates who scored 10.5 marks and above, were able to solve for values of r and p in part (a) and determine the value of x in part (b) after detecting the angles which are equal and the corresponding sides. Extract 13.1 is a sample of a candidate's correct responses given in question 13 (a) and (b).

B@Required	r and P.
$\overline{AD} = \overline{BC}$.	
$2r - 5 = 3.7\text{cm}.$	
$2r = 3.7 + 5.$	
$\frac{2r}{2} = \frac{8.7}{2}.$	
$r = 4.35\text{cm}.$	
\therefore The value of $r = 4.35\text{cm}.$	
$2r - P = 23$	(Because of Alternate angles. Alternating angles.)
But $r = 4.35.$	
$8.7 - P = 23.$	
$P = -14.3.$	
\therefore Value of $P = -14.3.$	
b) from.	$\frac{PQ}{QS} = \frac{PR}{ST}.$
$3X = 4X.$	
$2X \quad 1.2\text{cm}.$	
By crossing multiplication	
$3X (1.2\text{cm}) = 4X (2X)$	
$3.6X\text{cm} = 8X^2.$	
$X \quad X.$	
$3.6\text{cm} = 8X.$	
$8 \quad 8.$	
\therefore The value of $X = 0.45\text{cm}.$	

Extract 13.1: Illustrates the correct response of a candidate in question 13.

Among 38 candidates (10.9%) who scored from 0 to 5.5 marks, some of them got incorrect answers to this question because they did not know the corresponding sides and angles in part (a) which are $\overline{AD} = \overline{BC}$ and $BAC = ACD$ also the corresponding sides in part (b) which are $\frac{PQ}{QS} = \frac{PR}{ST}.$

Therefore, they were unable to solve for values for r , p and x as shown in extract 13.2.

13. a/ $\triangle ABC$ is congruent to $\triangle CDA$. Find the value of r and P .

$2r - p = 23^\circ$
 $\therefore r = \frac{23^\circ}{2} = 11.5^\circ$
 \therefore The value of $r = 11.5^\circ$ and $p = 11.5^\circ$

5/ $\triangle PQR \sim \triangle STU$. Find the value of x .
 from $\sin = \frac{\text{opposite}}{\text{Hypotenuse}}$
 where opposite = $8x$
 Hypotenuse = 1.2 cm

Extract 13.2: Is a sample of candidate's incorrect response given in question 13(a) and (b).

2.3 Section C: Essay Answer Questions (Pedagogy)

2.3.1 Question 14: Assessment in Mathematics

The candidates were required to justify the statement that "Mathematics is a mother of various disciplines" by giving five points.

This question examined candidates' ability to express the application of Mathematics in various fields of life. It was attempted by 224 candidates (53.3%). The analysis of data shows that 22 candidates (9.8%) scored from 0 to 5.5 marks, 92 (41.1%) scored from 6 to 10 marks while 110 candidates (49.1%) scored from 10.5 to 15 marks. The general performance in this question was good because 90.2 percent of the candidates scored from 6 to 15 marks. This performance is indicated in figure 9.

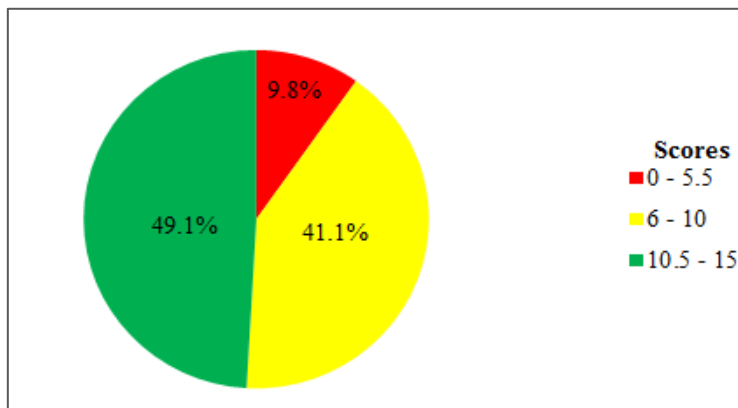


Figure 9: The performance of candidates in question 14.

The candidates who answered this question correctly had wide knowledge on the application of mathematics in their daily life. This implies that they were able to explain the applications of mathematics in their daily life. An example of a correct response is shown in extract 14.1.

14. Mathematic is the subject which deal with abstracting idea, critical thinking, and reasoning at given a particular decision making. The mathematic is also the subject deal with development of mental ability of the people and making correct decision making of a particular phenomena.

The mathematic is a mother of various disciplines as shown below:

Science subject like chemistry, physics, and other subject of art like geography it need depend to some extent the student to have the knowledge of mathematics. Such as knowledge of mathematical operation like addition, subtraction, multiplication, and division it more needed to be applicable for the field of various subject like physics, chemistry and geography.

Surveying activities, is another field or discipline which need the knowledge of mathematics application, the collection of data and performing different measurement during survey time, the mathematic calculation is needed in order to ensure the accuracy.

of collection of data.
Business and trade, is the field or discipline which need the knowledge of mathematics. This because all transactions is operated through trade it base on buying and selling the goods and the media which used to conduct is using the money, hence the trader needed to having the skill of mathematics in order to simplify their transaction.
Engineering activities, such as construction of road, railway, settlement plan, and other it needed the the constructor to and after it flow work to having the knowledge and idea of mathematics operation in order to enable them to perform some calculation during the measurement and ensuring the correct collection of the data.
Accountancy activities, this are field or discipline which base on exchange the money, saving money, and other the such activities conducted through bank it need the accountant to having the knowledge of the mathematics and the understand how to integrate the concept of mathematics with the activities base on bank system.
Generally, the mathematics is another master of various discipline because it's subject which guide the the every people through daily activities and all every environment.

In Extract 14.1: Is a sample of candidate's correct response in question 14.

However, 9.8% of the candidates who attempted this question failed to do it correctly. The reasons for the failure were lack of knowledge and ability to explain the application of mathematics in different fields in their daily life. Extract 14.2 shows a poor response from one of the candidate who failed to answer correctly this question.

14	<p>Mathematics this is an art which deals with the study of logic, number, theory, principles in various aspects. Mathematics is a mother of various disciplines. The following are the points in order to justify this statement</p> <p>Science: Mathematics also is a science which used systematically to solve different problems. Many science subject depend on Mathematics for example chemistry and Physics</p> <p>Theory: Mathematics comprises different theories concern about different concepts which explain, predict the truth of a certain concept. For example the sum of two interior angles of a triangle is equal to the one opposite exterior angle of a triangle.</p> <p>Number: Also Mathematics always show or explain various concept by using numbers. This can be natural numbers, whole numbers, real numbers and integers</p> <p>Logic: This subject is a mother of other disciplines this is because of using reasoning, it means also Mathematics deals with the study of reasoning that is how, why, if $a=0$ then $a>-1$</p> <p>Systematic: Mathematics solve different problems systematically by following the procedure of solving a certain problem</p> <p>The above are the point which justify the statement that Mathematics is a mother of various discipline. Also Mathematics have got many application in our daily life. For example</p>
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Extract 14.2: Sample of incorrect response of a candidate in question 14.

2.3.2 Question 15: Planning and Preparation for Teaching Mathematics

The candidates were required to describe five criteria for selection of teaching and learning techniques in the teaching and learning of Mathematics.

This question assessed candidates' knowledge about Planning and Preparation for Teaching Mathematics. It required the candidates to recall and describe the criteria for selection of teaching and learning techniques.

A total of 251 candidates (65.6%) scored from 10.5 to 15 marks, 128 (33.5%) scored from 6 to 10 marks while only 3 candidates (0.9%) scored from 0 to 5.5 marks. Therefore the performance for this question was generally good.

The candidates who got the correct answer were able to remember and describe clearly the criteria for selection of teaching and learning techniques in the teaching and learning of Mathematics as indicated in extract 15.1.

15	Criteria for selection of teaching and learning technique in teaching and learning of mathematics
	Teaching is the process of facilitating learning but learning is the relative permanent change of behaviour which results from experiences. Therefore teaching and learning technique implies or means devices or plans used in the process of teaching and learning. When facilitating teaching and learning mathematics there are criteria to consider. The following below are criteria for selection of teaching and learning techniques in the teaching and learning of mathematics.
	Firstly, objective of the lesson or subject matter. Before you plan to teach a professional teacher must consider the objective of the topic to be taught. A teacher teach that subject for which reason. Here the term why is considered. A teacher teaches what intended to be taught and to be covered in a lesson.
	Secondly, cognitive ability of the learner. The learner or student are able to respond the lesson. This is because you can teach a lesson which is above the cognitive level or ability of the learner or student.
	Thirdly, class level or developmental level of the student. The student can master what you intend or need to teach. If the lesson taught is above his or her class level he or she can not master it effectively. The learning result will be difficult.

Extract 15.1: Is a sample of a correct answer given in question 15.

However, 3 candidates (0.9%) answered this question incorrectly due to lack of knowledge about criteria used for selection of teaching and learning techniques in the teaching and learning of Mathematics. Some of them just provided the definition of a teacher and described the terms like audience, behaviour condition and degree as extract 15.2 reveals.

15	Teaching, refers to the transmission of knowledge from one people to another, while teaching techniques are methods used by a teacher to facilitates a learning process effectively, also Learning techniques are methods used by the students given by their teacher to enhance them to participate well in teaching and learning process, The following are some of the criteria for selection of teaching and learning techniques in the teaching and learning of mathematics. Audience, Means that the population of the participant in a particular learning Behavioral, means the characteristics of the participants in a particular learning of the course. Condition, means an environment that the learning take place. Degree, Means ^{to} what the facilitator expected to implement to the learners Time, means of what facilitator prepare to implement to the learners.
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Extract 15.2: A sample of a candidate's incorrect response in question 15.

2.3.3 Question16: Assessment in Mathematics

In this question, the candidates were required to briefly describe three functions of an effective mathematics teacher.

This question was attempted by 229 candidates (54.5%). Out of 229 candidates who attempted this question, 196 (85.7%) failed by scoring from

0 to 5.5 marks. This is the question which was poorly performed in this paper as can be seen in figure 10.

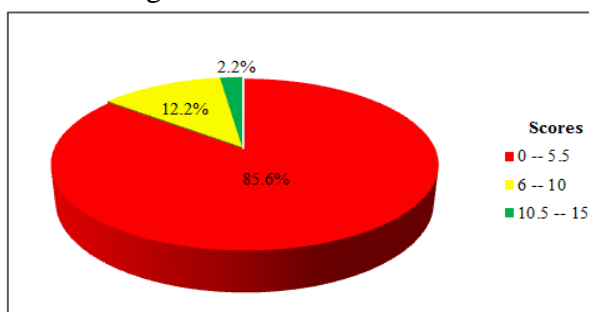


Figure 10: The performance of candidates in question 16.

The candidates who answered incorrectly this question failed to distinguish between the function of a mathematics teacher and the qualifications of a teacher. They wrote the definitions of a mathematics teacher and other requirements of a teacher when teaching in the classroom as shown in Extract 16.1.

16.	Mathematics Teacher
	are few one that facilitate
	the Mathematics Concepts
	to learners the following
	are three functions of
	Mathematics Teacher, as
	Effective Teacher.
	To be good solver
	of problem of the students in
	Mathematics in which he
	should want to be with
	Enough knowledges to
	Facilitate Mathematics.
	to organise learn
	methods and Technique
	in facilitator learning process

Extract 16.1: illustrates the candidate's incorrect response in question 16.

On the other hand, there were only 5 candidates (2.2%) who managed to score from 10.5 to 14 marks. This shows that there were very few candidates who had adequate knowledge on the topic of assessment in

mathematics. Extract 16.2 shows the response of a candidate who managed to answer this question correctly.

16.	Effective mathematics teacher - is the one who is academically and professionally well prepared teacher in mathematics Subject.
	The following are the three functions of an effective mathematics teacher.
	Executive Functions - an effective mathematics teacher have a leadership role or function in a class, he/she has a duty to lead, facilitate for effective teaching and learning.
	Interactive function - an effective mathematics teacher has a function to employ various methods, technique and strategies for effective interaction between student and materials and student to teaching and learning situation (teacher).
	Organization Function - also an effective mathematics teacher has to make sure he/she work and cooperate with other teacher and students and the community to bring better environment for teaching and learning mathematics.
	Generally apart from those functions the government should also ^{support} an effective mathematics teacher such as with incentives so that he/she can accomplish his/her tasks.

Extract 16.2: Is a sample of the candidate's correct response in question 16.

Generally the performance of the candidates in this question was poor, which means that this topic was not understood by most of the candidates before they sat for the examination.

3.0 THE ANALYSIS OF CANDIDATES PERFORMANCE PER TOPIC

The topics that were examined in the 2019 mathematics examination paper were; *Foundations of Mathematics*, *Logic*, *Similarity and Congruence*, *Analysis of Mathematics Curriculum Materials*, *Probability*, *Coordinate Geometry II*, *Hyperbolic Functions*, *Vectors*, *Algebra*, *Assessment in Mathematics* and *Planning and Preparations for Teaching Mathematics*.

The analysis of candidates' performance per topic shows that, four topics had good performance, namely; *Planning and Preparation for Teaching Mathematics* (99.1%), *Analysis of Mathematics Curriculum Materials* (94.9%), *Similarity and Congruence* (78.4%) and *Logic* (71.3%).

In addition to that, there were four topics with average performance, namely; *Hyperbolic Functions* (61.6%), *Vectors* (59.2%), *Coordinate Geometry II* (53.1%) and *Assessment in Mathematics* (52.3%).

However, the data shows that the candidate had poor performance on three topics which are; *Foundations of Mathematics* (38.1%), *Probability* (31.9%) and *Algebra* (28%). This poor performance was due to lack of skills and competence of the candidates on these topics.

Further analysis was done on each topic by finding the average of the percentages of the candidates who correctly answered the questions in a particular topic. The analysis of the performance in each topic was categorised in three groups according to the average of the percentages of the candidates who correctly answered the questions. These groups were 70–100, 40–69 and 0–39 percent for good, average and poor performance respectively.

4.0 CONCLUSION

The general performance for 740-Mathematics subject, was average by an overall average of 55.7%. The analysis of performance per topics reveals that the reasons that contributed to the topics having an average and weak performances include; making errors while performing mathematical operations, lack of skills on various concepts which were examined in the questions, applying incorrect formulae and failure to identify the requirements of the questions by the candidates.

5.0 RECOMMENDATIONS

In order to improve the performance of mathematics subject for Diploma in Secondary Education in future, the following opinions are recommended:

- (a) Tutors should make sure that all topics in the syllabus are clearly taught and covered so as to equip the candidates with wide skills in answering the examination questions.
- (b) Candidates should be given enough exercise in order to improve their capacity in answering examination questions.
- (c) Candidates should be encouraged to read various materials such as books and pamphlets of mathematics in order to increase their knowledge and ability on different mathematics concepts particularly in the topics with poor performance.
- (d) Tutors should provide monthly tests especially in the poorly performed topics in order to improve the performance.

APPENDIX

ANALYSIS OF PERFORMANCE OF CANDIDATES IN EACH TOPIC

S/N	Topic	Number of question	Performance of Candidates in Percentage		Remarks
			Percentage for Each Question	Average	
1	Planning and Preparation for Teaching Mathematics	15	99.1	99.1	Good
2	Analysis of Mathematics Curriculum Materials	5	94.9	94.9	Good
3	Similarity and Congruence	4	67.6	78.4	Good
		13	89.1		
4	Logic	3	80.3	71.3	Good
		12	62.3		
5	Hyperbolic Functions	9	61.6	61.6	Average
6	Vectors	10	59.2	59.2	Average
7	Coordinate Geometry II	7	39.4	52.7	Average
		8	66.0		
8	Assessment in Mathematics	14	90.2	52.3	Average
		16	14.4		
9	Foundations of Mathematics	1	36.1	38.1	Poor
		2	40.1		
10	Probability	6	31.9	31.9	Poor
11	Algebra	11	28	28	Poor

