THE UNITED REPUBLIC OF TANZANIA
MINISTRY OF EDUCATION, SCIENCE AND TECHNOLOGY
NATIONAL EXAMINATIONS COUNCIL OF TANZANIA

CANDIDATES' ITEM RESPONSE ANALYSIS REPORT FOR DIPLOMA IN SECONDARY EDUCATION EXAMINATION (DSEE) 2021

MATHEMATICS

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740 MATHEMATICS

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FOREWORD

The National Examinations Council of Tanzania is pleased to issue this report on Candidates' Item Response Analysis (CIRA) on the Diploma in Secondary Education Examination (DSEE) 2021. This report has been prepared in order to provide feedback to tutors, students, policy makers, educational administrators and other educational stakeholders on the candidates' performance in the subject.

The report in the Mathematics subject highlights the factors that made the candidates perform well in the examination. The factors include; ability to interpret the demand of the questions and to follow instructions as well as sufficient knowledge about the concepts and principles related to the subject. The report indicates that some of the candidates scored low marks because they failed to interpret the questions requirement and they lacked sufficient knowledge and skills about the mathematical concepts which were examined, making errors while performing mathematical operations, failure to use basic formulae and applying incorrect formulae.

The feedback provided in this report is expected to enable the educational stakeholders to take appropriate measures to improve teaching and learning in this subject. This will eventually improve the candidates' performance in the future examinations.

Finally, the National Examinations Council of Tanzania would like to extend sincere appreciation to everyone who participated in the preparation of this report.

Dkt. Charles E. Msonde

EXECUTIVE SECRETARY

1.0 INTRODUCTION

This report provides the candidates response in Mathematics for the candidates who sat for the DSEE. It gives feedback to educational stakeholder on the strengths and weakness of candidates' performance. A total of 429 candidates were registered in the 2021 DSEE in Mathematics subject out of which 426 (99.3%) candidates sat for the Examination.

The paper had a total of sixteen (16) questions that were divided into three sections; A, B and C. Section A consisted of 10 short answer questions where candidates were required to answer all questions. Each correct answer had 4 marks, making a total of 40 marks. Section B and C consisted of three (3) essay questions each where candidates were required to answer 2 questions from each section. Each correct answer had 15 marks, making a total of 60 marks.

The analysis on the performance for each question in section A had three categories of marks as follows: 3 - 4 marks; high marks, 2 - 2.5 marks; average marks and 0 - 1.5 marks; low marks. In sections B and C, the performance analysis for each question was also categorised into three groups of marks as follows: 10.5 - 15 marks; high marks, 6 - 10 marks; average marks and 0 - 5.5 marks; low marks. Also the analysis of performance was categorised in three groups. The groups are 70%–100%, 40%–69% and 0%–39% for good, average and weak performance respectively.

The analysis of candidates' responses in each question was done by using data, figures and extract of sample of answers from the candidates. In the figures of analysis on performance presented in this report, there are three colours which are used to represent the performance as follows:

Good performance, Average performance and Weak performance.

2.0 ANALYSIS OF CANDIDATES' RESPONSES IN EACH QUESTION

2.1 Section A: Short Answer Questions

2.1.1 Question 1: Differentiation

This question examined candidates' ability to apply knowledge of differentiation in determining the turning point of the given curve. The question instructed candidates to find the turning point on the curve $y = x^2 - 2x$.

A total of 406 (95.3%) candidates attempted this question. 330 (81.3%) candidates passed by scoring from 2 to 4 marks. Therefore, the general performance of candidates in this question was good. Figure 1 shows performance of the candidates.

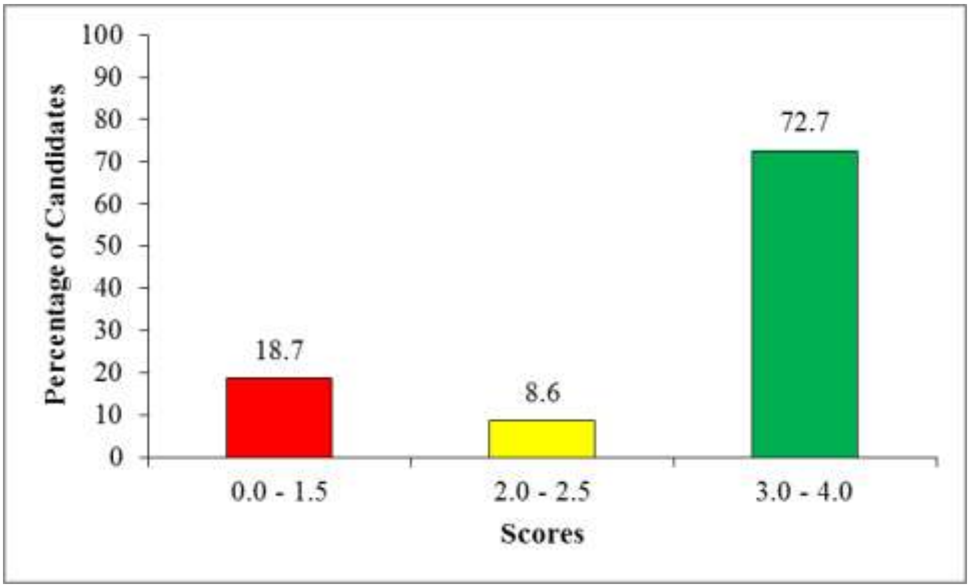
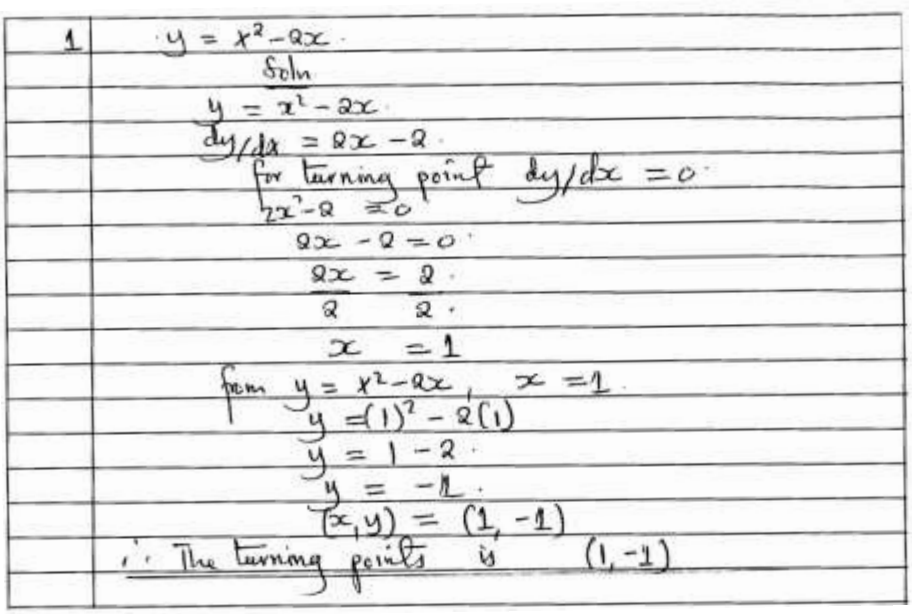


Figure 1: *The performance of candidates on question 1*

The data reveals further that 295 (72.7%) candidates scored from 3 to 4 marks, 35 (8.6%) candidates scored from 2 to 2.5 marks, and 76 (18.7%) candidates scored from 0 to 1.5 marks.

The candidates who scored full marks correctly applied the derivative method. They realized that the abscissa of the turning point of a curve is obtained at $\frac{dy}{dx} = 0$. Therefore, they determined the derivative of $y = x^2 - 2x$ and computed correctly the abscissa and the *y*-coordinate of the turning point, as shown in Extract 1.1.

Some candidates applied the formula for calculating the turning point of the quadratic function $y = ax^2 + bx + c$ which is $T(x, y) = \left(\frac{-b}{2a}, \frac{4ac - b^2}{4a} \right)$. These candidates replaced a, b and c in the formula with 2, -2 and 0 respectively and performed basic operations correctly to get $T(x, y) = (1, -1)$. There were also some candidates who used the graphical method to answer this question.



1 $y = x^2 - 2x$

Soln

$y = x^2 - 2x$

$dy/dx = 2x - 2$

for turning point $dy/dx = 0$

$2x - 2 = 0$

$2x - 2 = 0$

$\frac{2x}{2} = \frac{2}{2}$

$x = 1$

from $y = x^2 - 2x$, $x = 1$

$y = (1)^2 - 2(1)$

$y = 1 - 2$

$y = -1$

$(x, y) = (1, -1)$

$\therefore$ The turning points is $(1, -1)$

Extract 1.1: A sample of correct response to question 1.

On the other hand, a total of 76 (18.7%) candidates scored low marks. They failed to recall correctly the condition $\frac{dy}{dx} = 0$ that gives abscissa of the turning point. Also, there were candidates who used incorrect formula for finding the turning point of the quadratic equation $ax^2 + bx + c = 0$. The commonly observed incorrect formula was $T(x, y) = \left(\frac{-b}{2a}, \frac{4ac}{b^2} \right)$. Other candidates worked out to find *x*-intercepts. They assumed *y* = 0, hence developed an equation $x^2 - 2x = 0$ and solved it to get $x = 0$ or $x = 2$. Then they replaced *x* in $y = x^2 - 2x$ with 0 and 2 to get *y* = 0. Therefore, they wrote that the turning point is (0, 0) or (2, 0).

Other candidates established the value of the second derivative as *x*-coordinate and substituted it in the given curve to find the *y*-coordinate. See Extract 1.2.

1. $y = x^2 - 2x$.
required turning point.

$y = x^2 - 2x$

$dy/dx = 2x$.

$\frac{d^2y}{dx^2} = 2$.

$y = (2)^2 - 2 \times 2$

$= 4 - 4$

$= 0$

∴ Turning point will be $(2, 0)$.

Extract 1.2: A sample of incorrect response to question 1.

In Extract 1.2, the candidate computed incorrectly the abscissa by finding $\frac{d^2y}{dx}$ from $y = x^2 - 2x$ and resulted into an incorrect turning point.

2.1.2 Question 2: Coordinate Geometry II

This question assessed candidates' ability to derive an equation of a parabola. In this question, the candidates were required to find the focus and directrix of the parabola; $y^2 - 4y - 12x + 16 = 0$.

A total of 376 (83.3%) candidates attempted the question, whereby 230 (61.1%) candidates scored from 2 to 4 marks. Hence, the question was averagely performed. Figure 2 is a summary of candidates' performance in this question.

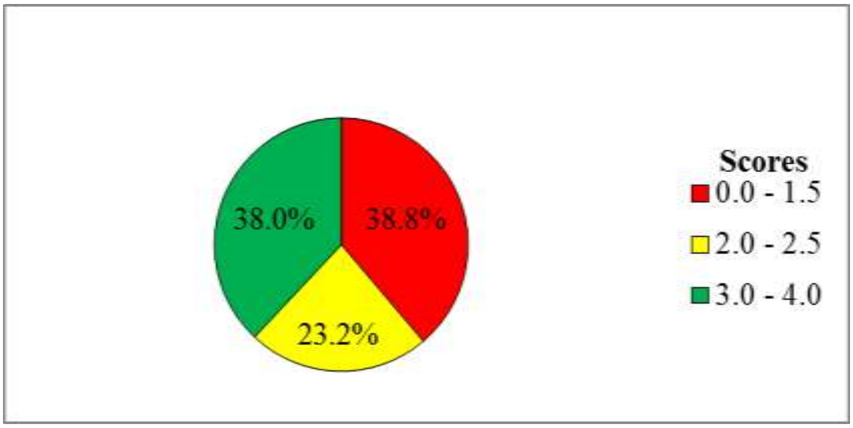


Figure 2: *The performance of candidates on question 2*

The analysis of data shows that, 143 (38.0%) candidates scored from 3 to 4 marks, 87 (23.1%) candidates scored from 2 to 2.5 marks and 146 (38.8%) candidates scored from 0 to 1.5.

The candidates who scored all 4 marks allotted to this question expressed correctly the given equation in standard form; $(y-2)^2 = 12(x-1)$. This indicates that they were competent on the concept of completing the square. Then, they compared to the general standard equation $(y-k)^2 = 4a(x-h)$ to get $a = 3, k = 2$ and $h = 1$. By applying correctly the formulae for Focus$=(a+h, k)$ and Directrix $x = -a+h,$ the candidates substituted correctly the values and computed to get the required answer. Extract 2.1 shows the situation.

2. $y^2 - 4y - 12x + 16 =$

$y^2 - 4y = 12x - 16$

$(y-2)^2 = 12x - 16 + 2^2$

$(y-2)^2 = 12x - 16 + 4$

$(y-2)^2 = 12x - 12$

$(y-2)^2 = 12(x-1)$

Compare with

$(y-k)^2 = 4a(x-h)$

$k = 2$

$h = 1$

$\frac{4a}{4} = \frac{12}{4}$

$a = 3$

$\therefore$ focus $(a,0) \xrightarrow{h,k} (a+h, k)$

$(3,0) = (3+1, 2)$

$= (4,2)$

Focus $= (4,2)$

Directix line $x = -a+h$

$\therefore x = -3+1$

$x = -2$

$\therefore$ Focus $= (4,2)$

Directrix line $x = -2$.

Extract 2.1: A sample of a correct response to question 2.

However, 146 (38.8%) candidates got low marks. It seemed that most of them had inadequate knowledge of completing the square as they failed to write the given equation in standard form $(y-k)^2 = 4a(x-h)$. As a result, they got incorrect values of *a*, *h* and *k*. which led to incorrect answers for focus and directrix. Extract 2.2 shows a response of a candidate who interchanged the components of the translating factor by writing (k,h) instead of (h,k).

2. $(y-2)^2 = 12\left(x-\frac{14}{12}\right)$

$(y-2)^2 = 12\left(x-\frac{7}{6}\right)$

then

Focus $= (a+k,\ 0)$

from

$(y-h)^2 = 4a(x-k)$

$\frac{4a}{4} = \frac{12}{4}$

$a = 3$

$k = \frac{7}{6}$

$\therefore$ Focus $= \left(3+\frac{7}{6},\ 2\right)$.

$= \left(4\frac{1}{6},\ 2\right)$

directrics $= -a$

$= -3$

$\therefore$ Directrics $= -3$

Extract 2.2: A sample of an incorrect response to question 2.

2.1.3 Question 3: Probability

This question examined candidates' ability to apply permutation to solve real life problems. Candidates were asked to find the number of arrangements that can be formed using the letters of the words (a) EQUATION and (b) TUMBAKU.

The question was attempted by 377 (88.5%) candidates whereby 228 (60.5%) scored from 2 to 4 marks. Therefore, the general performance of the candidates in this question was average. Figure 3 shows the percentage of candidates who scored low, average and high marks.

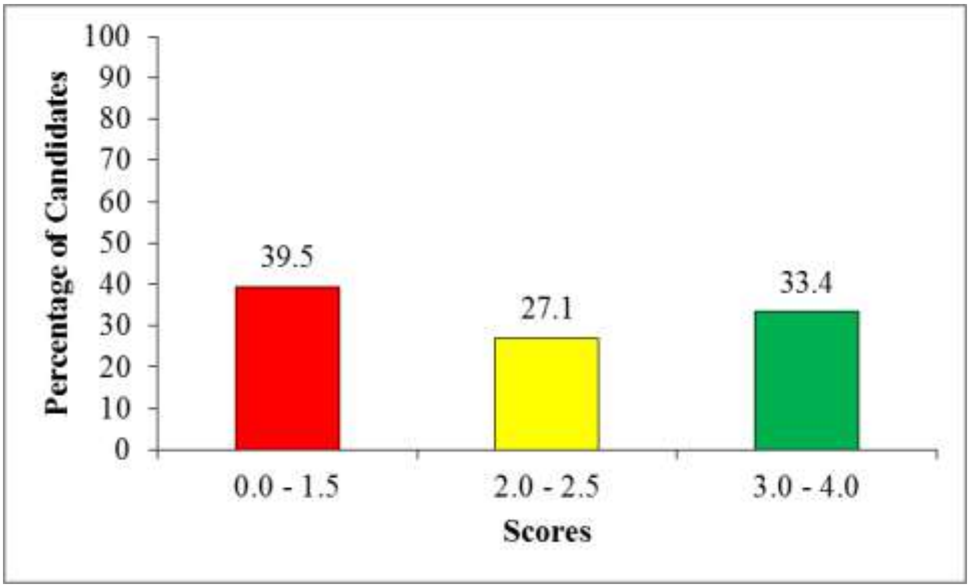
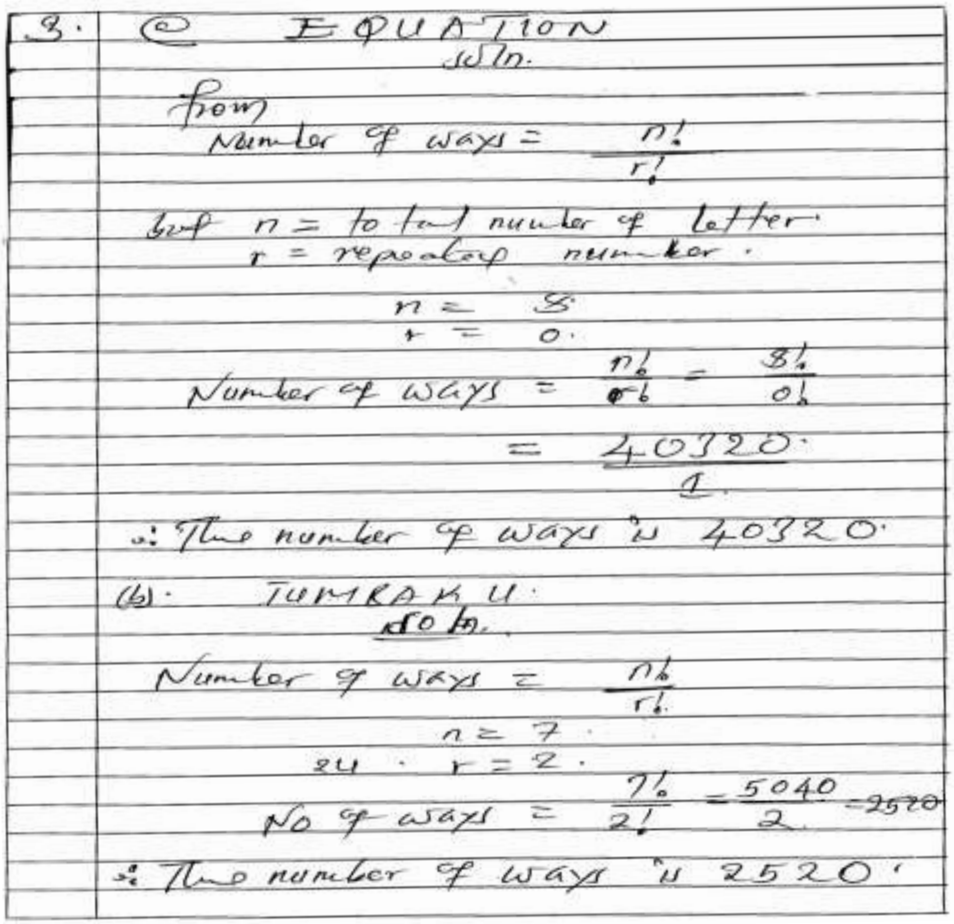


Figure 3: *The performance of candidates on question 3*

Further analysis shows that 126 (33.4%) candidates scored from 3 to 4 marks, 102 (27.1%) candidates scored from 2 to 2.5 marks and 149 (39.5%) candidates scored from 0 to 1.5 marks.

There were 111 (29.4%) candidates who answered the question correctly. These candidates realized that the word EQUATION contains eight (8) different letters. Therefore, they computed eight factorial (8!) correctly to get 40,320 arrangements. Similarly, the candidates identified that letter U in the word TUMBAKU is repeated. Therefore, they used the formula which is; $\text{number of arrangements} = \frac{n!}{r!}$ correctly and got a correct answer as shown in Extract 3.1.



3. (a) EQUATION

soln.

from

Number of ways $= \frac{n!}{r!}$

but n = total number of letter.

r = repeating number.

$n = 8$

$r = 0$.

Number of ways $= \frac{n!}{r!} = \frac{8!}{0!}$

$= \frac{40320}{1}$

∴ The number of ways is 40320.

(b). TUMRAKU.

soln.

Number of ways $= \frac{n!}{r!}$

$n = 7$

2u . $r = 2$.

No of ways $= \frac{7!}{2!} = \frac{5040}{2} = 2520$

∴ The number of ways is 2520.

Extract 3.1: A sample of a correct response to question 3.

On the other hand, some candidates got zero. Many candidates used inappropriate formula. In part (a) many candidates applied inappropriate formula like; ${}^{8}P_{0} = \frac{8!}{(8-0)!}$ to get 1 arrangement and ${}^{8}P_{1} = \frac{8!}{(8-1)!}$ to get 8 arrangements. Also, there were candidates who answered part (b) using the inappropriate formula $S = \frac{n!}{(n-r)!\text{r}!}$ as Extract 3.2 shows. This formula is for finding the number of selections and not arrangements.

3. (a) $^{8}C_{1} = 8$

8 words Can be Formed Using the letters of EQUATION.

Solution

(b) TUMBAKU.

$n = 7$

$r = 2$

$$^{n}C_{r} = \frac{n!}{(n-r)!r!}$$

$$^{7}C_{2} = \frac{7!}{(7-2)!2!}$$

$$^{7}C_{2} = \frac{7!}{5!2!}$$

$$^{7}C_{2} = \frac{5040}{120 \times 2}$$

$$^{7}C_{2} = \frac{5040}{240}$$

$$^{7}C_{2} = 21.$$

21 Word Can be Formed Using the letters of TUMBAKU.

Extract 3.2: A sample of an incorrect response to question 3.

In Extract 3.2, the candidate computed the number of combinations instead of permutations.

2.1.4 Question 4: Probability

This question aimed at assessing candidates' ability to apply Poisson Probability Distribution formula. The candidates were given the following problem: "Suppose the items processed on a certain machine are found to be 1% defective. Determine the probability of obtaining 4 defectives in a random sample batch of 80 such items".

A total of 280 (65.7%) candidates attempted this question, of which 3 (1.1%) candidates scored from 2 to 4 marks. Therefore, the general performance of candidates in this question was weak. Figure 4 gives a summary of candidates' performance in this question.

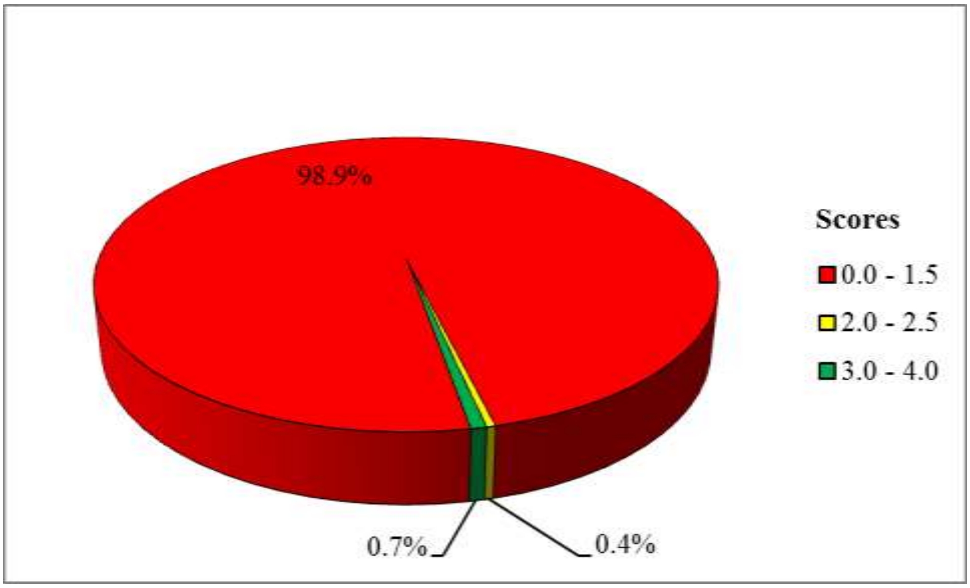


Figure 4: *The performance of candidates on question 4*

Although the question was compulsory, it was skipped by 146 (34.3%) candidates. Out of 280 candidates who attempted the question, 277 (98.9%) scored from 0 to 1.5 marks. Furthermore, 1 (0.4%) candidate scored 2 marks and 2 (0.7%) scored 4 marks.

Moreover, 277 (98.9%) candidates obtained low marks. Most of these applied inappropriate formulae. For instance, some candidates applied the formula for calculating the number of combinations as they wrote $^{80}C_4 = \frac{80!}{(80-4)!4!} = 1{,}581{,}580$, while other candidates computed $\frac{80!}{4!}$. Majority took 4 and 80 as number of event and 80 sample spaces respectively and applied inappropriate formulae $P(E) = \frac{n(E)}{n(S)}$ which resulted into an incorrect answer $\frac{1}{20}$, as shown in Extract 4.1. This indicates that the candidates failed to realise that the data are appropriate to Poison Distribution formula.

4. Give
item to be 1% defective.
4 defective? of 80 such items.
<u>solution</u>
from
number of sample space = 80
number of event = 4.
from the formula
$P(E) = \frac{n(E)\ \text{event}}{n(S)\ \text{sample space}}$

$P(E) = \frac{4}{80}$

$P(E) = \frac{1}{20}$.

∴ probability of obtaining 4 defectives fr'a random sample batch of 80 item is $\frac{1}{20}$

Extract 4.1: A sample of an incorrect response to question 4.

In Extract 4.1, the candidate ignored the given probability of an item being defective when answering a particular question.

Despite the weak performance, 2 (0.7%) candidates answered the question correctly. These candidates applied the Poisson Probability Distribution formula $P(x=r) = \frac{e^{-\mu}\mu^r}{r!}$ and computed to get correct answer as shown in Extract 4.2. They made a correct substitution of the given information that is $\mu = np$, where $n = 80$, $p = 1\% = 0.01$ to get the correct solution.

.

4 from: $P(X=x) = \frac{e^{-\lambda}\cdot\lambda^{x}}{x!}$

$n = 80$

$p = 0.01$

$x = 4$

from, $\lambda = np$

$\lambda = 80 \times 0.01$

$\lambda = 0.8$

$P(X=4) = \frac{e^{-0.8} \times (0.8)^{4}}{4!}$

$P(X=4) = 7.6685 \times 10^{-3}$

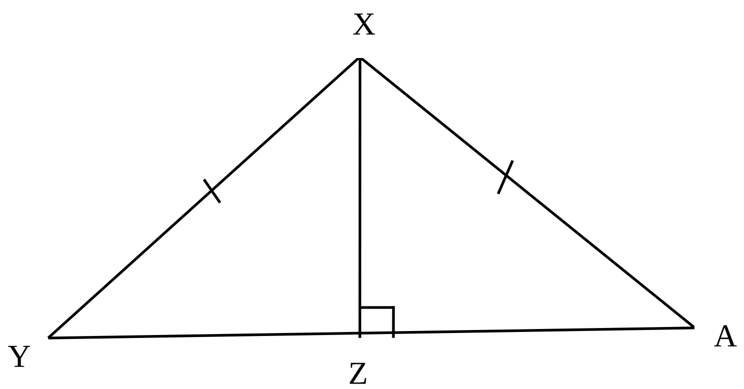
$\therefore$ Probability of obtaining 4 defective is 7.6685×10^{-3} or 0.7668%.

Extract 4.2: A sample of correct response to question 4.

In Extract 4.2, the candidate interpreted correctly all data and substituted them into the correct formula.

2.1.5 Question 5: Similarity and Congruence

This question assessed candidates' ability to use the congruence theorem and identify the common or given lines in the figure. They were given the following figure and were required to prove that $\triangle XYZ$ is congruent to $\triangle XAZ$.



A total of 417 (97.9%) out of 426 candidates attempted this question, 339 (81.3%) of the candidates passed by scoring from 2 to 4 marks. So, the general performance of candidates was good. Figure 5 shows the performance of candidates in this question.

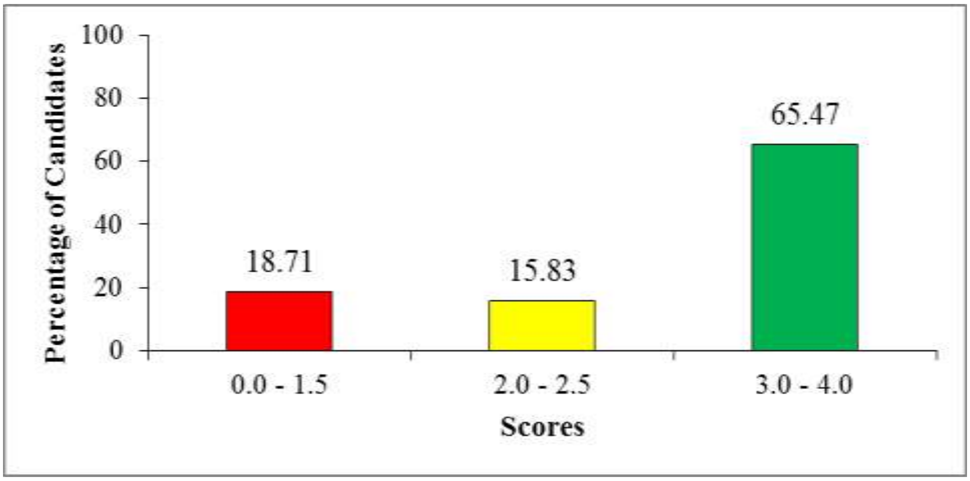
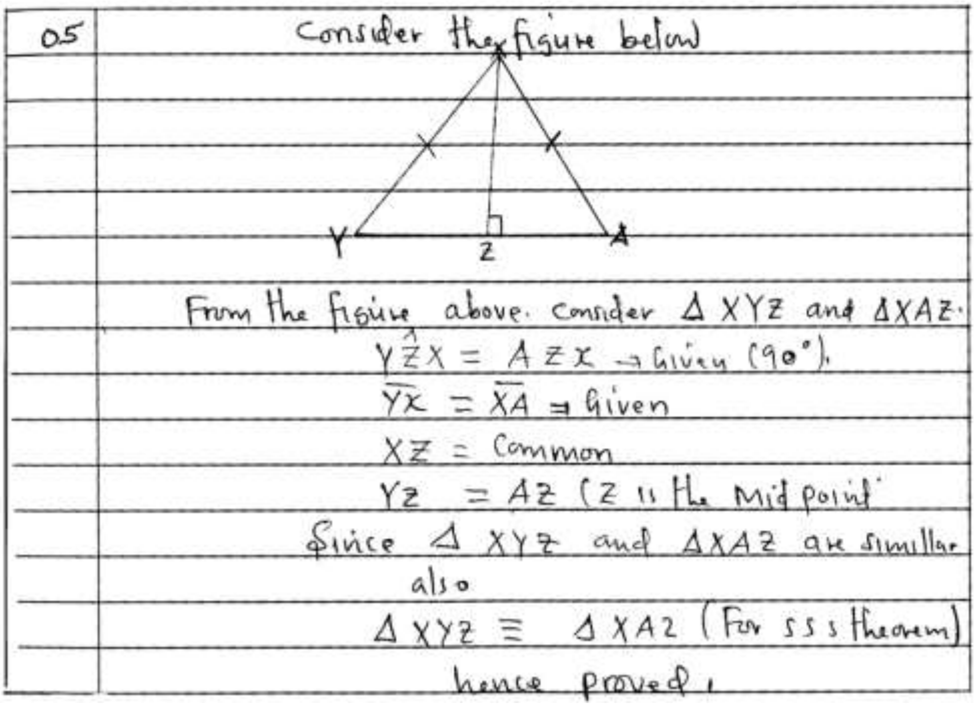


Figure 5: *The performance of candidates on question 5*

The analysis of data in this question indicates that 78 (18.7%) candidates scored from 0 to 1.5 marks, 66 (15.8%) scored from 2 to 2.5 marks while 273 (65.5%) candidates scored from 3 to 4 marks.

The candidates who correctly answered this question by scoring full marks applied the congruence theorems and identified the given conditions that helped them to prove the required circumstances as revealed in Extract 5.1.


05 Consider the figure below

From the figure above. consider Δ XYZ and ΔXAZ.

$Y\hat{Z}X = AZX$ → Given (90°).

$\overline{YX} = \overline{XA}$ → Given

XZ = Common

YZ = AZ (Z is the Mid point

Since Δ XYZ and ΔXAZ are similar

also

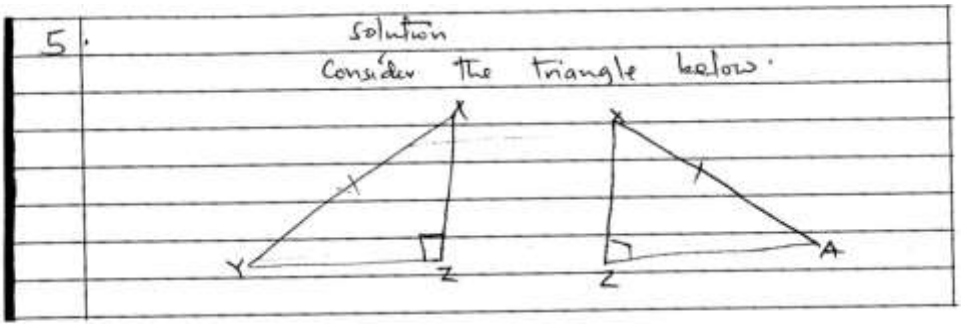
Δ XYZ ≡ Δ XAZ (For SSS theorem)

hence proved.

Extract 5.1: A sample of a correct response to question 5.

On the other hand, there were 78 (18.7%) candidates who scored low marks from 0 to 1.5. These candidates failed to remember and use properly the congruence theorem.

Some candidates drew a triangle without naming its edges and assumed it to be the final proof; others wrote the equations like $\overline{YX} = \overline{AX}$ and $XZY = XZA = 90^0$ without stating the reason. Also there were candidates who drew two separate triangles and assumed to have proved the condition as shown in Extract 5.2.



Extract 5.2: A sample of an incorrect response to question 5.

2.1.6 Question 6: Planning and Preparation for Teaching Mathematics

The question assessed candidates' ability to apply knowledge of preparation of a lesson plan. They were required to outline any four qualities of a well stated specific objective in Mathematics lesson plan.

A total of 423 (99.3%) candidates attempted this question. 331 (78.3%) candidates scored from 2 to 4 marks. Hence, the general performance of candidates in this question was good. Figure 6 shows percentage of candidates in this question.

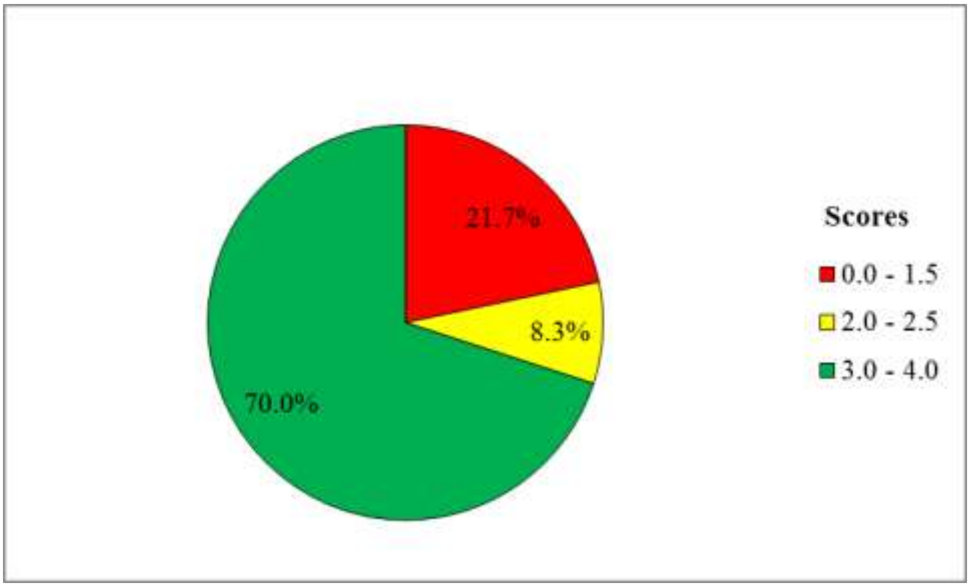


Figure 6: *The performance of candidates on question 6*

The analysis of data shows that, 92 (21.7%) of the candidates scored from 0 to 1.5 marks, 35 (8.3%) scored from 2 to 2.5 marks and 296 (70.0%) scored 3 to 4 marks.

The candidates who managed to get the correct answer had knowledge about the qualities of a well stated specific objective in a lesson plan. Extract 6.1 shows the response of a candidate.

06. (i) Audience. - It has started to whom the objective be delivered, i.e students.

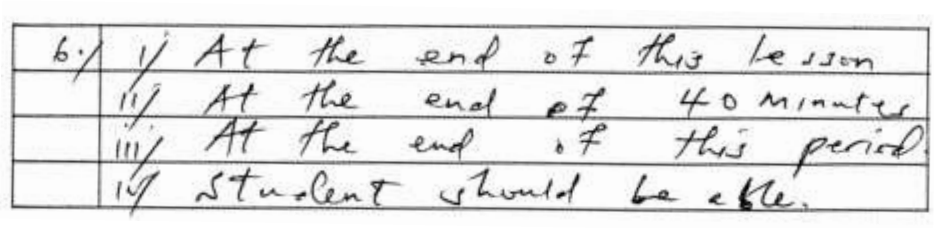
(ii) Time bound. - It indicates the time for completion of specific objective, i.e 40/80 minutes.

(iii) Degree of perfomance.

iv) Behavioural change - It indicate the behavioural change by using action verbs such as Mention, List, etc.

Extract 6.1: A sample of correct response to question 6.

On the other hand, the candidates who failed to respond correctly to this question lacked knowledge about the requirement of the question see Extract 6.2. Candidates in this group defined lesson plan and concluded while, others mentioned parts of a lesson plan.



6.	i/ At the end of this lesson
	ii/ At the end of 40 minutes
	iii/ At the end of this period
	iv/ Student should be able.

Extract 6.2: A sample of an incorrect response to question 6.

2.1.7 Question 7: Integration

This question was intended to examine candidates' ability to evaluate the integrals of the hyperbolic function. The candidates were required to evaluate $\int \sinh^3\theta \, d\theta$.

A total of 364 (85.4%) candidates attempted this question, whereby 191 (52.4%) candidates passed by scoring from 2 to 4 marks. This means that, the general performance of candidates in this question was average. Figure 7 displays the performance of the candidates in question 7.

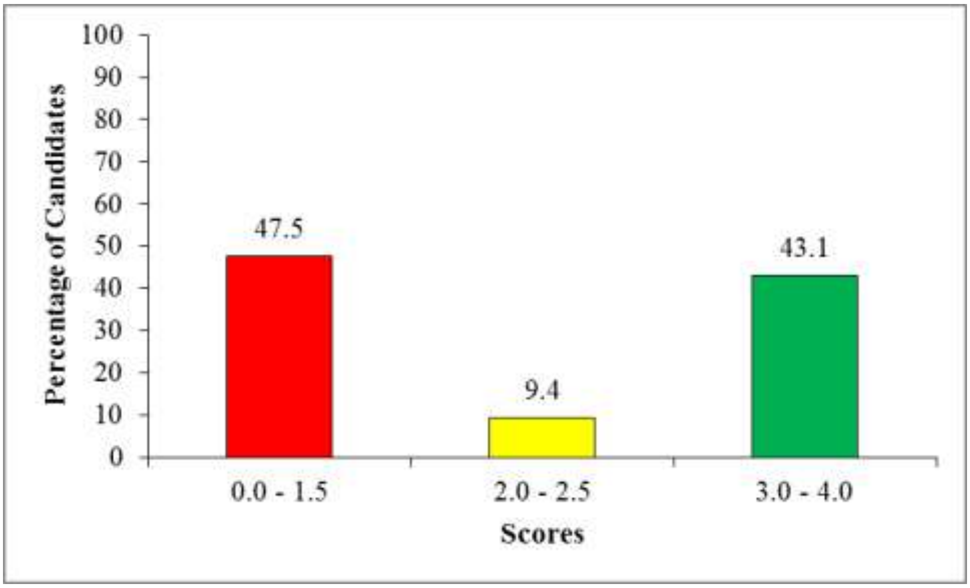
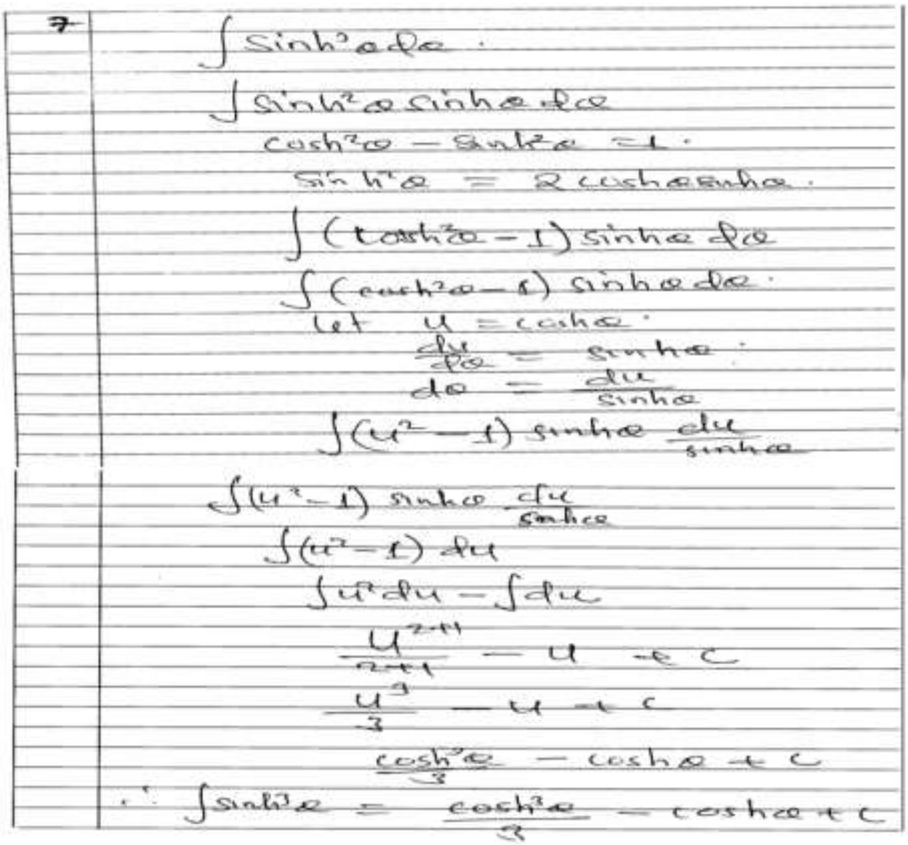


Figure 7: *The performance of candidates on question 7*

The analysis of data shows that 173 (47.5%) candidates scored from 0 to 1.5 marks, 34 (9.3%) candidates scored from 2 to 2.5 marks and 157 (43.1%) candidates scored from 3 to 4 marks.

The candidates who answered the question correctly expressed $\sinh^3\theta$ as $\sinh^2\theta\sinh\theta$. Then, they applied identity $\cosh^2\theta - \sinh^2\theta = 1$ to express $\sinh^2\theta$ as $\cosh^2\theta - 1$. Therefore, they wrote $\int \sinh^3\theta\,\mathrm{d}\theta$ as $\int \cosh^2\theta\sinh\theta\,\mathrm{d}\theta - \int \sinh\theta d\theta$. Under this form, the candidates applied the standard integral for $\int \sinh\theta\,\mathrm{d}\theta$ and the technique of function and its derivative for $\int \cosh^2\theta\sinh\theta\,\mathrm{d}\theta$; and resulted to the required integral $\frac{1}{3}\cosh^3\theta - \cosh\theta + c$. Extract 7.1 shows one of the candidate's correct responses in question 7.



7

$\int \sinh^3\alpha\, d\alpha$.

$\int \sinh^2\alpha \sinh\alpha\, d\alpha$

$\cosh^2\alpha - \sinh^2\alpha = 1$.

$\sinh^2\alpha = 2\cosh\alpha\sinh\alpha$.

$\int (\cosh^2\alpha - 1)\sinh\alpha\, d\alpha$

$\int (\cosh^2\alpha - 1)\sinh\alpha\, d\alpha$.

let $u = \cosh\alpha$.

$\frac{du}{d\alpha} = \sinh\alpha$.

$d\alpha = \frac{du}{\sinh\alpha}$

$\int (u^2 - 1)\sinh\alpha \frac{du}{\sinh\alpha}$

$\int (u^2 - 1)\sinh\alpha \frac{du}{\sinh\alpha}$

$\int (u^2 - 1)\, du$

$\int u^2 du - \int du$

$\frac{u^{2+1}}{2+1} - u + c$

$\frac{u^3}{3} - u + c$

$\frac{\cosh^3\alpha}{3} - \cosh\alpha + c$

$\therefore \int \sinh^3\alpha = \frac{\cosh^3\alpha}{3} - \cosh\alpha + c$

Extract 7.1: A sample of correct response to question 7.

On the other hand, 173 (47.5%) candidates got low marks. Some wrote $\sinh^3\theta = \frac{1}{4}(\sinh 3\theta - 3\sinh\theta)$. Many candidates used incorrect identity $\cosh^2\theta + \sinh^2\theta = 1$ instead of $\cosh^2\theta - \sinh^2\theta = 1$. As a result, they ended up with incorrect expression $\int (1-\cosh^2\theta)\sinh\theta d\theta$ instead of $\int (\cosh^2\theta - 1)\sinh\theta d\theta$. Also, there were candidates who applied definition of $\sinh\theta$. Most of these candidates failed to work out the exponential expression produced because the approach involved tedious work on exponents. This indicates that they lacked knowledge of exponents.

Other candidates straggled to express $\sinh^3\theta$ in terms of $\sinh 3\theta$ however, they failed to recall the correct triple angle formula. Most of them wrote $\sinh 3\theta = 4\sinh^3\theta - 3\sinh\theta$ instead of $\sinh 3\theta = 4\sinh^3\theta + 3\sinh\theta$. Moreover, few candidates changed the variable by letting $\mathrm{u} = \sinh\theta$. Such candidates ended up with an expression containing both u and $\cosh\theta$ including $\int \frac{u^3}{\cos\theta} du$. So they got a complicated integral instead of solving it.

7. $\int \sinh^3\theta \, d\theta$.

let $u = \sinh\theta$.

$\frac{du}{d\theta} = \cosh\theta$.

$d\theta = \frac{du}{\cosh\theta}$.

$d\theta = \frac{du}{\cosh\theta}$.

$= \int u^3 \, d\theta$.

$= \int u^3 \frac{du}{\cosh\theta}$.

$= \frac{1}{\cosh\theta} \int u^3 \, du$.

$\frac{1}{\cosh\theta} \cdot \frac{1}{4} u^4$.

but $u = \sinh\theta$.

$= \frac{1}{4\cosh\theta} \cdot \sinh\theta^4$.

$= \frac{1 \sinh\theta^4}{4\cosh\theta} = \frac{1}{4} \tanh\theta \sinh^3\theta$.

$\therefore \int \sinh^3\theta \, d\theta = \frac{1}{4} \tanh\theta \sinh^3\theta$.

Extract 7.2: A sample of an incorrect response to question 7.

In Extract 7.2, the candidate assumed $\sinh\theta$ is equal to u and substituted into the integral to become $\int \sinh^3\theta d\theta = \int u^3 d\theta$ which cannot be integrated.

2.1.8 Question 8: Coordinate Geometry II

This question assessed candidates' knowledge about the application of the general formula for ellipse. The candidates were required to find the equation of an ellipse with foci $(\pm 1, 0)$ and directrices $x = \pm 4$.

The question was attempted by 348 (81.7%) candidates. 218 (62.6%) candidates scored from 0 to 1.5 marks. Hence, the general performance in this question was weak. Figure 8 shows percentage of candidates who got low, average and high marks.

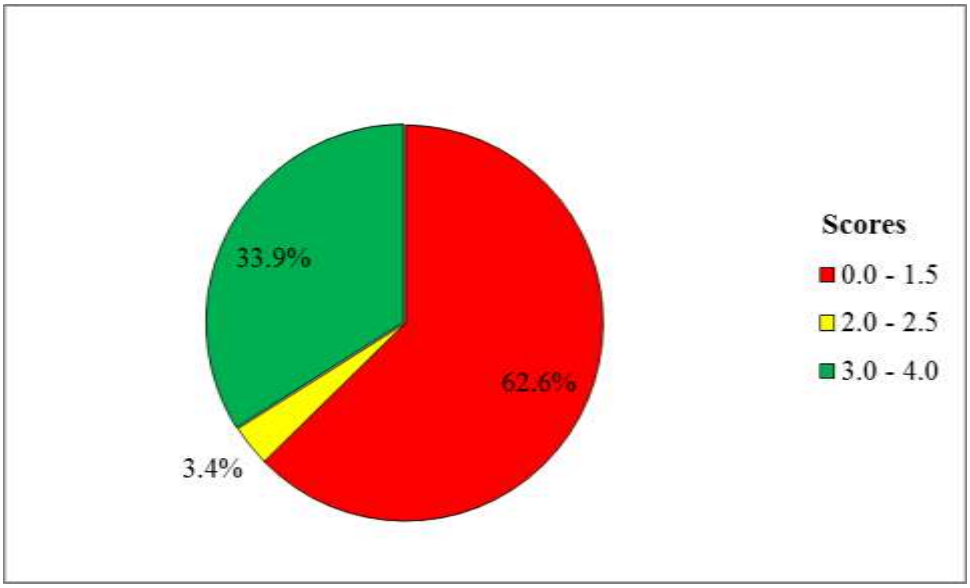


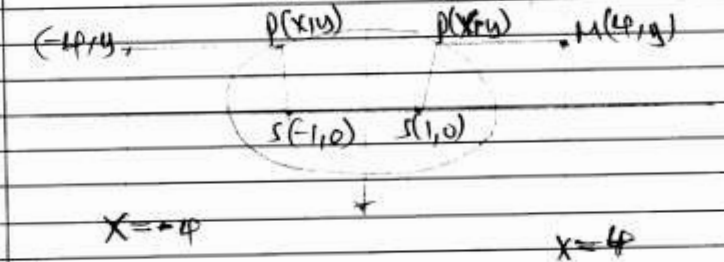
Figure 8: *The performance of the candidates on question 8*

The analysis shows that, 218 (62.6%) candidates scored from 0 to 1.5 marks, 12 (3.4%) scored from 2 to 2.5 marks and 118 (33.9%) scored from 3 to 4 marks. The question was skipped by 78 (18.3%) candidates.

Out of 218 (62.6%) candidates who scored between 0 and 1.5 marks in this question, 159 (45.7%) candidates scored zero. This failure was due to inability to remember and use the general formula of the ellipse, foci and directrices. There were candidates who drew the ellipse and wrote the equation of a circle $x^2 + y^2 = 0$. These candidates failed to know the difference between the ellipse and a circle.

Others were writing the general equation $a^2 + e^2 = 2ae$. They remembered letters used when leaning the ellipse but failed to recall the general formula used. Also, some of the candidates drew the ellipse, indicating the foci and found the required equation by applying the distance formula as indicated in extract 8.1.

8. Required Equation of the Ellipse.
given foci $(\pm 1, 0)$ and directrix $x = \pm 4$.
Consider figure below



From
$SP = ePM$
From
distance formular
$d =$

$\sqrt{(x-1)^2+(y-0)^2} = e\sqrt{(x-4)^2+(y-y)^2}$

$(x-1)^2+y^2 = e^2(x-4)^2+0$

$(x-1)^2+y^2 = e^2(x-4)^2$

$x^2-2x+1+y^2 = e^2(x^2-8x+16)$

$x^2-2x+1+y^2 = e^2x^2-8xe^2+16e^2$

$(x-1)^2+y^2 = e^2(x-4)^2$

$\frac{(x^2-1)^2}{e^2(x-4)^2}+\frac{y^2}{e^2(x-4)^2} = \frac{e^2(x-4)^2}{e^2(x-4)^2}$

let $e^2(x-4)^2 = b^2$

$\therefore \frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$ as required.

Extract 8.1: A sample of an incorrect response to question 8.

However, there were 117 (33.6%) candidates who managed to get the correct answer; these were able to remember and use the general formula for the ellipse, foci and directrices. They managed to show that the general formula for the ellipse is given by $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, whereby the foci is defined at point $(\pm ae, 0)$ and the directrices is given by the equation $x = \pm\frac{a}{e}$. From this information, the candidates were able to compute and get the correct answer, as shown in extract 8.2.

8. foci $(\pm 1, 0)$ directrices $x = \pm 4$.

Soln.

ellipse eqn:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

foci $(\pm ae, 0)$.

directrix $x = \pm a/e$.

$$a/e = \pm 4. \text{ ——— (i)}$$

$$f(\pm 1, 0) = (\pm ae, 0) \text{ ——— (ii)}$$

$$\pm 1 = \pm ae$$

$$ae = 1.$$

$$a = 1/e. \text{ ——— (iii)}$$

$$a/e = \pm 4.$$

$$\frac{1}{e} \times \frac{1}{e} = \pm 4.$$

$$\frac{1}{e^2} = \frac{4}{1}$$

$$1 = 4e^2$$

$$\sqrt{1/4} = \sqrt{e^2}$$

$$\pm\frac{1}{2} = e.$$

eccentricity $= 1/2$

8. but

$$a = \frac{1}{e}$$

$$a = \frac{1}{\frac{1}{2}} = 2$$

$$\underline{\underline{a = 2}}, \quad a^2 = 4$$

but from

$$a^2(1-e^2) = b^2.$$

$$2^2\left(1-\left(\tfrac{1}{2}\right)^2\right) = b^2$$

$$4\left(1-\tfrac{1}{4}\right) = b^2$$

$$4\left(\frac{4-1}{4}\right) = b^2$$

$$\frac{4}{4}(3) = b^2$$

$$b^2 = 3.$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\therefore \underline{\underline{\frac{x^2}{4} + \frac{y^2}{3} = 1}} \quad \text{answer}$$

$\therefore$ The equation of an ellipse is $x^2/4 + y^2/3 = 1$

Extract 8.2: A sample of a correct response to question 8.

2.1.9 Question 9: Analysis of Mathematics Curriculum Materials

This question assessed knowledge of professional curriculum materials. The candidates were required to define the following terms as used in Mathematics lesson:

(a) Mathematics logbook.

(b) Lesson plan.

(c) Scheme of work.

A total of 426 (100%) candidates attempted this question. There were 418 (98.1%) candidates who scored from 2 to 4 marks, indicating good performance. Figure 9 is a summary of candidates' performance in this question.

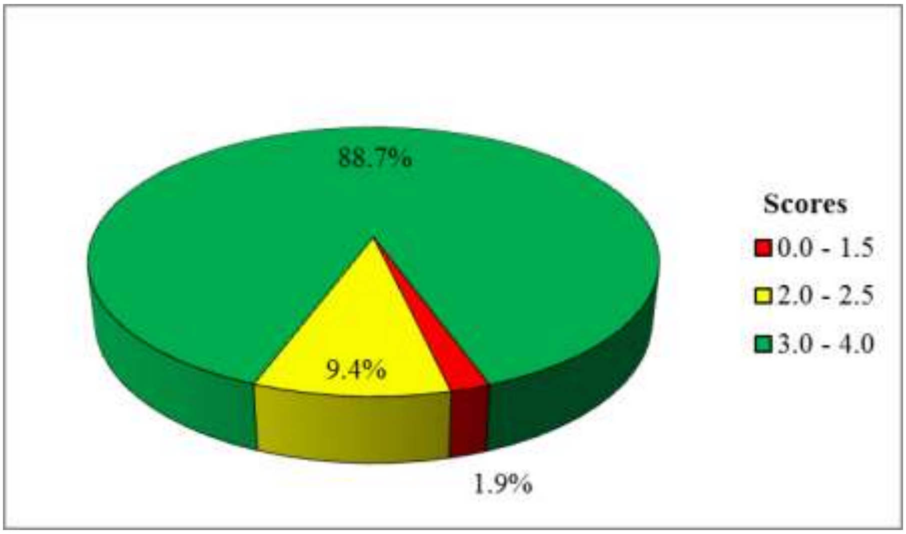


Figure 9: *The performance of the candidates on question 9*

There were 8 (1.9%) candidates who scored from 0 to 1.5 marks, 40 (9.4%) who scored from 2 to 2.5 marks and 378 (88.7%) candidates who scored from 3 to 4 marks.

Most of the candidates managed to answer this question correctly because the terms that were given to define are applied in their day to day activities at the college. Extract 9.1 is a sample answer of one of the candidates.

9. a) Mathematics logbook - Is the process which used by teacher Subjects after Complete the topic Such as General Information, Topic and Subtopics, Starting date, finishing date and Signiture of teacher and Academic.

b) lesson plan Is the plan of teaching used to help teacher in preparing the lesson and lesson notes in order to be Competent teacher Such as lesson development, Competence, Specific objective, Teaching and learning materials

c) Scheme of work - Is the planning of a teacher prepare from Syllybus in order to Know what it is tought used through teaching and learning process.

Extract 9.1: A sample of a correct response to question 9.

On the other hand, 8 (1.9%) candidates failed to get it correctly due to inability to define correctly the given terms. Extract 9.2 is a sample of a response from a candidate who failed to provide the proper definitions of the three terms.

9. a) Mathematic log book.
⇒ Is the book which used by teacher after the end of the topic at the specific subject to be filled what has been taught.

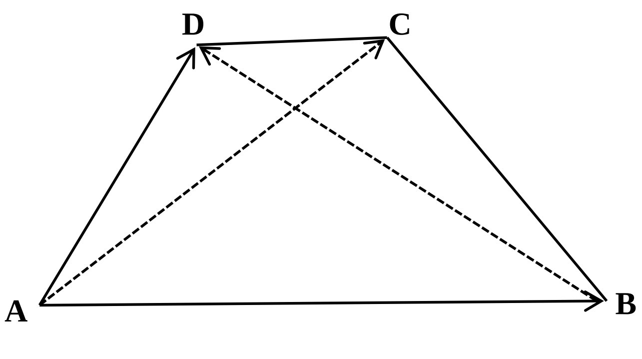
b) Lesson plan.
⇒ Is an instructional action plan.

c) Scheme of work.

Extract 9.2: A sample of an incorrect response to question 9.

2.1.10 Question 10: Vectors

This question examined candidates' ability to apply the cross product rule in vectors to determine the area of the quadrilateral. The question required the candidates to prove that the vector area of a quadrilateral *ABCD* with diagonals $\overline{AC}$ and $\overline{BD}$ is given by $\frac{1}{2}\left|\overline{AC} \times \overline{BD}\right|$ from the following figure;



The question was attempted by 307 (72.1%) candidates. The general performance of the candidates in this question was weak, because there were only 5 (1.6%) candidates who scored 2 marks. Figure 10 shows the performance of candidates in this question.

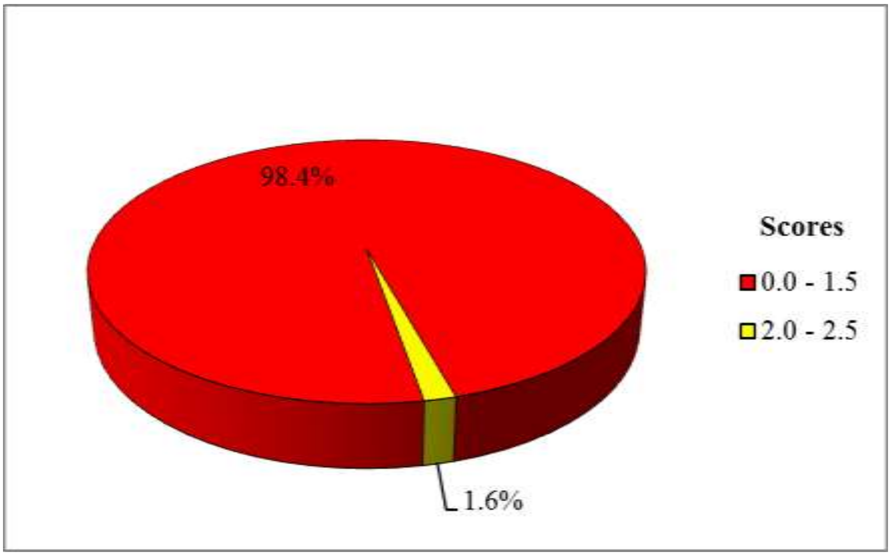


Figure 10*: The candidates' performance on question 10*

The analysis of data shows that 302 (98.4%) candidates scored from 0 to 1 mark and 5 (1.6%) candidates scored 2 marks. There was no candidate who scored from 2.5 to 4 marks in the entire group.

Most of these candidates failed to apply the cross product rule as used in vectors. They were supposed to use the formula;
$ABCD$ = (vector area of $\triangle ABC$) + (vector area of $\triangle ACD$), then apply the cross product rule to get; Area of $ABCD = \frac{1}{2}(\overrightarrow{AB} \times \overrightarrow{AC}) + \frac{1}{2}(\overrightarrow{AC} \times \overrightarrow{AD})$.
Some of the candidates wrote $A = \frac{1}{2}\overline{AC} \times \overline{BD} \sin\theta$ and then directly got $A = \frac{1}{2}|\overline{AC}|\,|BD| \sin\theta$.

There were also some candidates who used wrong formula $\text{Area} = \frac{1}{2} \times \text{base} \times \text{height}$ and assumed the base to be $|\overline{AD}|$ so that the required area is $A = \frac{1}{2}|AD| \times h$ where h is the height. Other candidates applied inappropriate knowledge of determinant by writing; $\text{Area} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$, as shown in Extract 10.1.

Recall.

10 Resultant Area $= \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_1 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

$$= \frac{1}{2}\, x_1\begin{vmatrix} y_2 & 1 \\ y_3 & 1 \end{vmatrix} - x_2\begin{vmatrix} y_1 & 1 \\ y_3 & 1 \end{vmatrix} + x_3\begin{vmatrix} y_1 & 1 \\ y_2 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\, x_1(y_2 - y_3) - x_2(y_1 - y_3) + x_3(y_1 - y_2)$$

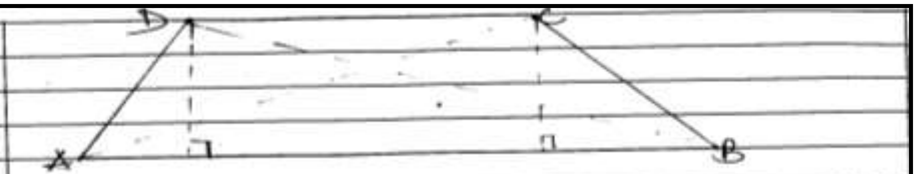
$$= \frac{1}{2}\left|\overrightarrow{AC} \times \overrightarrow{BD}\right|$$

hence proved

Extract 10.1: A sample of an incorrect response to question 10.

Meanwhile, there were 5 (1.6%) candidates who used the correct formula, which is; Area of $ABCD$ = (vector area of $\triangle ABC$) + (vector area of $\triangle ACD$) and manipulated to get $\text{Area} = \frac{1}{2}(\overrightarrow{AB} \times \overrightarrow{AC}) + \frac{1}{2}(\overrightarrow{AC} \times \overrightarrow{AD})$. However, they skipped some necessary steps. Therefore, they lost some marks. Extract 10.2 shows a sample of response of one of these candidates.

10.



Required to prove: Vector area of ABCD $= \frac{1}{2}(\overline{AC} \times \overline{BD})$

Proof: Area of ABCD = Area ABC + Area ACB = area ABD + Area ADC (Areas of triangle)

Area ABCD = Area ABC + Area ABD =

$= \left(\frac{1}{2} \times \overline{BC} \times \overline{AC}\right) + \frac{1}{2} \times \overline{AD} \times \overline{BD}$ (Area of triangle)

$= \frac{1}{2}\overline{BC} \times \overline{AC} + \frac{1}{2}\overline{AD} \times \overline{BD} =$

Since $\overline{BC} = \overline{AD}$ (Given)

Area ABCD $= \frac{1}{2}(\overline{AC} \times \overline{BD})$

Extract 10.2: A sample of response of average performance to question 10.

2.2 Section B: Essay Questions on Academic Content

2.2.1 Question 11: Algebra

The question assessed candidates' knowledge on application of sum of roots and product of roots in the problems involving roots of polynomial functions. The question had parts (a) and (b). The candidates were given that; "(a) The roots of a polynomial equation $2x^3 - 5x^2 + 7x - 8 = 0$ are α, β and γ". Then candidates were required to find the equation whose roots are: (i) $\frac{1}{\alpha\beta}$, $\frac{1}{\alpha\gamma}$ and $\frac{1}{\beta\gamma}$ and (ii) $\alpha - 1$, $\beta - 1$ and γ-1. (b) "The roots of the equation $x^2 + 2px + q = 0$ differ by 2", show that $p^2 = 1 + q$.

The question was attempted by 359 (84.3%) candidates, 178 (49.6%) candidates passed by scoring from 6 to 15 marks. Hence the general performance was average. Figure 11 shows the performance of candidates in this question.

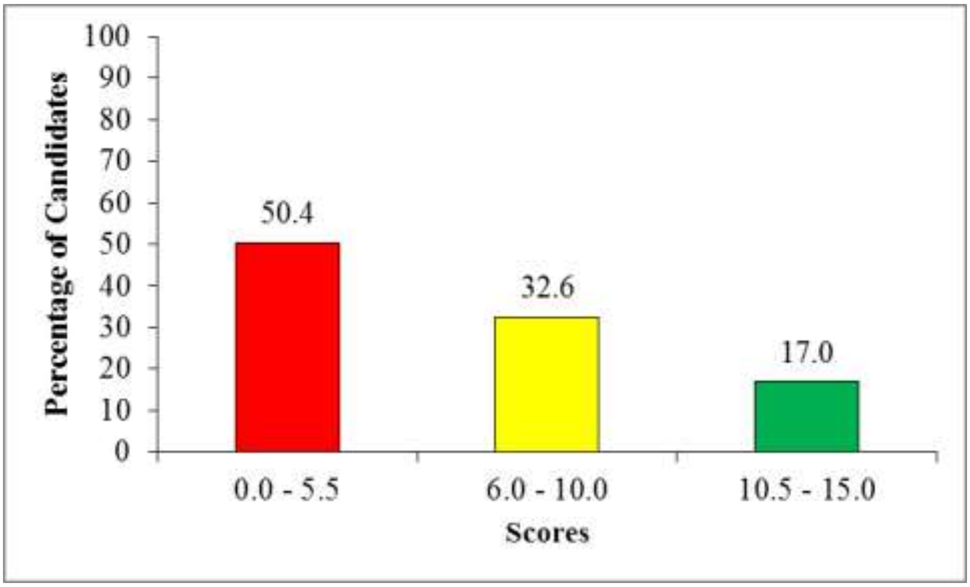


Figure 11*: The general performance of candidates on question 11*

The analysis of data shows that 50.4% of the candidates scored from 0 to 5.5 marks, 32.6% scored from 6 to 10 marks and 17.0% of the candidates scored from 10.5 to 15 marks.

In part (a) (i), the candidates were knowledgeable on how cubic equation is formed from its roots, that is;

$x^3 - (\text{sum of roots})\, x^2 + (\text{sum of products of pairs of roots})\, x - (\text{product of roots}) = 0$. These candidates realized that α, β and γ being roots of $2x^3 - 5x^2 + 7x - 8 = 0$, therefore, $\alpha + \beta + \gamma = \frac{5}{2}$, $\alpha\beta + \alpha\gamma + \beta\gamma = \frac{7}{2}$ and $\alpha\beta\gamma = 4$. Also, these candidates recognized that the equation whose roots are $\frac{1}{\alpha\beta}$, $\frac{1}{\alpha\gamma}$ and $\frac{1}{\beta\gamma}$ could be simplified to get $x^3 - \left(\frac{\alpha + \beta + \gamma}{\alpha\beta\gamma}\right)x^2 + \left(\frac{\alpha\beta + \alpha\gamma + \beta\gamma}{(\alpha\beta\gamma)^2}\right)x - \left(\frac{1}{(\alpha\beta\gamma)^2}\right) = 0$. Thereafter, the candidates performed appropriate substitutions and simplifications to get $x^3 - \frac{5}{8}x^2 + \frac{7}{32}x - \frac{1}{16} = 0$. These candidates also used the same knowledge and skills to answer part (a) (ii), as Extract 11.1 shows. Similarly, in part (b), candidates were knowledgeable on how the quadratic equation could be formulated using its roots, that is, $x^2 - (\text{sum of roots})x^2 + (\text{product of roots}) = 0$. These candidates formed the equation by describing two roots which differ by 2 and applied knowledge of sum and product of roots to verify that $p^2 = 1 + q$ as shown in Extract 11.1.

1(a) (ii) Required equation whose roots are:-

$\alpha-1, \quad \beta-1, \quad \gamma-1.$

$\Rightarrow$ Sum of the roots $= (\alpha-1)+(\beta-1)+(\gamma-1)$.

$= (\alpha+\beta+\gamma)-3$.

But: $\alpha+\beta+\gamma = {-b}/{a}$

$= -(-5/2)$

$= 5/2$.

$= 5/2 - 3 = -1/2$.

$\Rightarrow$ Sum of product of roots.

$= (\alpha-1)(\beta-1)+(\alpha-1)(\gamma-1)+(\beta-1)(\gamma-1)$.

$= (\alpha\beta-\alpha-\beta+1)+(\alpha\gamma-\alpha-\gamma+1)+(\beta\gamma-\beta-\gamma+1)$

$= \alpha\beta-\alpha-\beta+1+\alpha\gamma-\alpha-\gamma+1+\beta\gamma-\beta-\gamma+1$.

$= \alpha\beta+\alpha\gamma+\beta\gamma-2\alpha-2\beta-2\gamma+3$

$= (\alpha\beta+\alpha\gamma+\beta\gamma)-2(\alpha+\beta+\gamma)+3$.

But $(\alpha\beta+\alpha\gamma+\beta\gamma) = c/a = 7/2$

$(\alpha+\beta+\gamma) = -b/a = -(-5/2) = 5/2$

$\therefore$ Sum of product of roots $= 7/2 - 2(5/2)+3 = 3/2$

$\Rightarrow$ Product of roots: $(\alpha-1)(\beta-1)(\gamma-1)$

$= (\alpha\beta-\alpha-\beta+1)(\gamma-1)$

$= \alpha\beta\gamma-\alpha\beta-\alpha\gamma+\alpha-\beta\gamma+\beta+\gamma-1$

$= (\alpha\beta\gamma)-(\alpha\beta+\alpha\gamma+\beta\gamma)+(\alpha+\beta+\gamma)-1$

$= (-d/a)-(c/a)+(-b/a)+(-1)$.

$= -(-4)-(7/2)+(5/2)-1$

$= 4 - 7/2 + 5/2 - 1$

$= 2$

Therefore equation $\rightarrow$

11(a) (ii) $x^3 - (\text{sum of roots})x^2 + (\text{sum of product of roots})x - (\text{product of roots}) = 0$

$x^3 - (-\frac{1}{2})x^2 + (\frac{3}{2})x - 2 = 0$

$x^3 + \frac{1}{2}x^2 + \frac{3}{2}x - 2 = 0$

$2x^3 + x^2 + 3x - 4 = 0$

$\therefore 2x^3 + x^2 + 3x - 4 = 0.$

(b). Given: $x^2 + 2px + q = 0$ --- (i)

let α and β be roots of equation (i)

$ax^2 + bx + c = 0$ --- (ii)

Compare (i) and (ii)

$a = 1$, $b = 2p$, $c = q$.

Since $x^2 - (\text{sum of roots})x + (\text{product of roots}) = 0$

$x^2 - (b/a)x + (c/a) = 0$ is the standard equation.

Sum of roots $= (\alpha + \beta) = b/a$ and product $\alpha\beta = c/a$

But $\alpha - \beta = 2$

square both sides

$\alpha^2 + \beta^2 - 2\alpha\beta = 2.$

where

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta.$

then: $(\alpha^2 + \beta)^2 - 2\alpha\beta - 2\alpha\beta = 2$

(b). $(\alpha + \beta)^2 - 4\alpha\beta = 2.$

$(\frac{2p}{1})^2 - 4q = 2$

$4p^2 - 4q = 2.$

$2p^2 - 2q = 1.$

$2p^2 = 2 + 2q.$

$\frac{2p^2}{2} = \frac{2}{2} + \frac{2q}{2}.$

$p^2 = 1 + q.$ hence shown.

Extract 11.1: A sample of a correct response to question 11.

On the other hand, the 181 (50.4%) candidates scored from 0 to 5.5 marks. These candidates had inadequate knowledge about the application of the general formula for roots of polynomial functions.

In part (a), the challenge was on how to express the coefficients of intended sum and product of α, β and γ. This resulted from failure of candidates to use knowledge of factors and multiples. Some candidates failed to write the given equation in the standard form before doing comparison. Other candidates failed to multiply three factors of part (a) (ii). In part (b), many candidates failed to formulate an equation from statement “roots differ by 2”. As a result, they failed to produce correct equivalent equation containing sum and product of α and β that could allow them to make substitution of p and q for verification as shown in Extract 11.2.

11. (a) Given that $2x^3 - 5x^2 + 7x - 8 = 0$

Roots α, β and γ.

$(x-\alpha)(x-\beta)(x-\gamma) = 0$

$(x^2 - x\beta - \cancel{x\alpha} + \alpha\beta)(x-\gamma) = 0$

$x^3 - x^2\gamma - x^2\beta + x\beta\gamma - \alpha\beta x + \alpha\beta\gamma - \alpha\beta x - \alpha\beta\gamma = 0$

$\cancel{x^3 - (\gamma+\beta)}$

$x^3 - x^2\gamma - x^2\beta + x\beta\gamma - x^2\alpha + x\alpha\gamma + x\alpha\beta - \alpha\beta\gamma = 0$

$x^3 - (\gamma+\beta+\alpha)x^2 + (\beta\gamma+\alpha\gamma+\alpha\beta)x - \alpha\beta\gamma = 0$

This equation relates with

$x^3 - 5x^2 + 7x - 8 = 0$

$(\gamma+\beta+\alpha)x^2 = 5x^2$

$\gamma+\beta+\alpha = 5$ —— (i)

$\beta\gamma+\alpha\gamma+\alpha\beta = 7$ —— (ii)

$\alpha\beta\gamma = +8$ —— (iii)

Extract 11.2: A sample of an incorrect response to question 11.

2.2.2 Question 12: Linear Programming

This question assessed candidates' ability to solve the linear programming word problem and determine the optimal solution for the problem. The candidates were given the following word problem: "There two types of fertilizers F_1 and F_2. F_1 consists of 10% nitrogen and 6% phosphoric acid and F_2 consists of 5% nitrogen and 10% phosphoric acid. After testing the soil nutrient composition, a farmer found that she needs at least 14kg of nitrogen and 14kg of phosphoric acid for her crop. If F_1 costs 600 Tsh. per kilogram (kg) and F_2 costs 500 Tsh. per kilogram". From this, the candidates were required to: (a) determine how much of each type of fertilizer should be used so that the nutrient requirements are met at minimum cost; and (b) state the minimum cost.

The question was attempted by 357 (83.8%) candidates, of whom, 297 (83.2%) candidates scored from 6 to 15 marks. This means that the general performance in this question was good. Figure 12 shows the percentage of candidates who got low, average and high marks.

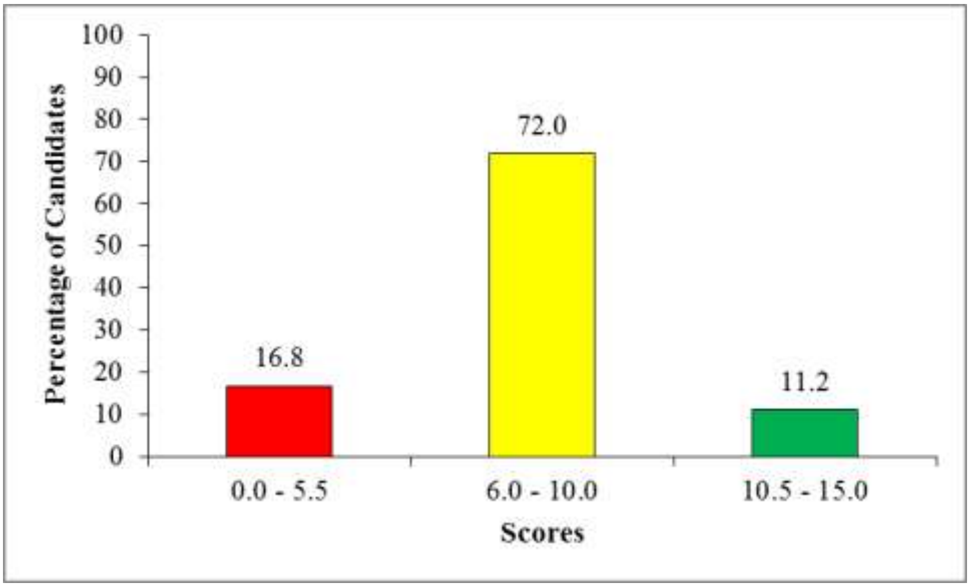


Figure 12: *The candidates' performance on question 12*

The data further show that 60 (16.8%) candidates scored from 0 to 5.5 marks, 257 (72.0%) from 6 to 10 marks and 40 (11.2%) candidates scored from 10.5 to 15 marks.

As Figure 12 shows, 11.2 per cent, equivalent to 40 candidates obtained high marks. They used x and y to represent number of fertilizer F_1 and fertilizer F_2 respectively. This enabled them to rewrite the given word problem into mathematical model, whereby the objective function is Maximize: $f(x, y) = 600x + 500y$ and the equivalent constraints are $2x + y \geq 280$, $3x + 5y \geq 700$, $x \geq 0$ and $y \geq 0$.

These candidates used graphical method to determine feasible region and its corner points as well as optimum point. Finally, they substituted the points into objective function to optimize the problem, as shown in Extract 12.1.

12

	F_1	F_2	Needs
Nitrogen	10%	5%	14kg
Phosphonic acid.	6%	10%	14kg

Let x be amount of F_1 fertilizer
y be amount of F_2 fertilizer

Objective function

$f(x, y) = 600x + 500y$ is subjected to the following constraints

$$\frac{10}{100}x + \frac{5}{100}y \geq 14$$

$$\frac{1}{10}x + \frac{1}{20}y \geq 14.$$

$$2x + y \geq 280 \text{ - - - - - (i)}$$

$$\frac{6}{100}x + \frac{10}{100}y \geq 14$$

$$\frac{6}{100}x \times 100 + \frac{10}{100} \times 100 \geq 14 \times 100$$

$6x + 10y \geq 1400$ Devide by 2

$$3x + 5y \geq 700 \text{ - - - - - - - - - (ii)}$$

$$x \geq 0 \text{ - - - - - (iii)}$$

$$y \geq 0 \text{ - - - - - (iv)}$$

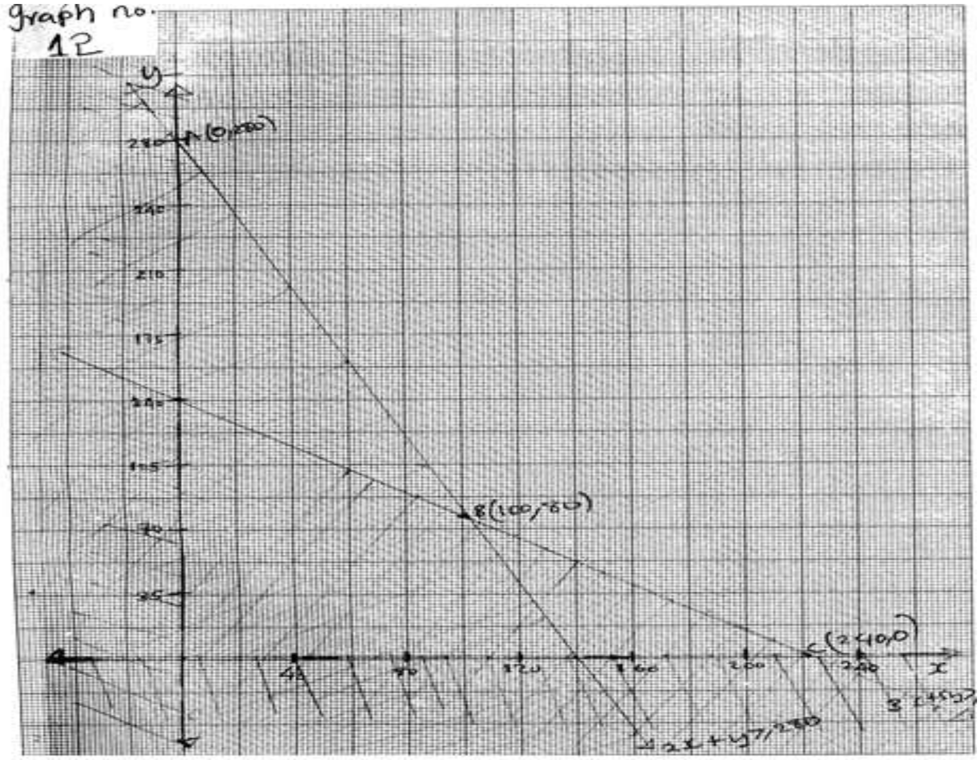
12.

$2x + y = 280$

x	0	140
y	280	0

$3x + 5y = 700$

x	0	$233\frac{1}{3}$
y	140	0



Consider the graph (12)

Point (x, y)	f(x, y) = 600x + 500y
(0, 280)	140,000
	100,00
(100, 80)	100,000
(240, 0)	610,000

(a) It should be used 100 kg of Fertilizer F_1 and 80 kg of firtilizer F_2 in order to met at minimum cost.

(b) Minimum cost will be 100,000Tsh.

Extract 12.1: A sample of a correct response to question 12.

However, 60 (16.8%) candidates scored 0 to 5.5 marks. Some of these candidates assigned variable to the incorrect quantities. They assumed x represent Nitrogen and y represent Phosphoric acid instead of representing F_1 and F_2 respectively. These candidates got incorrect constraints $0.1x + 0.06y \geq 600$ and $0.05x + 0.1y \geq 500$ as well as incorrect objective function $f(x, y) = 14x + 14y$. Others wrote incorrect constraints $2x + y \leq 280$ and $3x + 5y \leq 700$. This indicates that they wrongly interpreted the word "at least" as less than or equal instead of greater than or equal.

Further analysis shows that, there were candidates who failed to convert percentage into fraction or decimals. This led to incorrect constraints $10x+5y \geq 14$ and $6x+10y \geq 14$. Moreover, majority of this group drew incorrect graphs as they failed to use scale correctly. Extract 12.2 gives more another mistake.

12. Consider Data given.

	Nitrogen	phosphorus	costs
fertilizer F1	0.1	0.06	600
fertilizer F2	0.05	0.1	500
Required	14	14	

fertilizer F1 Nitrogen 10%

$\frac{10}{100} \times 14kg = 1.4\ kg$

Phosphorous $\frac{6}{100} \times 14kg = 0.84.\ kg$

fertilizer F2 Nitrogen 5% of

$\frac{5}{100} \times 14kg = 0.7\ kg$

Phosphorous 10%.

$\frac{10}{100} \times 14kg = 1.4\ kg.$

	Nitrogen	phosphorous	costs
F1	1.4	0.84	600
F2	0.7	1.4	500
Required	14	14	

let X replesent fertilizer F1

Y replesent fertilizer F2.

~~objective function~~ constraints

$1.4x + 0.7y \geq 14$

$0.84x + 1.4y \geq 14$

$x \geq 0$

$y \geq 0$

Objective function

$f(x,y) = 600x + 500y$

Extract 12.2: A sample of an incorrect response to question 12.

In Extract 12.2, the candidates computed percentage of nitrogen and phosphoric F_1 and F_2 and used the answer as coefficients of the constraints.

2.2.3 Question 13: Algebra

The question was set to examine the ability of candidates to apply the standard formula in sequence and series. They were required to:

(a) use standard results of $\sum r^2 = \frac{n}{6}(n+1)(2n+1)$ and $\sum r = \frac{n}{2}(n+1)$; to find the sum of the first 50 terms of the series $2+6+\ldots+\left(n^2-n\right)$.

(b) prove that $2b^2 = 9ac$ where a, b and c are real numbers, given that one root of the quadratic equation $ax^2+bx+c=0$ is twice the other.

(c) find an equation with integral coefficients whose roots are the cubes of the roots of the equation $2x^2+5x-6=0$.

The question was attempted by 129 (30.3%) candidates, of which, 74 (57.4%) scored from 6 to 15 marks. Therefore, the general performance of candidates in this question was average. Figure 13 displays candidates' performance in this question.

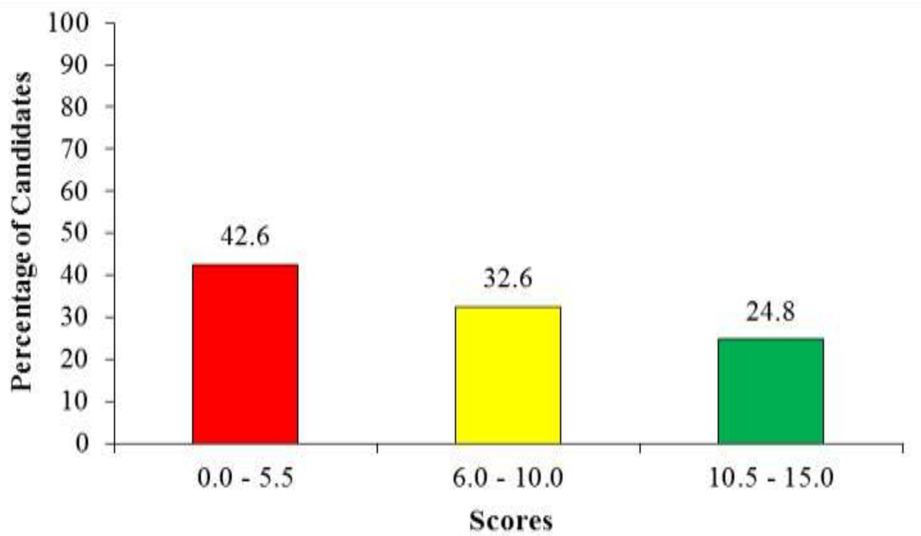


Figure 13: *The candidates' performance on question 13*

The analysis of data shows that 297 (69.7%) candidates skipped this question. About 55 (42.6%) candidates scored from 0 to 5.5 marks, 42 (32.6%) scored from 6 to 10 marks while 32 (24.8%) candidates scored from 10.5 to 15 marks.

The candidates who scored 10.5 marks and above were able to use correctly the sigma notation as well as standard result for summation of series of natural numbers.
These candidates recognised that the series is defined for all natural numbers greater than or equal to 2. Therefore, in order to get the sum of

first 50 terms, they substituted $n = 51$ into $S_n = \frac{n}{6}(n+1)(2n+1) - \frac{n}{2}(n+1)$ or its simplified form $S_n = \frac{n}{2}(n+1)(n-1)$ and computed to get the correct answer $S_{50} = 41,650$.

In part (b), they used properly the rules of sum and product of roots in quadratic equation to assign the values and substitute correctly. They realized that if α is one root of $ax^2 + bx + c = 0$ the other root could be 2α. Using the knowledge of sum and product of roots of quadratic equation, they identified that; $3\alpha = \frac{-b}{a}$ and $2\alpha^2 = \frac{c}{a}$. Then, they worked out to eliminate α by reducing the two equations into one equation containing a, b and c and arranged it to obtain $2b^2 = 9ac$.

In part (c), the candidates were aware that the intended equation could be $x^2 - (\alpha^3 + \beta^3)x + (\alpha\beta)^3 = 0$. Therefore, they computed correctly the numerical value of $\alpha^3 + \beta^3$ and $(\alpha\beta)^3$ from $\alpha + \beta$ and $\alpha\beta$ then substituted into the general form of the equation to get $8x^2 + 305x - 216 = 0$. Extract 13.1 shows an example of a correct response of a candidate.

13. (b) Given that

$ax^2+bx+c=0$

Required to prove that $2b^2=9ac$ when one root is twice the other

solution.

$ax^2+bx+c=0$

divide by a both sides.

$x^2+b/a\,x+c/a=0$ — (i)

$x^2-(\text{sum of roots})x+\text{product of roots}=0$

let β and α be the roots

$x^2-(\beta+\alpha)x+(\beta\alpha)=0$ — (ii)

By comparing the two equations.

$\beta+\alpha=-b/a$ — (iii)

$\beta\alpha=c/a$. — (iv)

But $\beta=2\alpha$.

~~From~~ From equation (iii) substitute $\beta=2\alpha$.

$(2\alpha+\alpha)=-b/a$.

$3\alpha=-b/a$.

$\alpha=\frac{-b/a}{3}.=\frac{-b}{3a}$

$\beta=2\left(\frac{-b}{3a}\right)$

Substitute the values of α and β into equation (iv)

13. (b) $\beta\alpha = \frac{c}{a}$.

$$2\left(\frac{-b}{3a}\right)\left(\frac{-b}{3a}\right) = \frac{c}{a}.$$

$$\frac{2b^2}{9a^2} = \frac{c}{a}.$$

$$2b^2 = \frac{c(9a^2)}{a}.$$

$2b^2 = 9ac$ Hence proved.

13 (c) Required to find the equation where the roots are the cubes of the roots of the equation

$$2x^2+5x-6=0.$$

divide by 2 both sides

$$x^2+\frac{5}{2}x-3=0 \longrightarrow (i)$$

From

x^2- (Sum of roots) $x+$ (product of roots) $=0$

let β and α be the roots.

$$x^2-(\beta+\alpha)x+(\beta\alpha)=0 \longrightarrow (ii)$$

By comparing equation ii and (i)

$\beta+\alpha = -\frac{5}{2}$ and $\beta\alpha = -3$

13 (c)

Sum of roots will be = $(\beta^3+\alpha^3)$

From $(\beta+\alpha)^3 = \beta^3+3\beta^2\alpha+3\beta\alpha^2+\alpha^3$.

$(\beta+\alpha)^3 = (\beta^3+\alpha^3)+3\beta\alpha(\beta+\alpha)$.

$(-5/2)^3 = (\beta^3+\alpha^3)+3(-3)(-5/2)$.

Sum of roots $(\beta^3+\alpha^3) = \frac{-125}{8} + \frac{45}{2}$

Sum of roots. $= -38\frac{1}{8} = -\frac{305}{8}$

product of roots = $\beta^3\alpha^3 = (\beta\alpha)^3$.

product of roots = $(-3)^3 = -27$

From $x^2-(\text{Sum of roots})x+\text{product of roots}=0$

$x^2+\frac{305}{8}x-27=0$.

Therefore, the equation for cubes of the roots will be

$x^2+\frac{305}{8}x-27=0$.

Extract 13.1: A sample of a correct response to question 13.

The 42.6 per cent of the candidates who attempted this question scored low marks because they were unable to use properly the rules of sum and product of roots in quadratic equation. Some candidates substituted 50 into the term n^2-n to get $50\times50-50=2,450$ in part (a). In part (b), some candidates derived the part of equation $ax^2+bx+c=0$ to the equation $2b^2=9ac$ after writing it as $2b^2-9ac=0$ that is; $2b^2+9ac\overline{)ax^2+bx+c}$.

In part (c), most of the candidates interpreted wrongly the word cube. They dealt with sums and product of cubic equation instead of quadratic one. There were candidates who solved the equation $2x^2 + 5x - 6 = 0$ to get the roots. Some candidates applied the inappropriate formula of summation in Arithmetic Progression instead of the standard formula for summing natural numbers as shown in extract 13.2.

13 (a) Solution

$2 + 6 + \cdots + (n^2 - n)$

But $n = 50$

$2 + 6 + \cdots + (50^2 - 50)$

$2 + 6 + \cdots + (2450)$

From

$S_n = \frac{1}{2} n (A_1 + A_n)$

$A_{50} = 2450, \quad n = 50$

$A_1 = 2$

$S_{50} = \frac{1}{2} \times 50 (2 + 2450)$

$= 25 \times (2452) = 61300$

$\therefore S_{50} = 61300$

Extract 13.2: A sample of an incorrect response to question 13.

2.3 Section C: Essay Questions on Pedagogy

2.3.1 Question 14: Planning and Preparation for Teaching Mathematics

This question examined candidates' competence to plan and prepare to teach the lesson. It required the candidates to explain the following components of a lesson plan as used in the teaching and learning of Mathematics:

(a) Preliminary information
(b) Objectives

(c) Lesson development

(d) Students' and teachers' evaluation.

The question was attempted by 402 (94.4%) candidates and among them, 395 (98.3%) candidates scored from 6 to 15 marks. Hence, the general performance of candidates in this question was good. Figure 14 illustrates performance of the candidates.

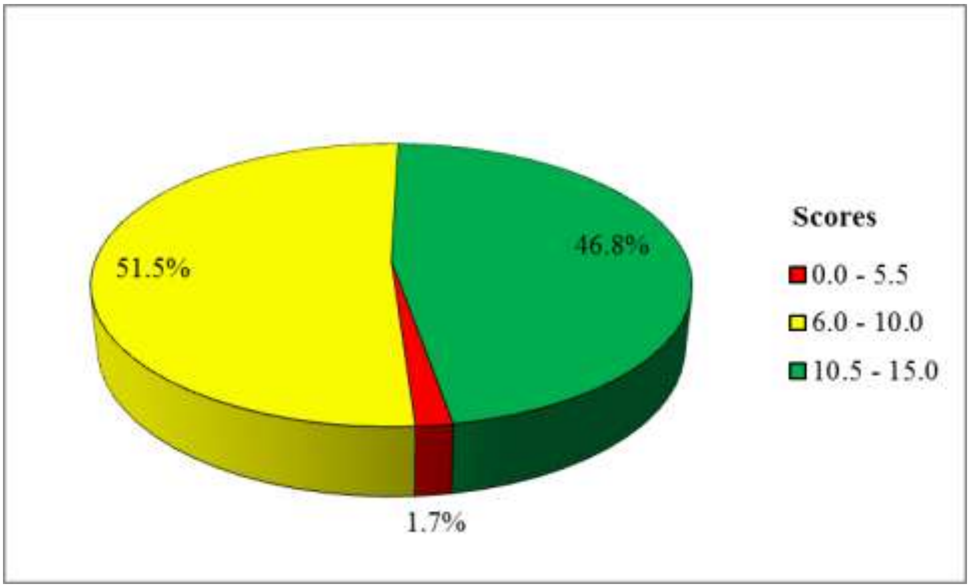


Figure 14: *The performance of candidates on question 14*

The analysis of data shows that 7 (1.7%) candidates scored from 0 to 5.5 mark, 207 (51.5%) scored from 6 to 10 marks and 188 (46.8%) scored from 10.5 to 15 marks.

Most of the candidates answered this question correctly because they were familiar with planning and preparation for teaching in their day to day activities. So, they were able to explain each component in detail because they practice them in their daily life. Extract 14.1 reveals this situation.

14 a) Preliminary information, Is the first par of lesson plan which consist name of t subject, date of the lesson to be taught, Period, time of the lesson to be taught and number of students registered in a specif class, number of students present when t lesson taught and number of students who miss the lesson. preliminary informa on on the lesson plan looks as follow

DATE	SUBJECT	CLASS	PERIOD	TIME	NUMBER OF STUDENTS							
					REGISTERED		PRESENT			ABSENT		
					M	F	T	M	F	T	M	F

b) Objectives, Is a plan of action to b achieved by the learner after a certain topic or lesson. and this plans are pla by the teacher. Objectives has been devid by into two where there is general obj ives and specific objectives. General objecti Is a plan to be achieved after a certain topic to be completed. forexample Ability t understand computation on numbers wh Specific objective Is a plan of action to be achieved after the the end of the lesson. forexample Within a period of 40 minutes each student of form one 'A' shou be able to add franctions correctly.

14 c) lesson development, is the part of lesson plans which consist five stages which are Introduction, presentation, Application, reflection and conclusion. Also lesson development consist time to be used in teaching every stage, teaching activities, learning activities and assessment. lesson development has the structure as follows

STAGES	TIME	TEACHING ACTIVITIES	LEARNING ACTIVITIES	ASSESSMENT
INTRODUCTION				
~~APPLICATIO~~ PRESENTATION				
APPLICATION				
REFLECTION				
CONCLUSION				

d) Student's' and teacher's' evaluation, These are the coments of the teacher and students about the lesson taught whether the lesson is well understood and what makes the lesson to be understood well and how many students understood well the lesson or the lesson have not understood well and why and where the teacher must improve so as to make lesson to be understood well iff possible to repeat the lesson.

Extract 14.1: A sample of a correct response to question 14.

On the other hand, there were 7 (1.7%) candidates who got low marks. This is due to lack of knowledge about planning and preparation to teach Mathematics. Some of them were mentioning the components of a lesson plan instead of explaining the given components as shown in extract 14.2.

14. component of a lesson plan
a) preliminary information: this are the first information used by a teacher of to giving student during teaching process taked place.

b) objectives: this are also the component of lesson plan which help the student teach or during prepare the lesson plan.

c) lesson development: this are also the component of lesson plan which help the student teacher to prepare well the lesson during teaching and learning of mathematics
d) student and teacher's evaluation: this are the information which the student and teacher be evaluated during teaching and learning process take place.

Extract 14.2: A sample of an incorrect response to question 14.

2.3.2 Question 15: Foundations of Mathematics

This question assessed candidates' ability to apply Maslow hierarchy of needs. Candidates were required to explain how the understanding and application of Maslow's hierarch of needs can promote better learning of Mathematics in schools.

The question was attempted by 76 (17.8%) out of 426 candidates, 68 (89.5%) candidates scored from 6 to 15 marks. This indicates that the performance of candidates in this question was generally good. Figure 15 shows percentage of candidates who got low, average and high marks.

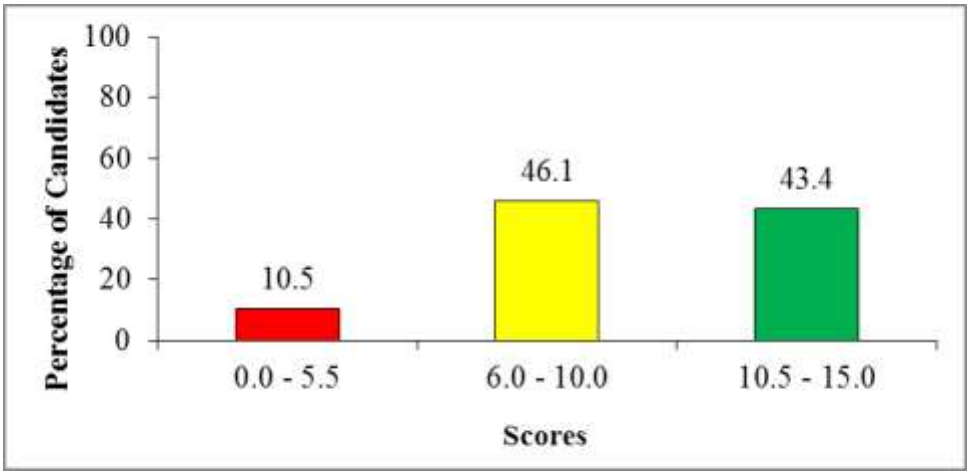


Figure 15: *The performance of candidates on question 15*

The analysis of data shows that 8 (10.5%) of the candidates who attempted it scored from 0 to 5.5 marks, 35 (46.1%) scored from 6 to 10 marks and 33 (43.4%) scored from 10.5 to 15 marks

The candidates who provided satisfactory explanation in this question had adequate knowledge about the physiological needs, safety belonging, esteem need and self-actualization. Extract 15.1 is a sample of the response of one of the candidates.

15 Maslow hierarchy of needs, Refers as the humanistic theory of learning that was introduced by Abraham Maslow for the purpose of ensuring effectiveness of teaching and learning process. Through Maslow hierarch y of needs its understanding and application it promote better learning of mathematics in schools include :-

Through physiological needs, It means in order a learner to learn better in mathemati cs should have fresh air (breathing fresh air (oxy gen), should have get enough food, should have get shelter and ~~clothes~~ clothes, In

15 this way students will be concentrating well in studying and result of effectiveness of learni ng process.

It promote through safety, According to Abraham Maslow a learner should have got enough security in his/her learning environment in order to be comfortable on lear ning process because he/she not afraid of anything since have enough security that result effectiveness of learning mathematics

Love and belonging it promote better lea rning of mathematics because it ensure good positive relationship in each other on cor paration, helping each other on the process of learning that result better learning of mathe matics in schools.

through self esteem, It means the respect of one another in all process of learni ng, hence it result of effective learning of mathematics due to no quarells, misunders tanding of any student toward another, hence through self actualization esteem it lead to active learning to occur.

through self actualization, It means the demands of higher needs of learner to reach, therefore through maslow hierarchy of needs it ensure the learner to have decision making due to all the ~~att~~ necessary condition are available to him/her.

Generally Maslow hierarchy of needs can promote better learning of mathematics due to physiological needs, safety, Love and belongs

Extract 15.1: A sample of a correct response to question 15.

On the other hand, 8 (10.5%) candidates scored low marks ranging from 0 to 5.5. Some of them defined different terms like motivation, cooperation, security and love. This indicates that they failed to know the requirement of the question as shown in Extract 15.2.

15 Hierachy Maslow of needs is a need in which motivate a learner during the process of teaching and learning, this was introduced by the psychologist who is called Abraham Maslow, example of these are love, security and etc. The following are is the how Abraham marlow of need can promote the learner

Motivate the learner, in order the learner to well motivation is very imp ortant.

Bring cooperation to the learner, cooperation is very important ie during group discussion this can ~~[illegible]~~ make the learner to under stand well the subjet.

Bring security to the learner, security is very important it is because without security the learning will be bad.

Enlourage learner to love and respect each other, it is the

because this is among the Abraham
maslow hirachy of needs which encourage learner
to learn well
Bring love to the learner,
the love is very important, it
is because can bring piele and
avoid confuict, if there is
confdict the prolest learning will
be not suitable.
Abraham masslow is
very important psychologist, it
is because introduced his
theory of Abraham Maslow
Hierachy of needs which help
us during the process of
teaching and learning.

Extract 15.2: A sample of an incorrect response to question 15.

2.3.3 Question16: Planning and Preparation for Teaching Mathematics

The question examined the ability of candidates to remember and demonstrate their expected daily role as teachers. They were required to describe five methods of teaching Mathematics.

The question was attempted by 371 (87.1%) candidates, out these, 368 (99.0%) candidates passed by scoring from 6 to 15 marks. Therefore, the general performance of candidates in this question was good. Figure 16 indicates the performance of the candidates in this question.

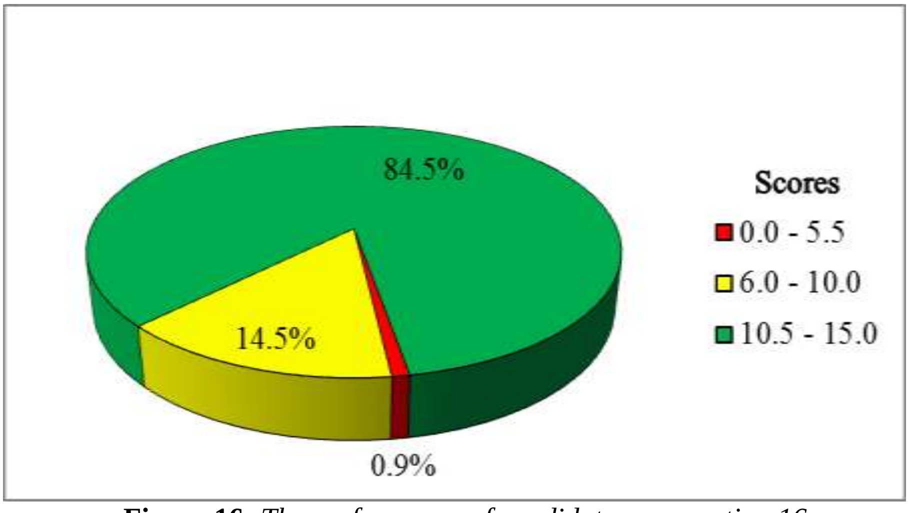


Figure 16: *The performance of candidates on question 16*

The analysis of data shows that 3 (0.9%) of the candidates who attempted it scored from 0 to 5.5 mark, 54 (14.5%) scored from 6 to 10 marks while 314 (84.5%) scored from 10.5 to 15 marks.

The analysis of data shows that almost all candidates (99.0%) passed this question by describing correctly the methods of teaching Mathematics. This is because they always apply different methods while learning and teaching the subject during teaching practice. Extract 16.1 shows the response of a candidate who answered this question correctly.

16. Teaching method of Mathematics is the technique used by a teacher to teach his or her lesson according the nature of his or her classroom. A teacher can use different method in teaching and learning according to the situation of the class. The following are the methods of teaching Mathematics;

Group discussion, In this method a teacher can formulate groups of students who have different knowledge to discuss a certain concept of Mathematics. In this method teacher can give task and observe if Learners are able to answer the question.

Question and answer, Also this method can be used by a teacher in teaching mathematics. Forexample a teacher can ask questions and students answer the question, a teacher can observe and know the prior knowledge of learners.

16. Guest speaker, In teaching Mathematics a teacher can use a guest speaker to teach his or her learners. forexample if a teacher teach a topic or concept which need a people who is priffessiond such as Account. he or she use guest speaker.

Field trip, This is another method of teaching Mathematics where by a teacher can take his or her students to field in order to ~~learn~~ learn more a certain concept of Mathematics.

Role play. Also this is another method used in teaching Mathematics where by a teacher can use rule play to teach a certain topic or concept and his or her students play different role to learn a certain concept.

There fore. Teaching method of mathematics differ according to the nature of class, nature of learners and age of Learners. A teacher can use any method which is suitable to the students.

Extract 16.1: A sample of a correct response to question 16.

But, there were 3 (0.9%) candidates who scored from 2 to 5 marks due to lack of knowledge about the concept of methods of teaching mathematics. Some of them were explained bout the learning environments, procedures for teaching mathematics and techniques of teaching mathematics instead of describing about teaching methods as shown in Extract 16.2.

16 Teaching mattematics is the process, where by teacher faulitate during learning process so as for the learner to understand well the lesson. The following are the method of teaching mathematics, are as follow

Teaching from simple to complex: This is the one method where by help the learners to capture materials easily and make them to love the subject

Another method is activity based. this is the one another method where by after teaching the learner must find activities to do because practice make perfect.

Reflective assessment: this is where by during the lesson the teacher must make evaluation wheather the lesson is understood the the learners or not and if, not must find another way of helping the learners

Balance of conceptual and procedural knowledge: this method it help the learners to be creativity for them self going some where either looking for books and other resources for studying.

Condusive environment: this is very important because if the place are not good the effective objective it must face difficulties, therefore is very important in teaching mathematics

Generally the explained above are the methods of teaching mathematics so my advice to the teacher is to follow that way so as to make, sure that the learning are taking place.

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Extract 16.2: A sample of an incorrect response to question 16.

3.0 THE ANALYSIS OF CANDIDATES' PERFORMANCE PER TOPIC

The analysis done on candidates' performance per topic showed that six topics out of 11 topics that were examined had good performance. These topic are; *Analysis of Mathematics Curriculum Materials (*98.1%*), Planning and Preparations for Teaching Mathematics (*91.9%*), Foundations of Mathematics (*89.5%*), Linear Programming* (83.2%), *Differentiation* (81.3%) and *Similarity and Congruence* (81.3%).

However, three topics had an average performance, namely; *Algebra (*53.5%*), Integration (*52.4%*)* and *Coordinate Geometry II (*49.2%*)*. Also, the data show that the candidate had week performance in two topics which are *Vectors* (1.6%) and *Probability (*30.8%*)*. This weak performance was due to candidates' lack of skills and knowledge about the formula and technics required for calculating the given questions from these two topics.

Further analysis shows that the performance in two (2) topics which are; *Analysis of Mathematics Curriculum Materials* and *Planning and Preparations for Teaching Mathematics* has been good for three consecutive years. The questions which had good performance were Questions; 16 (99.0%), 14 (98.3%), 9 (98.1%), 15 (89.5%), 12 (83.2%), 1 (81.3%), 5 (81.3%), and question 6 (78.3%). Questions which had average performance were 2 (61.1%), 13 (57.4%), 7 (52.4%) and 11 (49.6%). On the other hand, the questions with weak performance were 8 (37.3%), 10 (1.6%), 4 (1.1%). The candidates scored low marks because they failed to interpret the questions' requirement and lacked sufficient knowledge and skills about the mathematical concepts which were examined; others made errors while performing mathematical operations.

4.0 CONCLUSION

The general performance for 740-Mathematics subject in 2021 examination has dropped by 3% compared to that of 2020 with an overall average of 64.8% while that of 2020 had an overall average score of 67.8%. The performance on Probability topic has been poor for three consecutive years from 2019 to 2021. In 2019, the performance was 31.9 per cent; in 2020, it was 32.8 per cent while in 2021 the average performance was 30.8 per cent. This problem could be attributed to the candidates' failure to interpret the questions and inadequate competence in applying the relevant formula in probability topic.

5.0 RECOMMENDATIONS

In order to improve the performance of prospective candidates, it is recommended that:

(a) Tutors are advised to teach the students various techniques on how to answer different questions and guide them on how to identify the requirements of the questions.

(b) Students should be encouraged to read various recommended readings including text books and reference books in order to acquire more knowledge and skills in Mathematics.

(c) The students should be provided with project on designing in and out of class activities that can motivate them to learn.

(d) Tutors should make a regular change of teaching and learning strategies in various topics, for example, guide group discussion and presentation, internet search, library search, pair reflection and others.

(e) Tutors should pay more attention on teaching probability with different techniques in order to raise it performance.

APPENDIX

SUMMARY OF THE CANDIDATES' PERFORMANCE IN MATHEMATICS SUBJECT

		2020				2021			
S/N	**Topic**	**Question Number**	**Performance in Each Question (%)**	**Average Performance Per Topic (%)**	**Remarks**	**Question Number**	**Performance in Each Question (%)**	**Average Performance Per Topic (%)**	**Remarks**
1	Analysis of Mathematics Curriculum Materials	4	85.8	84.4	Good	9	98.1	98.1	Good
		16	82.9						
2	Planning and Preparation for Teaching Mathematics	15	98.7	98.7	Good	6	78.3	91.9	Good
						14	98.3		
						16	99.0		
3	Foundations of Mathematics	1	95.6	95.6	Good	15	89.5	89.5	Good
4	Linear Programming	10	70.7	75.2	Good	12	83.2	83.2	Good
		11	79.9						
5	Differentiation	5	7.4	7.4	Weak	1	81.3	81.3	Good
6	Similarity and Congruence	6	62.3	62.3	Average	5	81.3	81.3	Good
7	Algebra	9	67.0	63.4	Average	11	49.6	53.5	Average
		13	59.8			13	57.4		

		2020				2021			
S/N	Topic	Question Number	Performance in Each Question (%)	Average Performance Per Topic (%)	Remarks	Question Number	Performance in Each Question (%)	Average Performance Per Topic (%)	Remarks
8	Integration	-	-	-	-	7	52.4	52.4	Average
9	Coordinate Geometry II	-	-	-	-	2	61.1	49.2	Average
						8	37.3		
10	Probability	7	52.8	52.8	Average	3	60.5	30.8	Weak
						4	1.1		
11	Vector	3	70.1	70.1	Good	10	1.6	1.6	Weak
12	Hyperbolic Functions	12	87.6	87.6	Good	-	-	-	-
13	Logic	2	85.5	85.5	Good	-	-	-	-
14	Assessment in Mathematics	14	83.6	83.6	Good	-	-	-	-
15	Teaching the Selected Topics	8	14.2	14.2	Weak	-	-	-	-