

THE NATIONAL EXAMINATIONS COUNCIL OF TANZANIA



**STUDENTS' ITEM RESPONSE ANALYSIS
REPORT FOR THE FORM TWO NATIONAL
ASSESSMENT (FTNA) 2015**

042 ADDITIONAL MATHEMATICS

THE NATIONAL EXAMINATIONS COUNCIL OF TANZANIA



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FOR THE FORM TWO NATIONAL ASSESSMENT
(FTNA), 2015**

042 ADDITIONAL MATHEMATICS

Published by

The National Examinations Council of Tanzania,
P.O. Box 2624,
Dar es Salaam.

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TABLE OF CONTENTS

FOREWORD.....	IV
1.0 INTRODUCTION.....	1
2.0 ANALYSIS OF STUDENTS’ PERFORMANCE PER QUESTION... 2	2
2.1 Question 1: Numbers.....	2
2.2 Question 2: Algebra.....	3
2.3 Question 3: Geometrical Constructions.....	5
2.4 Question 4: Numbers.....	6
2.5 Question 5: Algebra.....	8
2.6 Question 6: Algebra.....	9
2.7 Question 7: Logic.....	11
2.8 Question 8: Symmetry.....	13
2.9 Question 9: Algebra.....	14
2.10 Question 10: Variations.....	15
2.11 Question 11: Locus.....	17
2.12 Question 12: Coordinate Geometry.....	19
2.13 Question 13: Coordinate Geometry.....	21
2.14 Question 14: Geometrical Constructions.....	22
2.15 Question 15: Coordinate Geometry.....	24
2.16 Question 16: Logic.....	26
2.17 Question 17: Symmetry.....	27
2.18 Question 18: Algebra.....	28
2.19 Question 19: Algebra.....	30
2.20 Question 20: Numbers.....	31
2.21 Question 21: Algebra.....	33
2.22 Question 22: Sets.....	35
2.23 Question 23: Logic.....	38
2.24 Question 24: Coordinate Geometry.....	40
2.25 Question 25: Variations.....	43
3.0 ANALYSIS OF STUDENTS’ PERFORMANCE IN EACH TOPIC ...	45
4.0 CONCLUSION AND RECOMMENDATIONS.....	45
4.1 Conclusion.....	45
4.2 Recommendations.....	46
4.2.1 Recommendations to Students.....	46
4.2.2 Recommendations to Teachers.....	46
APPENDIX A.....	47
APPENDIX B.....	48
APPENDIX C.....	49

FOREWORD

Additional Mathematics Students' Items Response Analysis Report on Form Two National Assessment (FTNA) 2015 was written in order to provide feedback to students, parents, policy makers and other education stakeholders on the performance of students. This subject has an average performance. The report in this report therefore, reveals the factors that are attributed to students' average performance.

This assessment is a formative evaluation which shows the effectiveness of education delivery system during the first two years of Secondary Education. Essentially, students' responses to the assessment questions are indicators of what the education delivery system was able or unable to offer to the students in the first two years of secondary education. Also the assessment helps in making decision for the form two students whether to proceed to form three or not.

In this report the factors which contributed to the student's failure to answer the questions correctly have been analysed. Such factors include lack of knowledge and skills on the concepts related to the subject, poor English Language proficiency and failure to identify the task of the question. The report also analyses the reasons which made some of the students answer the questions correctly.

The feedback provided in this report will enable the education stakeholders to identify proper measures to be taken to improve students' performance in future assessments administered by the Council.

The National Examinations Council of Tanzania (NECTA) will highly appreciate comments and suggestions from students and the public in general that can be used to improve future assessment Reports.



Dr. Charles E. Msonde
EXECUTIVE SECRETARY

1.0 INTRODUCTION

The report on Additional Mathematics is based on the analysis of performance of students who sat for the Form Two National Assessment (FTNA) in November 2015. The paper was set basing on the 2010 Additional Mathematics Syllabus for Secondary Schools.

The paper consisted of sections A and B. Section A had 20 questions and section B had 5 questions. The students were required to answer all questions from both sections. Each question in section A weighed 3 marks while those in section B weighed 8 marks each.

In 2015, a total of 587 students sat for this assessment and 321 (54.68%) passed, while in 2014, a total of 281 students sat for the assessment and 156 (55.52%) passed. The performance of the students in the two years is shown in the following table:

year	Registered	Sat	A	B ⁺	B	C	D	E	F	Passed	
2014	Number	310	281	21	34	38	27	36	24	101	156
	%		90.6	7.5	12.1	13.5	9.6	12.8	8.5	35.9	55.52
2015	Number	627	587	58	76	61	62	64	75	191	321
	%		93.6	20.6	27	21.7	22.1	22.8	26.7	68	54.68

The registered students doubled in 2015 when compared to 2014, indicating a growing interest of students in this subject. However, the percentage of the students who passed the assessment has decreased by 0.84 percent.

This report explains the task which was expected to be done by the students in each question. It also identifies the students' strengths and weaknesses in attempting each of the questions. The extracts of both best and poor solutions for each question are included in the report as evidence of the descriptions of performance of the students.

The performance of the students is categorized into three groups, which are good, average and poor depending on the percentage of students who scored 30 percent or more of the marks allotted in a particular question. If 30 percent or more of the students scored from 50 to 100 percent, 30 to 49 percent and 0 to 29 percent, that performance was categorized as good, average and poor respectively.

2.0 ANALYSIS OF STUDENTS' PERFORMANCE PER QUESTION

2.1 Question 1: Numbers

The question required the students to write the next three numbers in the pattern: 4, 8, 12, 20, In this question, the students were supposed to recall that, in order to obtain the next number in the given pattern they should find the sum of the previous two numbers.

This question was attempted by 587 students of which 59.8 percent scored below 1.0 mark, 9.7 percent scored from 1.0 to 2.0 marks and 30.5 percent scored 3.0 marks. The performance of students in this question was average.

The majority of the students (59.8%) failed to get the correct response as they were unable to apply the rule of Fibonacci pattern. They found the differences between the two consecutive numbers and used various incorrect ideas to find the three numbers. Some of them wrote incorrectly the numbers either 32, 48 and 68; 28, 32 and 36; or 28, 40 and 52 as answers. Extract 1.1 is an example of the student's poor answer.

Extract 1.1

1. Write down the next three numbers in the following pattern;
4, 8, 12, 20, ...

solution

4 8 12 20 28 40 52

4 4 8 8 12 12

∴ The next three numbers are 28, 40 and 52

The image shows a handwritten student solution. It starts with the question: '1. Write down the next three numbers in the following pattern; 4, 8, 12, 20, ...'. The student has written 'solution' and then a zigzag line connecting the numbers 4, 8, 12, 20, 28, 40, and 52. Below the zigzag line, the differences between consecutive numbers are written: 4, 4, 8, 8, 12, 12. The numbers 28, 40, and 52 are underlined. At the bottom, the student concludes: '∴ The next three numbers are 28, 40 and 52'.

In Extract 1.1, the student used a single difference of two consecutive terms twice to find the next two terms.

On the other hand, the students who were able to provide good solutions wrote the sequence of numbers correctly by applying the rule of the Fibonacci Numbers. Extract 1.2 shows an example of a correct answer from the script of a student who was able to find the three numbers in the pattern.

Extract 1.2

1. Write down the next three numbers in the following pattern;
4, 8, 12, 20, ...

soln

From

4, 8, 12, 20, 32, 52, 84

\swarrow + \searrow \swarrow + \searrow \swarrow + \searrow

\therefore The three next numbers are 32, 52, 84 Ans.

In Extract 1.2, the student applied the concept of Fibonacci pattern by adding two previous consecutive terms to get each of the next terms correctly.

2.2 Question 2: Algebra

In this question, the students were required to make x the subject of the

$$\text{formula } m = \frac{y - b}{x - a}.$$

In order to answer the question the students were required to have knowledge and skills of expanding brackets and transposing formulae. They were required to apply the cross multiplication technique or eliminate the denominator $x - a$ in order to obtain the linear equation $m(x - a) = y - b$ before proceeding to other steps.

The question was attempted by 587 students of which 30.3 percent scored below 1.0 mark, 29.2 percent scored from 1.0 to 2.5 marks and 40.5 percent scored 3.0 marks. The students' performance in this question was good as 69.7 percent of them scored from 1.0 to 3.0 marks.

The students who performed well in this question, managed to transpose the given formula as it was expected. Extract 2.1 shows an example of one of the students' correct responses.

Extract 2.1

2. Make x the subject of the formula $m = \frac{y-b}{x-a}$.

$$\frac{m}{1} = \frac{y-b}{x-a}$$

$$mx - ma = y - b$$

$$\frac{mx}{m} = \frac{y-b+ma}{m}$$

$$x = \frac{y-b+ma}{m}$$

$$\therefore x = \frac{y-b+ma}{m}$$

Extract 2.1 shows that the student was able to cross multiply the terms correctly to get the linear equation $m(x-a) = y-b$ which enabled him/her to make x subject of the formula.

On the other hand, the students who did not provide the required solution lacked the knowledge of expanding the expressions after using cross multiplication procedure or eliminating the denominator to form a linear equation and proceed to other steps correctly. Extract 2.2 is a sample answer from one of the students who failed to answer the question correctly.

Extract 2.2

2. Make x the subject of the formula $m = \frac{y-b}{x-a}$.

solution

$$\frac{m}{1} = \frac{y-b}{x-a}$$

$$x-a \times m = y-b$$

$$x-am = y-b$$

$$x = y-b + am$$

$$\therefore x = y-b + am$$

In Extract 2.2, the student wrongly expanded the left hand side expression in step three to obtain $x-am = y-b$ instead of $mx-ma = y-b$ and consequently ended up with an incorrect answer.

2.3 Question 3: Geometrical Constructions

The students were required to find the value of x from the given interior angles of a quadrilateral. They were supposed to recall the meaning of quadrilateral as well as the formula for finding the sum of interior angles of a regular polygon.

The question was attempted by 587 students of which 61.5 percent scored below 1.0 mark, 10.7 percent scored from 1.0 to 2.5 marks and 27.8 percent scored 3.0 marks. The performance of students in this question was average as 38.5 percent scored from 1.0 to 3.0 marks.

The students who failed to give a correct solution for this question had inadequate understanding of quadrilaterals as they were unable to recall the meaning as well as the formula for the sum of interior angles of a quadrilateral. Extract 3.1 shows an incorrect solution provided by one of the students.

Extract 3.1

3. If the interior angles of a quadrilateral are $2x$, $2x-1^\circ$, $3x-10^\circ$ and $x-13^\circ$, find the value of x .

Soln

$$2x + 2x - 1 + 3x - 10 + x - 13 = 180^\circ$$
$$2x + 2x + 3x + x - 1 - 10 - 13 = 180^\circ$$
$$8x - 24 = 180^\circ$$
$$8x = 180^\circ + 24^\circ$$
$$\frac{8x}{8} = \frac{204^\circ}{8}$$
$$x = 13^\circ$$

In Extract 3.1, the student summed up the given angles and equated it to 180° instead of 360° and was unable to get the correct value of x .

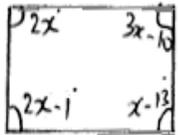
The students who scored full marks had good understanding of quadrilaterals as some of them were able to recall that the sum of interior angles of a quadrilateral is 360° . Others applied the formula for the sum of interior angles of a regular polygon which enabled them to proceed to other

steps of getting the correct value of x . Extract 3.2 shows an example of one of the best solutions provided by the students.

Extract 3.2

3. If the interior angles of a quadrilateral are $2x$, $2x-1^\circ$, $3x-10^\circ$ and $x-13^\circ$, find the value of x .

Soln.



$$2x + 2x - 1^\circ + 3x - 10^\circ + x - 13^\circ = 360^\circ$$

$$4x + 4x - 11^\circ - 13^\circ = 360^\circ$$

$$8x - 24^\circ = 360^\circ$$

$$8x = 360^\circ + 24^\circ$$

$$8x = 384^\circ$$

$$\frac{8x}{8} = \frac{384^\circ}{8}$$

$$x = 48^\circ$$

$$SI = (n-2)180^\circ$$

$$SI = (4-2)180^\circ$$

$$SI = 2 \times 180^\circ$$

$$SI = 360^\circ$$

\therefore The value of x is 48°

In Extract 3.2, the student used the formula to find the sum of interior angles of the quadrilateral and then summed up the given angles to form an equation which led to the correct value of x .

2.4 Question 4: Numbers

The students were required to use divisibility rule to show whether 47187 is divisible by 9 or not.

There were 587 students (100%) who attempted this question of which 46.2 percent scored below 1.0 mark, 9.3 percent scored from 1.0 to 2.0 marks and 44.5 percent scored 3.0 marks. Generally, the students' performance in this question was good as 53.8 percent scored from 1.0 to 3.0 marks that were allotted to the question.

The students who gave correct proofs of the question had adequate knowledge of divisibility rules on natural numbers. One of the good proofs is shown in Extract 4.1.

Extract 4.1

4. Use divisibility rule to show whether 47187 is divisible by 9 or not.

$$\begin{array}{l} \text{John:} \\ 47187 \\ = 4+7+1+8+7 \\ 11+9+7 \\ = \frac{27}{9} = 3 \\ \therefore 47187 \text{ is divisible by } 9 \end{array}$$

In Extract 4.1, the student summed up all digits in the given number and concluded that 47187 is divisible by 9 as the sum of the digits is divisible by 9.

On the other side, 46.2 percent of the students were unable to give the correct answer due to inadequate knowledge of divisibility rules. Some of them applied the concept of normal division of numbers instead of the required rule of divisibility; while others applied the divisibility rules of other numbers. Extract 4.2 is an example of a poor solution that was provided by one of the students.

Extract 4.2

4. Use divisibility rule to show whether 47187 is divisible by 9 or not.

$$\begin{array}{l} 47187 \text{ the last three digits are } 187 \\ 187 \div 9 = 20.777 \\ 187 \text{ is not divisible by } 9 \\ \therefore 47187 \text{ is not divisible by } 9 \end{array}$$

In Extract 4.2, the student divided the last three digits of 47187 by 9 and ended up with a wrong conclusion.

2.5 Question 5: Algebra

In this question, the students were asked to write the expressions (a) $7m - 2n + 6 - 5m + 7n + 3$ and (b) $\frac{72a^2b}{8a} - 8ab$ in the simplest forms.

A total of 587 students attempted the question of which 29.6 percent scored 0 mark, 37.4 percent scored from 1.0 to 2.5 marks and 33.0 percent scored 3.0 marks. The data shows that the students' performance in this question was good as 70.4 percent scored from 1.0 to 3.0 marks.

The students who did well were able to collect like terms of the expression, perform algebraic operations and simplify expressions that involved fractions. Extract 5.1 is an example of a good solution.

Extract 5.1

5. Write each of the following expressions in the simplest form.

(a) $7m - 2n + 6 - 5m + 7n + 3$,

$$7m - 2n + 6 - 5m + 7n + 3$$

$$= 7m - 5m - 2n + 7n + 6 + 3$$

$$= 2m + 5n + 9$$

$$\therefore 7m - 2n + 6 - 5m + 7n + 3 = 2m + 5n + 9$$

(b) $\frac{72a^2b}{8a} - 8ab$.

$\frac{72a^2b}{8a} - 8ab$ $= \frac{72a^2b}{8a} - \frac{8ab}{1}$ $= \frac{72a^2b - 8a(8ab)}{8a}$	$= \frac{72a^2b - 64a^2b}{8a}$ $= \frac{8a^2b}{8a}$ $= \frac{8 \times a \times a \times b}{8 \times a}$ $= ab$ $\therefore \frac{72a^2b}{8a} - 8ab = ab$
---	--

In Extract 5.1, the student simplified correctly the expression in part (a) by collecting the like terms and in part (b) wrote the expression in a single denominator which led to the correct simple form.

The students who failed to give the correct solutions in this question were unable to collect the like terms of the expression and perform the operations correctly in part (a). They also had inadequate knowledge and skills of simplifying expressions that involve fractions as they were unable

to write the expression with a single denominator in part (b). Extract 5.2 illustrates one of the poor solutions.

Extract 5.2

5. Write each of the following expressions in the simplest form.

(a) $7m - 2n + 6 - 5m + 7n + 3$.

$$7m - 2n + 6 - 5m + 7n + 3$$

$$7m + 5m = 12 \quad 12m + 10n - 3 \text{ Answer}$$

$$2n + 7n = 10$$

$$6 - 3 = 3$$

(b) $\frac{72a^2b}{8a} - 8ab$.

Soln

$$\frac{72a^2b}{8a} - 8ab = 72a^2b - b$$

$$= 72a^2$$

$$\therefore \frac{72a^2b}{8a} - 8ab = 72a^2$$

In Extract 5.2 the student lacked the knowledge and skills of simplifying expressions.

2.6 Question 6: Algebra

The students were asked to find $x : y$ given that $(x + y) : (2x + y) = 4 : 5$.

The data analysis shows that 587 students attempted the question of which 57.1 percent scored from 0 to 0.5 mark, 22.8 percent scored from 1.0 to 2.5 marks and 20.1 percent scored 3.0 marks. The data indicates that the performance of students in this question was average as 42.9 percent scored from 1.0 to 3.0 marks allotted to the question.

Most students (57.1%) performed poorly in this question due to inadequate knowledge and skills of Algebra. Some of them expressed the given ratio of the equations as fractions but failed to formulate the two equations: $x + y = 4$ and $2x + y = 5$ so that they could solve them simultaneously. Others substituted the variables x and y of them equation by 4 and 5

respectively and hence got the wrong ratio. Extract 6.1 shows one of the wrong solutions given by the students.

Extract 6.1

6. If $(x+y):(2x+y)=4:5$, find $x:y$.

Given, $(x+y):(2x+y)=4:5$ find $x:y$

$$(4+5):(2 \times 4+5)$$

$$(4+5):(8+5)$$

$$(32+20):(40+25)$$

$$52:65$$

$\therefore x:y = 117$

In Extract 6.1, the student provided a wrong answer for he/she wrongly substituted 4 and 5 for x and y respectively.

The students who performed well in this question were able to compare the terms of the given ratios to form two simultaneous equations. They were also able to find the values of x and y from the formulated equations so as to obtain the required ratio. One of the good solutions is illustrated in Extract 6.2.

Extract 6.2

6. If $(x+y):(2x+y)=4:5$, find $x:y$.

$$\begin{cases} x+y=4 \\ 2x+y=5 \end{cases}$$

$$-x = -1$$

$$x = 1$$

$x+y=4$ but $x=1$

$$1+y=4$$

$$y=3$$

$\therefore x:y = 1:3$

In Extract 6.2, the student formulated two equations which were then solved simultaneously to get the required ratio.

2.7 Question 7: Logic

The question had parts (a) and (b). In part (a), the students were asked to write the symbol against each of the connectives used in logic as shown in the following table:

Connective	Symbol
Equivalence	
Disjunction	
Condition	
Conjunction	

In part (b), the students were required to write the following statement in symbolic form: “If 2 is an even number, then 5 is an odd number.”

The question was attempted by 587 students of which 30.0 percent scored from 0 to 0.5 mark, 54.2 percent scored from 1.0 to 2.5 marks and 15.8 percent scored 3.0 marks. The data analysis shows that 70.0 percent of the students scored from 1.0 to 3.0 marks, which is an indication of good performance.

The students who did well in this question had good understanding of Logic. Extract 7.1 illustrates one of the best solutions that was provided by the students.

Extract 7.1

7. (a) The table below shows the connectives used in logic. Write the symbol used for each connective.

Connective	Symbol
Equivalence	\longleftrightarrow
Disjunction	\vee
Condition	\longrightarrow
Conjunction	\wedge

- (b) Write the following statement in symbolic form;
 "If 2 is an even number, then 5 is an odd number."

Let 2 is an even number be a
 and 5 is an odd number be b $a \rightarrow b$

In Extract 7.1, the student provided the appropriate symbol against each of the connectives and was able to write the correct symbolic statement.

The students who did wrongly in this question lacked understanding of the meaning of connectives and symbols used in Logic. They wrote various symbols which are not related to the given connectives. Extract 7.2 is an example of an incorrect solution written by one of the students.

Extract 7.2

7. (a) The table below shows the connectives used in logic. Write the symbol used for each connective.

Connective	Symbol
Equivalence	$\cdot \cdot$
Disjunction	$\wedge \cdot$
Condition	$\cdot \cdot$
Conjunction	$\cdot \cdot$

- (b) Write the following statement in symbolic form;
 "If 2 is an even number, then 5 is an odd number."

$$2/5 (5/2) = 7/4 = (4/1)$$

In Extract 7.2 the student wrote inappropriate symbol against each of the given connectives in part (a) and in part (b) he/she wrote fractions which were not related to the given logical statement.

2.8 Question 8: Symmetry

The question had parts (i) and (ii). The students were asked to (i) draw all lines of symmetry, and (ii) state the number of lines of symmetry in the following figure:

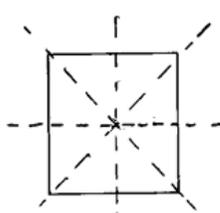


The analysis of data shows that 587 students attempted this question of which 29.6 percent scored from 0 to 0.5 mark, 23.7 percent scored from 1.0 to 2.5 marks and 46.7 percent scored 3.0 marks. The performance in this question was good as 70.4 percent of the students scored from 1.0 mark and above.

In this question, the students who performed well were able to apply the concepts of symmetry in drawing lines of symmetry and ultimately provided appropriate number of the drawn lines. An example of a correct answer is shown in Extract 8.1.

Extract 8.1

8. (i) Draw all lines of symmetry on the following figure by using dotted lines.



(ii) State the number of lines of symmetry in (i) above.

∴ There are Four lines of symmetry

In Extract 8.1, the student drew and stated the number of lines of symmetry correctly.

Some of the students who did poorly in this question lacked knowledge of symmetry. While others failed to identify the demand of the question. Extract 8.2 shows one of the incorrect solutions provided by one of the students.

Extract 8.2

8. (i) Draw all lines of symmetry on the following figure by using dotted lines.



- (ii) State the number of lines of symmetry in (i) above.

There are three (3) ^{solv} number of lines.

In Extract 8.2, the student drew another figure with three lines in the given figure which did not portray the meaning of lines of symmetry.

2.9 Question 9: Algebra

In this question the students were required to find the value of t in the

$$\text{equation } \frac{1}{2}t + \frac{2}{5} = t - \frac{4}{5}.$$

The question was attempted by 587 students of which 47.0 percent scored below 1.0 mark, 12.5 percent scored from 1.0 to 2.5 marks and 40.5 percent scored 3.0 marks. The students' performance in this question was good as 53.0 percent scored from 1.0 to 3.0 marks.

The students who got the correct answer managed to apply the knowledge of fractions and algebra to simplify the algebraic equations. Extract 9.1 is an example of the best answer.

Extract 9.1

9. Find the value of t in the equation $\frac{1}{2}t + \frac{2}{5} = t - \frac{4}{5}$.

solution

$$\begin{aligned} \frac{1}{2}t + \frac{2}{5} &= t - \frac{4}{5} \\ \frac{2}{5} + \frac{4}{5} &= t - \frac{1}{2}t \\ \frac{2+4}{5} &= \frac{2t-t}{2} \end{aligned}$$

$$\begin{aligned}
 2 \times \frac{6}{5} &= \frac{1}{5}t \times 2 \\
 \frac{12}{5} &= t \\
 \therefore t &= 2\frac{2}{5}
 \end{aligned}$$

In Extract 9.1 the student collected like terms and was able to manipulate the fractions correctly in solving the algebraic equation.

The students who performed poorly in this question had inadequate skills of solving algebraic equations and lacked skills of manipulating the fractions. Extract 9.2 is an example of a poor solution.

Extract 9.2

9. Find the value of t in the equation $\frac{1}{2}t + \frac{2}{5} = t - \frac{4}{5}$.

Soln:

$$\begin{aligned}
 \frac{1}{2}t + \frac{2}{5} &= t - \frac{4}{5} \\
 \frac{3}{7}t &= t - \frac{4}{5} \\
 t &= \frac{4}{5} - \frac{3}{7} \\
 t &= \frac{1}{6} \\
 \therefore t &= \frac{1}{6} \text{ value}
 \end{aligned}$$

In Extract 9.2, the student wrongly collected the like terms together which led to a wrong answer.

2.10 Question 10: Variations

The question required the students to find the length of a pendulum when the period is $1.2s$, given that, the period T of a simple pendulum varies

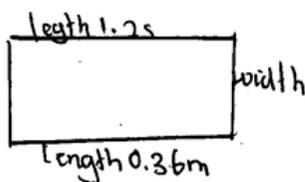
directly with the square root of length l of the pendulum and when the period is $1.2s$ the length is $0.36m$.

The question was attempted by 587 students of which 52.0 percent scored from 0 to 0.5 mark, 26.0 percent scored from 1.0 to 2.5 marks and 22.0 percent scored 3.0 marks. The data shows an average students' performance, as 48.0 percent scored from 1.0 to 3.0 marks.

The students who performed poorly in this question did not understand the requirements of the question. They were not able to translate the word problem mathematically as $T = k\sqrt{l}$ and thereafter solve this equation to find the length of pendulum. Extract 10.1 illustrates one of the worst solutions provided by one of the students.

Extract 10.1

10. The period T of a simple pendulum varies directly with the square root of length l of the pendulum. If the period is $1.2s$ when the length is $0.36m$, find the length when the period is $1.2s$.



Soln.

(Up up) $\times 2$

$1.2s + 0.36m$

$0.36m$

$1.20s$

$\frac{1.20s}{1.565m} = 1.565m$

In Extract 10.1, the student used the concept of perimeters to find the length of a rectangle instead of the length of a pendulum.

The students who did well in this question had adequate knowledge of the terms used in variation, and good mastery of English Language. They were able to translate the question mathematically as illustrated in Extract 10.2.

Extract 10.2

10. The period T of a simple pendulum varies directly with the square root of length l of the pendulum. If the period is 1.2s when the length is 0.36m, find the length when the period is 1.2s.

Soln

Given
 $T_1 = 1.2$ $L_1 = 0.36$
 $T_2 = 1.2$ $L_2 = ?$

From;

$$T \propto \sqrt{L}$$
$$T = k\sqrt{L}$$
$$k = \frac{T}{\sqrt{L}}$$
$$\frac{T_1}{\sqrt{L_1}} = \frac{T_2}{\sqrt{L_2}}$$
$$\frac{1.2}{\sqrt{0.36}} = \frac{1.2}{\sqrt{L}}$$
$$\frac{1.2}{0.6} = \frac{1.2}{\sqrt{L}}$$
$$2 = \frac{1.2}{\sqrt{L}}$$
$$2\sqrt{L} = 1.2$$
$$\frac{2\sqrt{L}}{2} = \frac{1.2}{2}$$
$$\sqrt{L} = 0.6$$
$$(\sqrt{L})^2 = (0.6)^2$$
$$L = 0.36$$

\therefore The length is 0.36m

In Extract 10.2, the student changed the word problem into the variation equation $T = k\sqrt{l}$ and proceeded to find the value of constant k and the length of the pendulum correctly.

2.11 Question 11: Locus

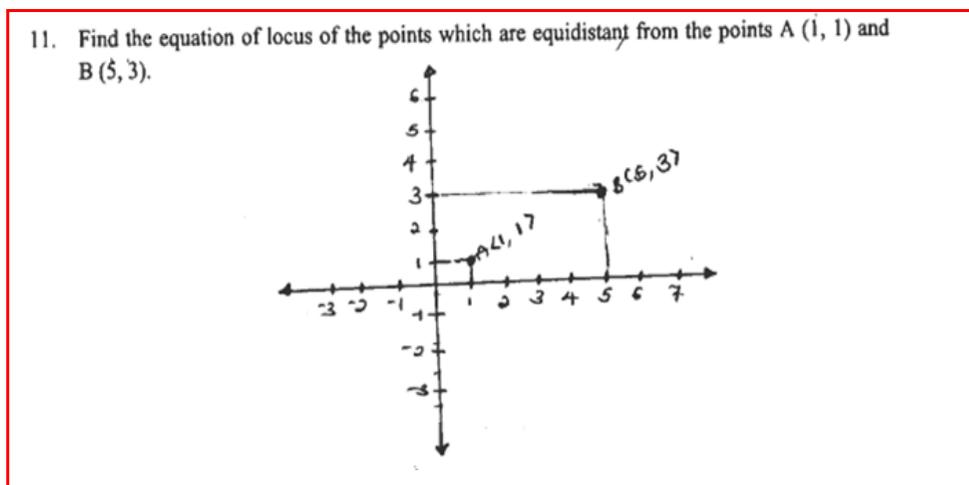
In this question the students were required to find the equation of locus of the points which are equidistant from the points $A(1,1)$ and $B(5,3)$.

The data analysis shows that 587 students attempted this question of which 73.8 percent scored below 1.0 mark, 11.0 percent scored from 1.0 to 2.5 marks and 15.2 percent scored 3.0 marks. The performance in this question was poor as 73.8 percent of the students scored below 1.0 mark.

The students who failed to answer this question lacked knowledge of locus and the use of the distance formula. Some of them confused the ideas of locus with those of Coordinate Geometry while others provided solutions

which were not related to the demand of the question. Extract 11.1 is an example of a poor solution provided by a student.

Extract 11.1



In Extract 11.1, the student plotted points A and B contrary to the demand of the question.

The students who were able to find the required locus of the points had good understanding of locus concepts. They applied the distance formula correctly in answering the question. Extract 11.2 illustrates one of the best solutions that were provided by the students.

Extract 11.2

11. Find the equation of locus of the points which are equidistant from the points A (1, 1) and B (5, 3).

Let $P(x, y)$ be the point
 $PA = PB$ $A(x_1, y_1)$, $B(x_2, y_2)$

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \text{Distance}$$

$$\left(\sqrt{(x-1)^2 + (y-1)^2} \right)^2 = \left(\sqrt{(x-5)^2 + (y-3)^2} \right)^2$$

$$x^2 - 2x + 1 + y^2 - 2y + 1 = x^2 - 10x + 25 + y^2 - 6y + 9$$

$$-2x + 10x + 1 - 9 - 2y + 6y + 1 - 25 = 0$$

$$8x - 8 + 4y - 24 = 0$$

$$8x + 4y - 32 = 0$$

$$\frac{4y}{4} = \frac{-8x + 32}{4}$$

$$y = -2x + 8$$

∴ The locus of the points is $y = -2x + 8$ which is linear equation

In Extract 11.2, the student applied correctly the distance formula to find the locus of the points correctly.

2.12 Question 12: Coordinate Geometry

In this question the students were asked to find the coordinates of the mid-point of the line segment which joins the points $P(3,7)$ and $R(5,9)$. The students were required to recall and apply the formula for calculating the coordinates of the midpoint by using the given points in a line segment.

The question was attempted by 587 students of which 85.3 percent scored below 1.0 mark, and only 11.4 percent scored all the 3.0 marks indicating that this question was poorly performed.

The students who scored low marks had inadequate knowledge of how to find a midpoint of a line segment. Some students used incorrect formula while others made wrong substitution of the given coordinates in the formula. Extract 12.1 shows a poor solution from one of the students.

Extract 12.1

12. Find the coordinates of the mid-point of the line segment which joins the points $P(3,7)$ and $R(5,9)$.

Solution:

Points $P(3,7)$

$R(5,9)$

Then.

$$\text{Mid-point} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\begin{aligned} \text{Mid points} &= \frac{9-7}{5-3} \\ \text{Mid-points} &= \frac{2}{2} \\ \text{Mid-points} &= 1 \\ \therefore \text{Coordinates of the mid-points} &= 4 \end{aligned}$$

In Extract 12.1, the student used the formula for finding the slope of a line instead of the formula to find the midpoint of a line segment.

However, the students who were able to provide the correct response, applied correctly the midpoint formula to get the required coordinates. Extract 12.2 is a sample answer illustrating how the students answered this question.

Extract 12.2

12. Find the coordinates of the mid-point of the line segment which joins the points P (3, 7) and R (5, 9).

Soln

$$\begin{aligned} \text{Mid point} &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ &= \left(\frac{3+5}{2}, \frac{7+9}{2} \right) \\ &= \left(\frac{8}{2}, \frac{16}{2} \right) \\ &= (4, 8) \\ \therefore \text{Mid point is } &(4, 8) \end{aligned}$$

In Extract 12.2 the student determined the required coordinates by using the midpoint formula.

2.13 Question 13: Coordinate Geometry

The question had two parts, (a) and (b). In part (a), the students were asked to define the term ‘Non-Collinear points’ and in part (b), they were asked to find k if the points $R(3, 4)$, $S(k, 1)$ and $T(15, -5)$ are collinear.

A total of 587 students attempted this question of which 58.4 percent scored below 1.0 mark, 23.5 percent scored from 1.0 to 2.5 marks and 18.1 percent scored 3.0 marks. The general performance in this question was average as 41.6 percent of students scored from 1.0 to 3.0 marks.

The majority of the students (58.4%) lacked knowledge of non collinear points as they were unable to give a correct definition of such points, and use the concept of slope of a line to find the value of k . Extract 13.1 illustrates an example of a poor response from one of the students.

Extract 13.1

13. (a) Define the term “Non Collinear points”
Non collinear points are the point that are not going to the same direction

(b) Find k if the points $R(3, 4)$, $S(k, 1)$ and $T(15, -5)$ are collinear.
Soln
$$3 + k + 15 = 4 + 1 - 5$$
$$18k = +5 - 5$$
$$\frac{18k}{18} = \frac{0}{18}$$
$$\therefore k = 0$$

In Extract 13.1, the student gave an incorrect definition of collinear points and incorrectly assumed that the sum of values of x is equal to the sum of values of y in finding the value of k .

On the other hand, a few students (18.1%) gave a correct answer to this question as they had adequate knowledge of collinear points. They were able to use the concept of slope of a line to find the value of k . Extract 13.2 shows one of the correct responses from the students.

Extract 13.2

13. (a) Define the term "Non Collinear points".

Non collinear points are points which do not lie on the same line

(b) Find k if the points R (3, 4), S (k , 1) and T (15, -5) are collinear.

$$\begin{aligned} M_1 &= M_2 \\ M &= \frac{y_2 - y_1}{x_2 - x_1} \\ M_1 &= \frac{1 - 4}{k - 3} \\ M_1 &= \frac{-3}{k - 3} \\ M_2 &= \frac{-5 - 1}{15 - k} \\ M_2 &= \frac{-6}{15 - k} \end{aligned}$$
$$\begin{aligned} M_1 &= M_2 \\ \frac{-3}{k - 3} &= \frac{-6}{15 - k} \\ -45 + 3k &= -6k + 18 \\ 3k + 6k &= 18 + 45 \\ 9k &= 63 \\ k &= 7 \end{aligned}$$

In Extract 13.2, the student gave a correct definition of non collinear points and used the concept of slope of a line through collinear points correctly to find the value of k .

2.14 Question 14: Geometrical Constructions

In this question the students were required to find the number of sides of the polygon whose sum of interior angles is 540° .

The data analysis shows that, 587 students attempted this question of which 60.6 percent scored below 1.0 mark, 9.4 percent scored from 1.0 to 2.5 marks and 30.0 percent scored 3.0 marks. The performance in this question was average because 39.4 percent of the students scored from 1.0 to 3.0 marks.

The responses of students who scored low marks showed that they lacked the knowledge of geometry; and they had no ability of using the correct formula to find the number of sides of polygons. Some students used the formula which relates the number of sides of a polygon and the exterior

angle while others used other wrong approaches. Extract 14.1 shows an example of an incorrect response provided by one of the students.

Extract 14.1

14. The sum of the interior angles of a polygon is 540° . Find the number of the sides of the polygon.

Soln

$$\text{Interior angles} = \frac{(n-2)}{n} \times 180^\circ$$

$$= \frac{(540^\circ - 2)}{540^\circ} \times 180^\circ$$

$$= 538^\circ$$

$\therefore 538^\circ$

In Extract 14.1, the student used incorrect formula to find the number of sides of the given polygon.

The responses from students who scored high marks in this question show that they had adequate knowledge of finding the required number of sides in the polygon. Extract 14.2 illustrates an example of a correct response from one of the students.

Extract 14.2

14. The sum of the interior angles of a polygon is 540° . Find the number of the sides of the polygon.

Soln.

$$\text{Sum of the interior angle} = 180^\circ (n-2)$$

$$180^\circ (n-2) = 540^\circ$$

$$180^\circ n - 2 = 540^\circ$$

$$180n - 360 = 540^\circ$$

$$\begin{array}{r} 180n^\circ = 540^\circ \\ + 360^\circ \\ \hline 900 \end{array}$$

$$\frac{180n^\circ}{180^\circ} = \frac{900}{180} \quad n = 5$$

\therefore Number of the sides = 5

In Extract 14.2, the student wrote the formula for sum of interior angles of a polygon and substituted correctly the given angle, and obtained the required number of sides of the polygon.

2.15 Question 15: Coordinate Geometry

The students were asked to find the points of intersection of the curve $y = x^2$ and the line $y = 2x + 3$.

The data analysis shows that, 587 students attempted this question of which 65.2 percent scored below 1.0 mark, 20.3 percent scored from 1.0 to 2.5 marks and 14.5 percent scored 3.0 marks. The students' performance in this question was average because 34.8 percent scored from 1.0 to 3.0 marks.

The majority of students (65.2%) were unable to answer the question correctly due to lack of knowledge and skills of algebra especially to solve simultaneous equations. Extract 15.1 is an example of a response from one of the students illustrating this case.

Extract 15.1

15. Find the points of intersection of the curve $y = x^2$ and the line $y = 2x + 3$.

Solution

$$y = x^2$$

$$\frac{5x^2}{x}$$

$y = x^2$	$\frac{5x^2}{x}$
Line $y = 2x + 3$	$y = 5x$
$y = 2x + 3$	$y = x^2$
$2x + 3$	$\frac{5x^2}{x}$
$= 5x$	$y = 5x$
$y = x^2$	

In Extract 15.1, the student provided a meaningless solution indicating lack of knowledge of Coordinate Geometry.

A few students (14.5%) had knowledge of solving simultaneous equations as they were able to find the points of intersection by using either graphical or substitution method. Extract 15.2 shows an example of a response from one of the students who provided a correct solution.

Extract 15.2

15. Find the points of intersection of the curve $y = x^2$ and the line $y = 2x + 3$.

Given

$$\begin{cases} y = x^2 & \text{--- (i)} \\ y = 2x + 3 & \text{--- (ii)} \end{cases}$$

To substitute eqn (ii) into (i)

$$y = x^2$$

$$2x + 3 = x^2$$

$$x^2 - 2x - 3 = 0$$

By General formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4 \times 1 \times -3}}{2 \times 1}$$

$$x = \frac{2 \pm \sqrt{4 + 12}}{2}$$

$$x = \frac{2 \pm \sqrt{16}}{2}$$

$$x = \frac{2 \pm 4}{2}$$

$$x = \frac{2+4}{2} \text{ or } \frac{2-4}{2}$$

$$x = 3 \text{ or } -1$$

from

$$y = 2x + 3$$

If $x = 3$ or -1

$$y = 2 \times 3 + 3 \text{ or } 2 \times (-1) + 3$$

$$y = 6 + 3 \text{ or } -2 + 3$$

$$y = 9 \text{ or } 1$$

\therefore The points of intersection is $(3, 9)$ and $(-1, 1)$

In Extract 15.2, the student substituted $y = x^2$ in the equation $y = 2x + 3$ to get the quadratic equation, which he/she solved correctly.

2.16 Question 16: Logic

The question had two parts (a) and (b). In part (a), the students were asked to define the term “Contradiction” as it is used in logic, and in part (b), they were required to write the contrapositive of the proposition $(p \vee q) \rightarrow \sim q$.

The data analysis shows that, 587 students attempted this question of which 83.6 percent scored 0 and 1.9 percent scored 3.0 marks, showing that this question was poorly done.

The students who scored 0 mark failed to define the term contradiction and to write the contrapositive of the proposition $(p \vee q) \rightarrow \sim q$. They lacked knowledge of logic especially in converse, contradiction and contrapositive of propositions. Extract 16.1 shows a poor solution from one of the students.

Extract 16.1

16. (a) Define the term “Contradiction” as it is used in logic.

Contradiction are the symbols which show the means of less than, equal than or greater than.

(b) Write the contrapositive of the proposition $(p \vee q) \rightarrow \sim q$.

$(p \text{ is less than } q) \rightarrow \text{is equal to } q$

In Extract 16.1, the student gave a wrong definition of contradiction and could not write the contrapositive of the given proposition.

On the other hand, students who did well in this question had adequate knowledge of logic. They defined the term contradiction correctly and determined the contrapositive of the compound statement $(p \vee q) \rightarrow \sim q$ by following the required steps. Extract 16.2 illustrates the correct solution written by one of the students.

Extract 16.2

16. (a) Define the term “Contradiction” as it is used in logic.

Contradiction is when in a truth table the result are false only

(b) Write the contrapositive of the proposition $(p \vee q) \rightarrow \sim q$.

$$\begin{aligned} \text{Soln} \\ (p \vee q) &\rightarrow \sim q \\ \sim q &\rightarrow (p \vee q) \\ \sim(\sim q) &\rightarrow \sim(p \vee q) \\ q &\rightarrow \sim p \wedge \sim q \end{aligned}$$

In Extract 16.2, the student defined the term contradiction and wrote the converse and contrapositive of the given proposition correctly.

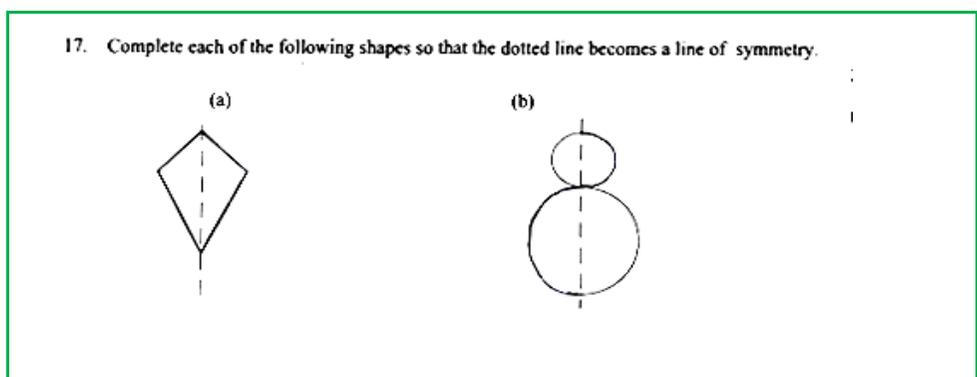
2.17 Question 17: Symmetry

This question had two parts; (a) and (b). In both parts, the students were asked to complete the shapes of the provided figures which contained lines of symmetry indicated by dotted lines.

There were 587 students who attempted this question of which 41.6 percent scored 0, 11.2 percent scored 1.5 marks and 47.2 percent scored 3.0 marks. The performance in this question was good as 58.4 percent scored above 1.0 mark.

The students who scored 3.0 marks were able to apply the knowledge of lines of symmetry to complete drawing the figures correctly. Extract 17.1 shows an example of a good solution from one of the students.

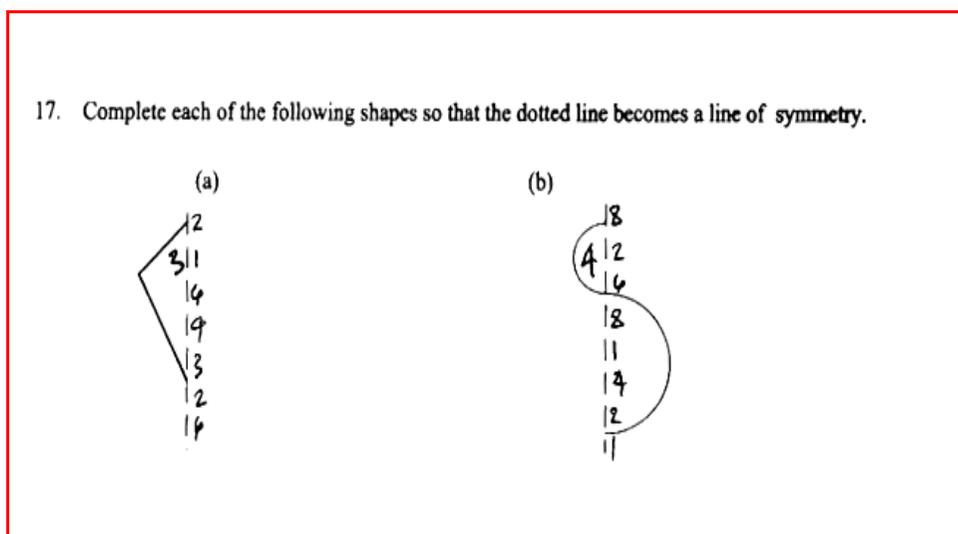
Extract 17.1



Extract 17.1 shows that, the student was able to draw the symmetrical figures in parts (a) and (b) as required.

The students who performed poorly in this question lacked the knowledge of symmetry. Some students wrote answer which were meaningless as they drew lines which had no relation with the lines of symmetry and the given figures. Extract 17.2 is a sample answer showing how the students failed to answer this question.

Extract 17.2



In Extract 17.2, the student wrote numbers did not make sense along the lines of symmetry.

2.18 Question 18: Algebra

In this question, the students were required to find the stationary point on the quadratic function $y = x^2 + 10x + 10$.

The data analysis shows that, 587 students attempted this question of which 97.6 percent scored 0, indicating that the students failed to provide the correct answer. These students were unable to find the stationary point on the quadratic function $y = x^2 + 10x + 10$. They either failed to factorize the equation by completing the square in order to obtain the stationary point or failed prepare correct table of values in order to sketch the graph and locate the point. For instance some students determined the x -intercepts instead of finding the stationary point on the curve, as shown in Extract 18.1.

Extract 18.1

18. Find the stationary point on the quadratic function $y = x^2 + 10x + 10$.

$$x^2 + 10x + 10 = 0$$

by using quadratic formula

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-10 \pm \sqrt{100 - 40}}{2}$$

$$\frac{-10 \pm \sqrt{60}}{2}$$

$$x = -10 + \frac{\sqrt{60}}{2} \text{ or } -10 - \frac{\sqrt{60}}{2}$$

In Extract 18.1, the student wrongly used the formula for solving quadratic equations to find the value of x at which the function is zero instead of finding the stationary point.

A few students (0.7%) who scored full marks had adequate knowledge of stationary points. They solved the question algebraically and obtained the values of x and y which were needed. Extract 18.2 is an example of a solution from a student who performed well in this question.

Extract 18.2

18. Find the stationary point on the quadratic function $y = x^2 + 10x + 10$.

$$x = \frac{-b}{2a}$$

$$x = \frac{-10}{2}$$

$$x = -5$$

$$(x, y) = (-5, -15)$$

$$y = \frac{4ac - b^2}{4a}$$

$$y = \frac{(4 \times 1 \times 10) - 10^2}{4}$$

$$y = \frac{40 - 100}{4}$$

$$y = -15$$

In Extract 18.2, the student used the general formula for solving quadratic equations to determine correctly the stationary point.

2.19 Question 19: Algebra

In this question the students were given that $a * b = ab^2 - 2b$, and they were required to find $(2 * 3) * 5$.

The data indicates that, 587 students attempted the question of which 40.5 percent scored below 1.0 mark, 13.0 percent scored from 1.0 to 2.5 marks and 46.5 percent scored 3.0 marks. In this question the performance was good as 59.5 percent of the students scored from 1.0 to 3.0 marks.

About a half of the students (46.5%) performed well in this question as they had enough knowledge of binary operations. They were able to make correct substitutions of the number in the definition of the binary operation, and they computed correctly the required answer. Extract 19.1 illustrates an example of a good solution.

Extract 19.1

19. If $a * b = ab^2 - 2b$, find $(2 * 3) * 5$

$a * b = ab^2 - 2b$
 $(2 * 3) * 5 = ?$
If $a * b = ab^2 - 2b$
 $2 * 3 = 2 \times 3^2 - 2 \times 3$
 $= 2 \times 9 - 6$
 $= 18 - 6$
 $= 12$
 $2 * 3 = 12$

Then:
 $12 * 5$
 $a * b = ab^2 - 2b$
 $12 * 5 = 12 \times 5^2 - 2 \times 5$
 $= 12 \times 25 - 10$
 $= 300 - 10$
 $= 290$
 $\therefore (2 * 3) * 5 = 290$

In Extract 19.1, the student substituted correctly the values of a and b in the given rule to obtain the correct solution.

The students who scored low marks in this question lacked the knowledge of carrying out binary operations. Others considered binary operation as a multiplication or addition sign. Some of them were able to do the first part of the expression $(2 * 3) * 5$ but failed to complete the second part.

They multiplied, added or factorized the numbers in the given expression. Extract 19.2 illustrates this case.

Extract 19.2

19. If $a * b = ab^2 - 2b$, find $(2 * 3) * 5$.

$$\begin{aligned} & (2 * 3) * 5 \\ & = 2 * 3 * 5 \\ & = 2 + 3 + 5 \\ & = 10 \end{aligned}$$

In Extract 19.2, the student interpreted the symbol “*” as addition sign and then simplified the given expression which led to an incorrect answer.

2.20 Question 20: Numbers

In this question, the students were asked to find an average score of five subjects, given that the average score of a student in four subjects was 80 marks and the score in the fifth subject was 95 marks.

The data shows that 587 students attempted this question of which 74.3 percent scored from 0 to 0.5 marks, 4.7 percent scored from 1.0 to 2.5 marks and 21.0 percent scored 3.0 marks. The performance in this question was poor as 74.3 percent of the students scored below 1.0 mark.

Generally, the majority of students (74.3%) lacked the knowledge of finding the average of numbers. They were unable correctly use correctly the formula for finding average of numbers. Extract 20.1 is a sample answer to illustrate this case.

Extract 20.1

20. A student scored an average of 80 marks in four subjects. Find the average score of the student in five subjects if the score in the fifth subject was 95 marks.

Soln

$$\begin{aligned} \text{Four subject} &= 80 \\ \text{For four subject } \frac{320}{4} &= 80 \\ \text{Five subject will be} & \qquad 95 = \frac{320 + x}{5} \end{aligned}$$

$$\begin{aligned}
 95 \times 5 &= 320 + x \\
 475 &= 320 + x \\
 x &= 475 - 320 \\
 x &= 155
 \end{aligned}$$

Then

$$\begin{aligned}
 \text{Five student} \\
 &= \frac{320 + 155}{5} \\
 &= \frac{475}{5} \\
 &= 95
 \end{aligned}$$

∴ The average is 95

In Extract 20.1, the student wrongly considered 95 as an average of the five subjects and used it to find the marks of the fifth subject and the average score.

A few students (25.7%) had adequate knowledge of Numbers as they calculated the average by applying the correct rule. They determined the average score by dividing the total score by five. Extract 20.2 is a solution illustrating how the students performed well in this question.

Extract 20.2

20. A student scored an average of 80 marks in four subjects. Find the average score of the student in five subjects if the score in the fifth subject was 95 marks.

Let sum be k

$$\begin{aligned}
 \text{Average} &= \frac{\text{Sum}}{\text{Number of items}} \\
 80 &= \frac{k}{4} \\
 320 &= k
 \end{aligned}$$

$$\begin{aligned}
 \text{Sum} &= 320 + 95 \\
 &= 415 \\
 \text{Average} &= \frac{415}{5} \\
 &= 83 \\
 \therefore \text{Average is } &83
 \end{aligned}$$

In Extract 20.2, the student applied correctly the formula for finding the average in answering the question.

2.21 Question 21: Algebra

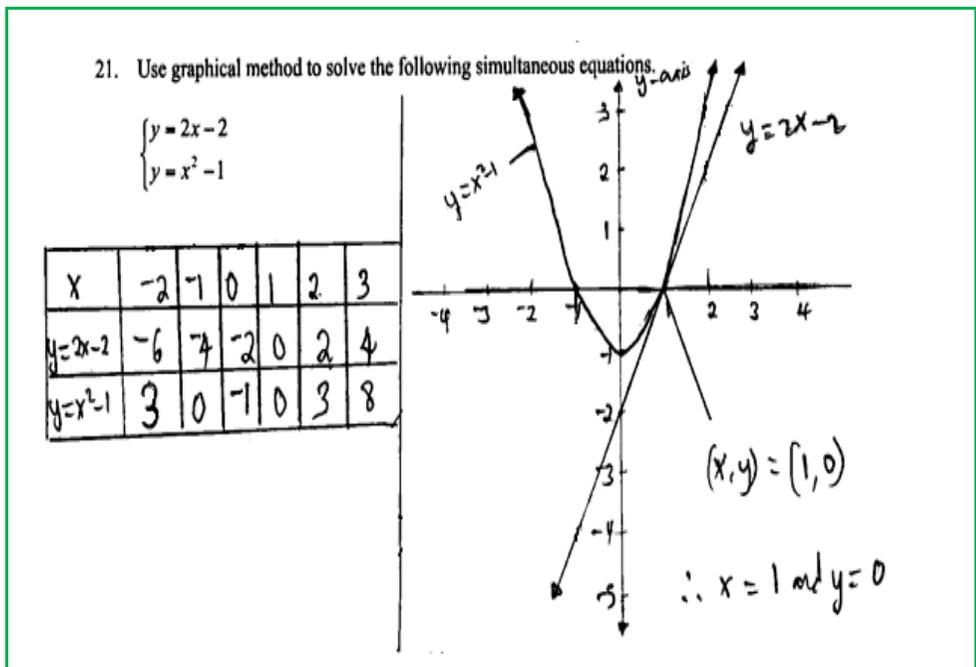
In this question the students were asked to solve the simultaneous

equations: $\begin{cases} y = 2x - 2 \\ y = x^2 - 1 \end{cases}$ by using graphical method.

The data shows that 587 students attempted this question of which 45.8 percent scored from 0 to 2.0 marks, 43.0 percent scored between 2.0 and 7.5 marks and 11.2 percent scored all the 8.0 marks. The performance in this question was good because 54.2 percent of students performed well.

The students who scored full marks in this question were able to answer either some or all parts of the question. They had skills of plotting the graphs of equations and applying them in solving simultaneous equations. They were able to prepare the table of values of x and y and plot to correctly the graphs which were required. Also, they located the point of intersection accurately on the graph. Extract 21.1 is a sample answer showing how these students answered this question correctly.

Extract 21.1



In Extract 21.1, the student prepared a table of values of the two equations, drew well labeled graphs and identified the point of intersection correctly.

On the other hand, 45.8 percent of the students scored very low marks in this question due to lack of knowledge to plot the graphs of simultaneous equations. They were unable to draw the graphs correctly and locate and join the points of the given equations on the graphs. Some students used substitution and elimination methods of solving simultaneous equations instead of graphical method. Extract 21.2 illustrates one of these cases.

Extract 21.2

21. Use graphical method to solve the following simultaneous equations.

$$\begin{cases} y = 2x - 2 \\ y = x^2 - 1 \end{cases}$$

soln.

$$y = 2x - 2$$

$$\frac{2x = -2 - y}{2}$$

$$x = \frac{-2 - y}{2}$$

$$y = x^2 - 1$$

$$y = \left(\frac{-2 - y}{2}\right)^2 - 1$$

$$y = \frac{-2 - y}{2} - \frac{-2 - y}{2} - y - 1 = 0$$

$$2x - \frac{2 - y}{2} - \frac{-2 - y}{2} - 1 \times 2 = 0$$

$$= (-2 - y) - (-2 - y) - 1 = 0$$

$$= -4 - 2y - 2y - y^2 - 1 = 0$$

$$= -4 - 4y - y^2 - 1 = 0$$

$$= y^2 - 4y - 1 - 4 = 0$$

$$= y^2 - 4y - 5 = 0$$

$$y^2 - 4y - 5 = 0$$

$$a = 1, b = -4, c = -5$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \times 1 \times (-5)}}{2 \times 1}$$

$$x = \frac{4 \pm \sqrt{16 + 20}}{2}$$

$$x = \frac{4 \pm \sqrt{36}}{2}$$

$$x = \frac{4 + 6}{2}$$

$$x = \frac{4 - 6}{2}$$

$$x = 5$$

$$x = -1$$

$x = 3$ or $x = 1$

In Extract 21.2, the student solved wrongly the simultaneous equations by using substitution method instead of graphical method.

2.22 Question 22: Sets

In this question, the students were given the following information, in a group of tourists, 37 like chicken, 48 like fish and 45 like beef, 15 like chicken and fish, 13 like fish and beef, 7 like chicken and beef and 5 like all the three” They were then required to (a) draw a Venn diagram to represent the given information, (b) find the number of tourists who like Beef only and (c) calculate the number of tourists in the group if each tourist has at least one choice.

The data shows that there were 587 students who attempted this question, of which 68.7 percent scored below 2.5 marks, 19.0 percent scored from 3.0 to 7.0 marks and 12.3 percent scored 8.0 marks. The performance of students in this question was average as 31.3 percent scored from 3.0 to 8.0 marks.

The majority of the students were unable to fill correctly the disjoint regions of the Venn diagram as they had inadequate knowledge of sets. Some of them were not able to differentiate between concepts of sets and statistics as they used the concept of statistics to answer the question on sets. Extract 22.1 shows an example of an incorrect solution that was provided by one of the students.

Extract 22.1

22. In a group of tourists, 37 like ¹chicken, 48 like ²fish and 45 like ³beef. 15 tourists like ⁴chicken and fish, 13 like fish and beef, 7 like chicken and beef and 5 like all the three.

(a) Draw a Venn diagram to represent this information.

(b) How many tourists like Beef only? Tourists like beef only is 72.

(c) If each tourist has at least one choice, calculate the number of tourists in the group.

Like chicken	Fish	beef	chicken and fish	fish and beef	chicken and beef	all the three
37	48	45	15	13	7	5

$$\begin{aligned}
 &37 + 48 + 45 + 15 + 13 + 7 + 5 \\
 &135 + 60 + 20 + 5 \\
 &195 + 25 \\
 &220 \\
 &= 220
 \end{aligned}$$

∴ The number of tourist in the group is 220

In Extract 22.1, the student used the data wrongly to draw a histogram and frequency distribution table instead of a Venn diagram.

However, a few students (12.3%) answered correctly all parts of the question as they had adequate knowledge and skills of sets. They were able to draw correctly the Venn diagram. Also they were able to work with joint sets as they calculated correctly the number of tourists who like beef only and the number of tourists in the group. Extract 22.2 is an example of a good solution from one of the students.

Extract 22.2

22. In a group of tourists, 37 like chicken, 48 like fish and 45 like beef. 15 tourists like chicken and fish, 13 like fish and beef, 7 like chicken and beef and 5 like all the three.

(a) Draw a Venn diagram to represent this information.

Data

let $C = \{\text{chicken}\}$
 $F = \{\text{fish}\}$
 $B = \{\text{beef}\}$

Hence

$n(C) = 37$
 $n(F) = 48$
 $n(B) = 45$
 $n(C \cap F) = 15$
 $n(F \cap B) = 13$
 $n(C \cap B) = 7$
 $n(C \cap F \cap B) = 5$

Soln

Venn diagram

(b) How many tourists like Beef only? Tourist who like beef only are 30

(c) If each tourist has at least one choice, calculate the number of tourists in the group.

Soln

By using Venn diagram

$n(C \cup F \cup B) = 30 + 20 + 25 + 2 + 8 + 10 + 5$
 $n(C \cup F \cup B) = 50 + 25 + 25$
 $n(C \cup F \cup B) = 50 + 50$
 $n(C \cup F \cup B) = 100$

∴ The number of tourists in the group was 100

In Extract 22.2, the student calculated correctly the number of elements in each disjoint set, then filled in the Venn diagram and determined the number of tourists in the group and those who like beef only.

2.23 Question 23: Logic

The question had three parts; (a), (b) and (c). In part (a) the students were required to write the converse of the statement “If x is a negative number then x^2 is positive.” In part (b), they were asked to show that the proposition $(\sim P \wedge Q) \wedge (Q \rightarrow P)$ is a contradiction by using a truth table. In part (c), they were asked to draw the electrical circuit of the compound statement $P \wedge R \wedge (P \vee S)$.

The data shows that there were 587 students who attempted this question of which 39.9 percent scored from 0 to 2.0 marks, 57.4 percent scored from 2.5 to 7.0 marks and 2.7 percent scored 8.0 marks. The performance in this question was good because many students (60.1%) scored from 2.5 to 8.0 out of 8.0 marks.

The students who scored high marks in this question were able to write the converse of the statement, show the propositions on the truth table and draw the electrical circuit correctly. Extract 23.1 shows an example of the correct answers.

Extract 23.1

23. (a) Write the converse of the statement “If x is a negative number, then x^2 is positive.”

Converse

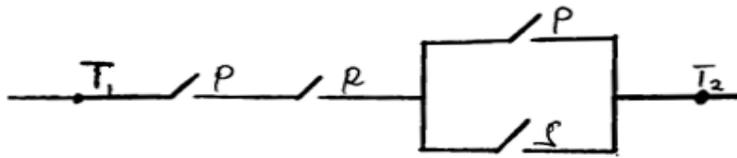
\therefore If x^2 is positive then x is negative

(b) By using a truth table, show that the proposition $(\sim P \wedge Q) \wedge (Q \rightarrow P)$ is a contradiction.

P	Q	$\sim P$	$\sim P \wedge Q$	$Q \rightarrow P$	$a \wedge b$
T	T	F	F	T	F
T	F	F	F	T	F
F	T	T	T	F	F
F	F	T	F	T	F

\therefore The proposition $(\sim P \wedge Q) \wedge (Q \rightarrow P)$ is a contradiction

(c) Draw the electrical circuit of the compound statement $P \wedge R \wedge (P \vee S)$



Extract 23.1 shows that the student wrote clearly the converse of the given statement, drew clearly the truth table to show that the proposition is a contradiction and drew a well labeled electrical circuit.

Despite the good performance, there were few students (19.8%) who performed poorly in this question. Some students lacked knowledge of logic as they were unable to write the converse of the given statement, prepare the truth table and draw the electric circuit. The students were unable to use the logical connectives to find the values of compound propositions while others were unable to relate the logical connectives in order to draw the electrical circuit. Extract 23.2 illustrates one among the poor responses provided by the students.

Extract 23. 2

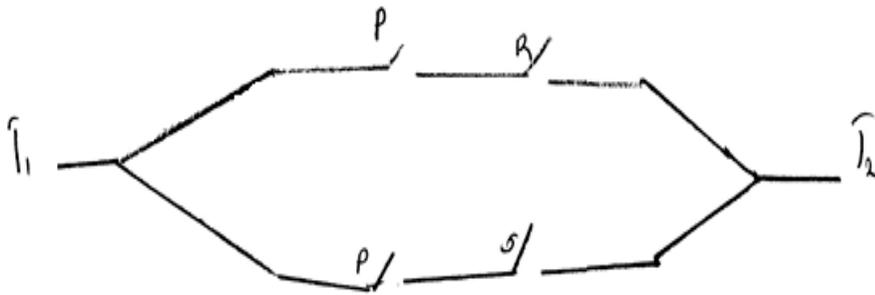
23. (a) Write the converse of the statement "If x is a negative number, then x^2 is positive."

Soln
 x - negative number
 x^2 - positive number
 -
 $x \approx x^2$

(b) By using a truth table, show that the proposition $(\sim P \wedge Q) \wedge (Q \rightarrow P)$ is a contradiction.

$(\sim P \wedge Q) \wedge (Q \rightarrow P)$	T	F	T	F
T	T	F	T	F
F	F	T	F	T
T	F	T	F	T
F	F	T	F	T

(c) Draw the electrical circuit of the compound statement $P \wedge R \wedge (P \vee S)$



In Extract 23.2, the student was unable to write the converse of the statement; use logical connectives to fill in the truth table and draw incorrect electrical circuit.

2.24 Question 24: Coordinate Geometry

The question had two parts; (a) and (b). In part (a), the students were given the line $y = 2x + 4$ which is parallel to another line which passes through the points $(k, 4)$ and $(4, 6)$ and they were asked to find the value of k . In part (b), the students were required to find the equation of a line which is perpendicular to the line $y = -\frac{3}{4}x + 4$ and passes through the point $(1, 4)$.

The data shows that 587 students attempted this question of which 53.7 percent scored from 0 to 2.0 marks, 37.3 percent scored from 2.5 to 7.5 marks and 9.0 percent scored 8.0 marks. The students performance in this question was average as 46.3 percent scored from 2.5 marks and above.

The students who failed to provide the correct solution lacked the knowledge of parallel lines and perpendicular lines. They were unable to apply the concept of slopes of parallel and perpendicular lines to find the value of k . Extract 24.1 shows an example of a student's incorrect response.

Extract 24.1

24. (a) The line $y = 2x + 4$ is parallel to the line which passes through points $(k, 4)$ and $(4, 6)$. Find the value of k .

$$k + 4 = 4 + 6$$

$$\frac{4k}{4} = \frac{10}{4}$$

$$\underline{k = \frac{10}{4}}$$

- (b) Find the equation of the line which is perpendicular to $y = -\frac{3}{4}x + 4$ and passes through the point $(1, 4)$.

$$4 = \frac{3}{4}x + 4$$

$$16 = 3x + 4$$

$$\frac{16}{7} = \frac{7x}{7}$$

$$x = \frac{16}{7}$$

In Extract 24.1, the student determined wrongly the value of k and could not find the equation of the line which is perpendicular to the given line.

The students who provided good solution had adequate knowledge of Coordinate Geometry. They were able to find the value of k and the equation of a line as required. Extract 24.2 shows an example of a correct solution from one of the students.

Extract 24.2

24. (a) The line $y = 2x + 4$ is parallel to the line which passes through points $(k, 4)$ and $(4, 6)$. Find the value of k .

<p><u>Data</u> $y = 2x + 4$ Points A $(k, 4)$ B $(4, 6)$ <u>Slope:</u> <u>Consider line 1</u> $y = 2x + 4$ $M_1 = 2$</p>	<p><u>From</u> $M_1 = M_2$ <u>Consider points</u> $m = \frac{y_2 - y_1}{x_2 - x_1}$ $2 = \frac{6 - 4}{4 - k}$ $\frac{2}{1} = \frac{2}{4 - k}$ $2 = 2(4 - k)$</p>	<p>$2 = 2(4 - k)$ $2 = 8 - 2k$ $2 - 8 = -2k$ $\frac{-6}{-2} = \frac{-2k}{-2}$ $3 = k$ <u>∴ The value of k is 3</u></p>
--	---	---

- (b) Find the equation of the line which is perpendicular to $y = -\frac{3}{4}x + 4$ and passes through the point $(1, 4)$.

<p><u>Data</u> $L_1: y = -\frac{3}{4}x + 4$ Point $(1, 4)$ <u>Consider L_1</u> $y = -\frac{3}{4}x + 4$ <u>Slope</u> $M_1 = -\frac{3}{4}$ <u>From</u> $M_1 M_2 = -1$</p>	<p>$M_1 M_2 = -1$ $-\frac{3}{4} M_2 = -1$ $-3 M_2 = -4$ $\frac{-3 M_2}{-3} = \frac{-4}{-3}$ $M_2 = \frac{4}{3}$ <u>Consider point</u> Eqn of the line $y = M(x - x_1) + y_1$ $y = \frac{4}{3}(x - 1) + 4$</p>	<p>$y = \frac{4}{3}x - \frac{4}{3} + 4$ $y = \frac{4}{3}x + \frac{8}{3}$ <u>∴ The equation of the line is $y = \frac{4}{3}x + \frac{8}{3}$</u></p>
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In Extract 24.2, the student used the relationship of slopes of parallel and perpendicular lines to find correctly the value of k in part (a) and the equation of the line in part (b) respectively.

2.25 Question 25: Variations

The students were given a variable y which varies directly proportional to the square of x and inversely proportional to z . Also, they were given that when $y = 12$, $x = 10$ and $z = 50$.

The analysis shows that there were 587 students who attempted this question of which 60.5 percent scored below 3.0 marks, 18.2 percent scored from 3.0 to 7.0 marks and 21.3 percent scored 8.0 marks. The students performance in this question was average because 39.5 percent scored 3.0 marks and above.

The majority of students (60.5%) performed poorly in this question. The responses show that, some students were unable to write the formula for joint variation which relates the variables that were given in the question. Others failed to differentiate between direct variation and joint variation because they lacked the knowledge of variations. Extract 25.1 shows an example of incorrect solution.

Extract 25.1

25. Given that y is directly proportional to the square of x and inversely proportional to z . If $y = 12$ when $x = 10$ and $z = 50$; find y when $x = 50$ and $z = 30$.

$$\begin{aligned}y &= 12 \text{ when } x = 10 \text{ and } z = 50 \\z &= 50 + 30 = x 50 + 10 = y 12 \\z &= 80 = x 60 = y 12 \\ \therefore z &= 80 \text{ Ans.} \\ \therefore x &= 60 \text{ Ans.} \\ \therefore y &= 12 \text{ Ans.}\end{aligned}$$

In Extract 25.1, the student found the sum of values of each variable instead of writing the equation relating the variables and finding the required value of y .

The students who provided good solutions in this question had adequate knowledge of joint variations as they were able to write correctly the

relationship of the variables that were given. They were able to show clearly all necessary steps which should be followed when solving problems about joint variations. Extract 25.2 shows the response from a student who provided the correct solution.

Extract 25.2

25. Given that y is directly proportional to the square of x and inversely proportional to z . If $y = 12$ when $x = 10$ and $z = 50$; find y when $x = 50$ and $z = 30$.

<u>Data given</u>	<u>Soln</u>
$y = 12, x = 10, z = 50$	
$y = ?, x = 50, z = 30$	

$$y \propto \frac{x^2}{z}$$

$$y = k \frac{x^2}{z}$$

$$12 = \frac{k(10)^2}{50}$$

$$12 = \frac{100k}{50}$$

$$\frac{12}{2} = \frac{2k}{2}$$

$$k = 6$$

$$y = k \frac{x^2}{z}$$

$$y = \frac{6(50)^2}{30}$$

$$y = \frac{6(2500)}{30}$$

$$y = \frac{15000}{30}$$

$$y = 500$$

∴ $y = 500$ Ans.

In Extract 25.2, the student was able to find the variation equation and made correct substitution of the values into it in order to find the constant and the value of y .

3.0 ANALYSIS OF STUDENTS' PERFORMANCE IN EACH TOPIC

In the FTNA 2015 Additional Mathematics, several questions were set from a single topic. The only exceptions were two topics: Sets and Locus, which had one question each. The students' performance in a topic is the average of the performance on the questions from that topic. The performance was good in the topics of Symmetry and Algebra. It was average in six topics, namely Logic, Variation, Numbers, Geometrical Constructions, Coordinate Geometry and Sets. It was poor in the topic of Locus. Moreover, the analysis shows that the students' performance was high in Symmetry, but it was low in Locus. However, the average performance per topic was 42.0 percent (see **Appendix A**).

4.0 CONCLUSION AND RECOMMENDATIONS

4.1 Conclusion

The analysis of the performance of students in Additional Mathematics paper for the Form Two National Assessment (FTNA) shows that, the performance was good in ten questions, average in ten questions and weak in five questions (see **Appendix A**). Generally, the overall performance of candidates in this subject was average.

The comparison of performance of the years 2014 and 2015 shows that the performance has increased in six topics, which are Symmetry, Algebra,

Numbers, Coordinate Geometry, Sets, and Locus (see **Appendix B** and **C**). On the other hand, the performance has decreased in three topics which are Logic, Variations, and Geometrical Constructions. The decrease in students' performance was caused by lack of knowledge and skills in different topics as well as failure to recall and apply mathematical rules, theorems and formulae in solving questions. Also the performance was low due to students' inability to understand and use English to identify the needs of the question.

4.2 Recommendations

4.2.1 Recommendations to Students

In order to improve the performance of the students in Additional Mathematics the students should:-

- (a) solve many questions in each topic in order to understand the concepts in different topics.
- (b) read books and do many exercises that will enhance their ability to recall and apply rules, theorems and formulae in answering questions.

4.2.2 Recommendations to Teachers

Teachers should make sure that they provide many exercises to students and guide them in solving the questions. They should also assist the students to make good preparation for assessments by encouraging them to read books and do many exercises.

The Summary of Students' Performance by Topics

S/N	Topic	Performance in Each Question		The Percentage of Students Performance who Scored 30% or More	Remarks
		Question Number	Percentage of Performance		
1	Symmetry	8	70.4	64.4	Good
		17	58.4		
2	Algebra	2	69.7	50.2	Good
		5	70.4		
		6	42.9		
		9	53.0		
		18	2.0		
		19	59.5		
3	Logic	7	70.0	48.8	Average
		16	16.4		
		23	60.1		
4	Variations	10	48.0	43.8	Average
		25	39.5		
5	Numbers	1	40.2	39.9	Average
		4	53.8		
		20	25.7		
6	Geometrical Constructions	3	38.5	39.0	Average
		14	39.4		
7	Coordinate Geometry	12	14.7	34.4	Average
		13	41.6		
		15	34.8		
		24	46.3		
8	Sets	22	31.3	31.3	Average
9	Locus	11	26.2	26.2	Weak
Overall Average Performance				42.0	Average

In this Appendix, green, yellow and red colours, represent good, average and weak average percentage of students' performance respectively.

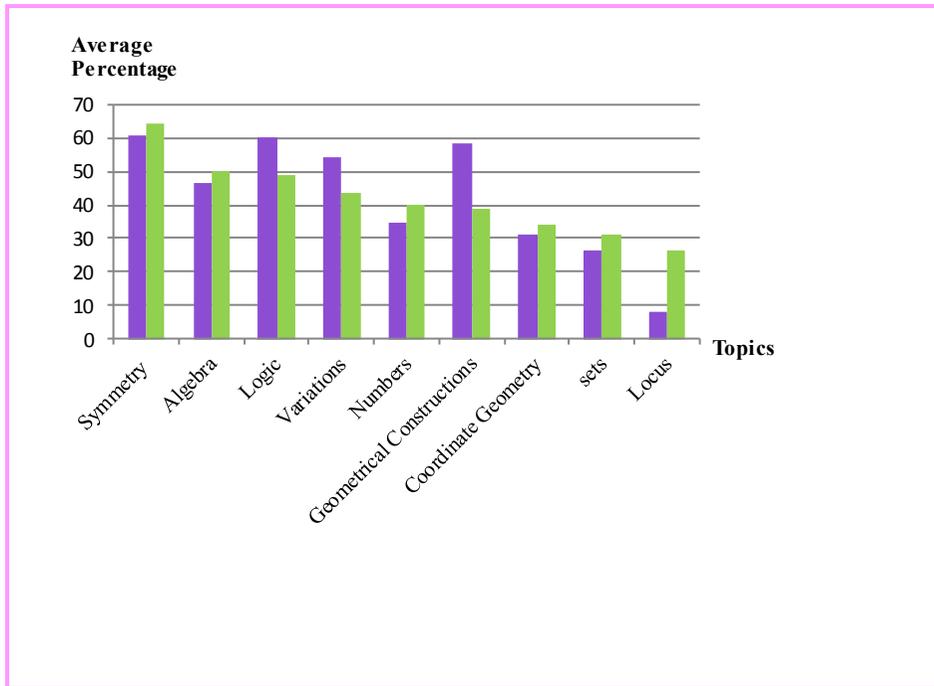
APPENDIX B

A Comparison of the Students' Performance in 2014 and 2015

S/N	Topic	Year	
		2014	2015
1	Symmetry	60.8	64.4
2	Algebra	46.4	50.2
3	Logic	59.9	48.8
4	Variations	54.3	43.8
5	Numbers	34.9	39.9
6	Geometrical Constructions	58.2	39
7	Coordinate Geometry	31.1	34.4
8	sets	26.6	31.3
9	Locus	7.9	26.2
	Average	42.2	42.0

This table shows the average performance of students in each topic in 2014 and 2015.

The Histogram showing the Comparison of the Student Performance for the Years 2014 and 2015



In this graph the blue and green bars represent the average performance of students in 2014 and 2015 respectively.

