

THE NATIONAL EXAMINATIONS COUNCIL OF TANZANIA



**STUDENTS' ITEMS RESPONSE ANALYSIS
REPORT FOR THE FORM TWO NATIONAL
ASSESSMENT (FTNA) 2018**

042 ADDITIONAL MATHEMATICS

THE NATIONAL EXAMINATIONS COUNCIL OF TANZANIA



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042 ADDITIONAL MATHEMATICS

Published by

The National Examinations Council of Tanzania,

P.O. Box 2624,

Dar es Salaam, Tanzania.

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FOREWORD

The Additional Mathematics Students' Items Response Analysis report for the Form Two National Assessment (FTNA) 2018 was written in order to provide feedback to students, teachers, parents, policy makers and other education stakeholders on how students answered the questions.

FTNA is the formative evaluation in Ordinary Level Secondary Education which is done at the end of second year. The significance of FTNA is to show the effectiveness of education system in general and education delivery system in particular to secondary school students in Form One and Form Two. Essentially, students' responses to the assessment questions are strong indicators of what the education system was able or unable to offer to students in the two years of Ordinary Level Education. Therefore, the 042 Additional Mathematics responses to the assessment questions reveal the strengths and weaknesses of students in achieving the objectives of the subject.

The students' performance in 042 Additional Mathematics was average. The analysis presented in this report is intended to contribute towards understanding the reasons behind the average performance of students. Such reasons include moderate knowledge and skills on most topics in 042 Additional Mathematics.

The feedback provided will enable education stakeholders to identify measures to be taken in order to improve students' performance in the future assessment.

Finally, the National Examinations Council of Tanzania would like to thank all who participated in the preparation of this report.



Dr. Charles E. Msonde
EXECUTIVE SECRETARY

1.0 INTRODUCTION

This report is the analysis of the performance of students in the Additional Mathematics for Form Two National Assessment (FTNA) in 2018. The paper was set to assess competences and knowledge acquired by students in accordance with 2018 assessment format and 2010 syllabus. The 042 Additional Mathematics paper consisted of ten compulsory questions, which carried 10 marks each.

In FTNA 2018 for Additional Mathematics there were a total of 701 registered students out of which 383 were boys and 318 girls. The analysis of data shows that 667 students sat for assessment, out of which 57 (8.5%) scored A, 59 (8.9%) scored B, 155 (23.2%) scored C, 124 (18.6%) scored D and 272 (40.8%) scored F. Generally, 395 (59.2%) students passed the assessment. Comparatively, in 2017 a total of 729 students sat for Additional Mathematics out of which 305 (46.5%) passed the assessment. This shows that the students' performance in 2018 has increased by 12.7 percent compared to 2017.

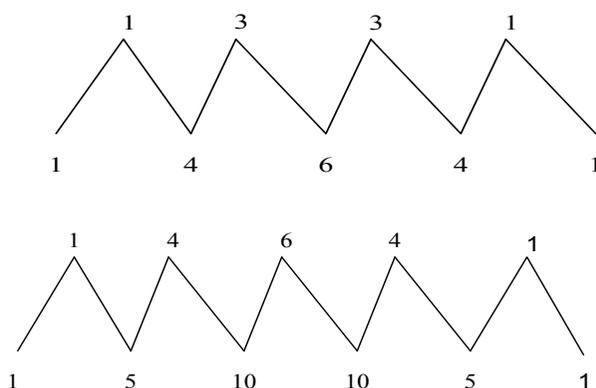
This report provides feedback to stakeholders on the performance of students by stating the strengths and weaknesses in responding to the questions. In analysing the students' performance in each question the performance is graded as good, average or weak if the percentage of the students who scored 30 percent or above of allotted marks to a question is 65-100, 30-64 or 0-29 respectively. In this report, the students' performance is presented in different charts and tables where red, yellow and green colours represent weak, average and good respectively.

The analysis of students' performance is done by showing the requirements of the question, the distribution of students' scores, strengths and weaknesses of the students in attempting the questions. Furthermore, Extracts for both good and poor responses from the students in each question have been included in the description in order to elaborate the cases.

2.0 ANALYSIS OF STUDENTS' PERFORMANCE PER QUESTION

2.1 Question 1: Numbers

This question consisted of three parts; (a), (b) and (c). In part (a), students were required to write the first four multiples of 17. In part (b), they were required to identify the numbers which are divisible by 3 among 8476, 942, 5181, 7124, 35768 and 91284. In part (c), they were required to predict the next two patterns of triangular numbers as follows:



The analysis of data shows that, 651 students attempted this question, out of which 30.6 percent scored from 0 to 2.5 marks, 26.5 percent scored from 3 to 6 marks and 42.9 percent scored from 6.5 to 10 out of 10 marks. Generally, this question had a good performance since 69.4 percent scored above 2.5 out of 10 marks. Figure 1 illustrates the performance of students in this question.

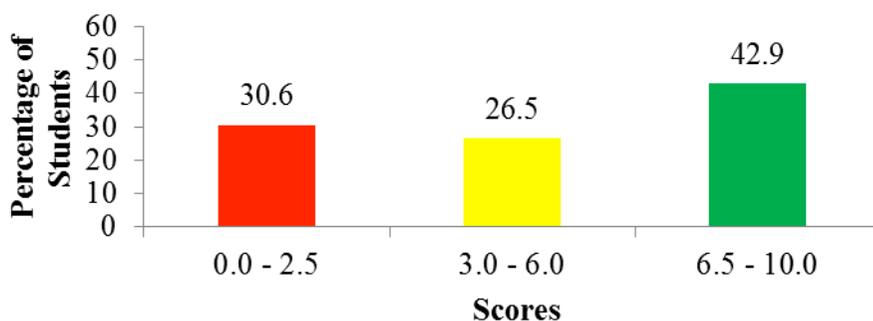


Figure 1: Students' performance in question 1.

The students who scored high marks were able to answer correctly two or all three parts of the question. In part (a), they were able to state clearly the

list of numbers which are multiples of 17. For instance, some students found the multiples of 17 by performing the addition process as follows:

Taking the first number 17, then the second number is $17 + 17 = 34$, the third number is $34 + 17 = 51$ and the fourth number is $51 + 17 = 68$. Other students computed the multiples of 17 as follows:

“ $17 \times 1 = 17$; $17 \times 2 = 34$; $17 \times 3 = 51$; $17 \times 4 = 68$ ”.

In general, students were required to list the multiples of 17 as 17, 34, 51 and 68.

In part (b), the students were able to use divisibility rule of 3 to determine whether the given numbers are divisible by 3 or not. The divisibility rule of 3 states that, “a number is divisible by 3 if the sum of the digits forming the number is divisible by 3”. Some students answered this part as follows:

“ $8476 = 8 + 4 + 7 + 6 = 25$

$2 + 5 = 7$ ”

which is not divisible by 3. Therefore, 8476 is not divisible by 3.

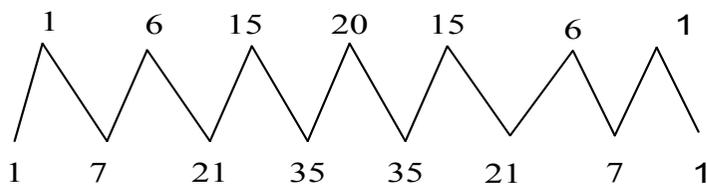
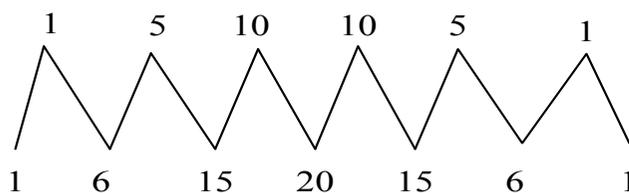
“ $942 = 9 + 4 + 2 = 15$

$1 + 5 = 6$ ”

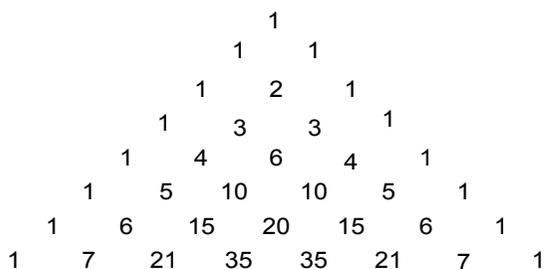
which is divisible by 3. Therefore, 942 is divisible by 3.

The same test was done on numbers 5181, 7124, 35768 and 91284.

In part (c), some students were able to correctly predict the next two patterns. They managed to get the second, third, fourth and fifth entries of the next two patterns. Some of the responses provided by students were as follows:



Others used Pascal's triangle instead of drawing the given patterns. It was presented as follows:



These responses indicate that the students had adequate knowledge on the topic of *Numbers*. Extract 1.1 shows a sample of a correct response from one of the students who performed well in this question.

Extract 1.1

1. (a) Solution
 $17 = 17, 34, 51, \cancel{68}, 68, 85, 102, \dots$
 The first four multiples of 17 = 17, 34, 51 and 68
 \therefore The first four multiples of 17 = 17, 34, 51 and 68

1. (b) Solution:
 Given 8476, 942, 5181, 7124, 35768 and 91284
 Required to finding numbers which divided by 3 exactly.
 From the rule of divisibility by 3 which state that;
 A number is ~~divisibly~~ divisible by 3 when the sum of its digits is divisible by 3.

$$\begin{array}{l}
 \underline{8476} \\
 = 8+4+7+6 \\
 = 25 \\
 = 2+5 \\
 = 7 \\
 \therefore 8476 \text{ is not divisible by 3 exactly.}
 \end{array}$$

$$\begin{aligned}
 &\underline{942} \\
 &= 9 + 4 + 2 \\
 &= 15 \\
 &= 1 + 5 \\
 &= 6
 \end{aligned}$$

$\therefore 942$ is ~~divisibly~~ divisible by 3 exactly.

$$\begin{aligned}
 &\underline{5181} \\
 &= 5 + 1 + 8 + 1 \\
 &= 15 \\
 &= 1 + 5 \\
 &= 6
 \end{aligned}$$

$\therefore 5181$ is divisible by 3 exactly.

$$\begin{aligned}
 &\underline{7124} \\
 &= 7 + 1 + 2 + 4 \\
 &= 14 \\
 &= 1 + 4 \\
 &= 5
 \end{aligned}$$

$\therefore 7124$ is not divisible by 3 exactly.

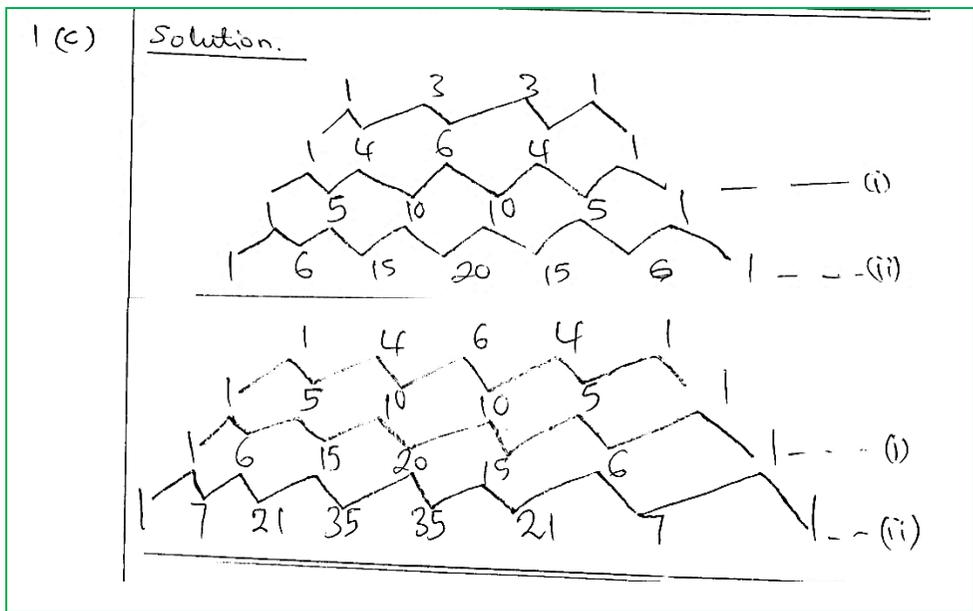
$$\begin{aligned}
 &\underline{35768} \\
 &= 3 + 5 + 7 + 6 + 8 \\
 &= 29 \\
 &= 2 + 9 \\
 &= 11 \\
 &= 2
 \end{aligned}$$

$\therefore 35768$ is not divisible by 3 exactly.

$$\begin{aligned}
 &\underline{91284} \\
 &= 9 + 1 + 2 + 8 + 4 \\
 &= 24 \\
 &= 2 + 4 \\
 &= 6
 \end{aligned}$$

91284 is divisible by 3 exactly.

\therefore Number which are divided by 3 exactly = 942, 5181
and 91284.



In Extract 1.1, the student was able to find multiples of 17, use divisibility rules for testing if the numbers are divisible by 3 and find the next two numbers in the given pattern.

On the other hand, 30.6 percent of students had poor performance in this question as they scored below 2.5 marks. The poor performance is attributed to lack of knowledge and skills that led to incorrect use of concepts of numbers in answering the question. For instance, in part (a), students had misconceptions on multiples of 17. They added two different numbers which resulted to 17 instead of finding multiples of 17 as required. Furthermore, there were students who had inadequate skills in arithmetic as they were unable to add the numbers correctly, for instance, one of these students wrote, “ $6 + 9 = 17$ ” instead of $6 + 9 = 15$. In part (b), they added the digits of the given numbers but could not use divisibility rule as required by the question.

In part (c), the students were not able to recognise the patterns thus failed to predict the next patterns. For instance, some of them found the new patterns by adding 1 to each intermediate numbers in the given pattern. Finally, the patterns obtained were 1, 5, 7, 5, 1 and 1, 6, 11, 11, 6, 1. This indicates that these students had inadequate knowledge on *Numbers*. Extract 1.2 shows an example of poor solutions from one of the students.

Extract 1.2

a/ The ^{soln} number which is divisible by 17 is

$$7 + 10 = 17$$

$$9 + 8 = 17$$

$$6 + 9 = 17$$

b/ soln

$$8 + 4 + 7 + 6 = 25$$

$$9 + 4 + 2 = 17$$

$$5 + 1 + 8 + 1 = 15$$

$$7 + 1 + 2 + 4 = 14$$

$$3 + 5 + 7 + 6 + 8 = 29$$

$$9 + 1 + 2 + 8 + 4 = 25$$

This number is not divisible by 3

a/ y

b/ y

In Extract 1.2, the student was unable to find the multiples of 17, use divisibility rules for testing whether the given numbers are divisible by 3 or not and use the given patterns to find the next two patterns.

2.2 Question 2: Algebra

This question had three parts; (a), (b) and (c). The students were required to make b the subject of the equation $\frac{1}{a} + \frac{1}{b} = \frac{1}{c}$ in part (a). In part (b), they were required to determine the value of n by using the formula $c = \frac{nE}{R + nr}$, where $c = 1.5$, $E = 3$, $R = 6.3$ and $r = 1.1$; and in part (c), they were required to show the inequalities (i) $x \geq -3$ and (ii) $-2 < x < 3$ on a number line.

The analysis of data shows that, 661 students attempted this question, out of which 31.2 percent scored from 0 to 2.5 marks, 18.9 percent scored from 3 to 6 marks and 49.9 percent scored from 6.5 to 10 marks. The students' performance in this question was good as 68.8 percent of students scored above 2.5 out of 10 marks. Figure 2 illustrates the performance of students in this question.

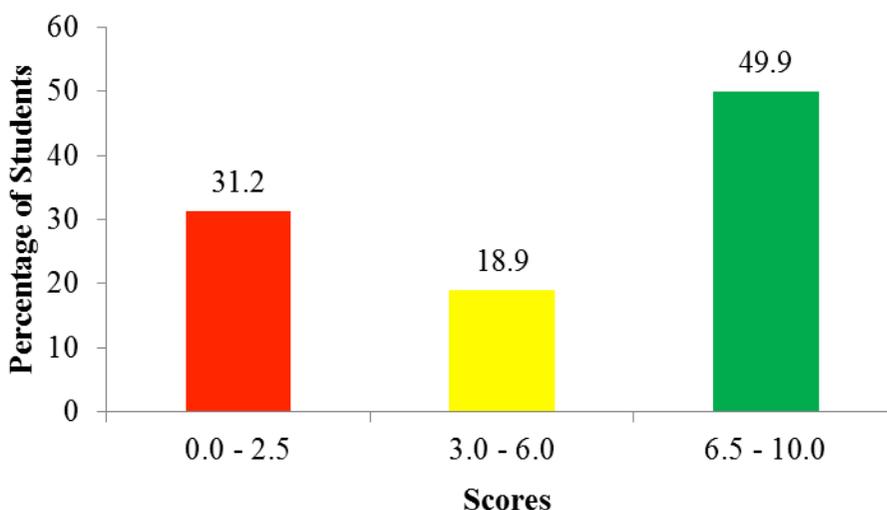


Figure 2: Students' performance in question 2.

The students who did well in this question had sufficient knowledge on the tested concepts. They managed to show all the necessary procedures that were required in answering this question. In part (a), the students were able to make b the subject of the given equation. They further used the correct manipulation and simplification rules in answering the question. In part (b), they used the rule of cross multiplication and later applied the steps similar

to part (a) to make n the subject. Then, they substituted the given values of c , E , R and r to determine the value of n . In part (c), they managed to indicate appropriately the regions defined by the given inequalities on the number lines. Moreover, they correctly used proper symbols to present the concepts of inclusive and exclusive numbers in inequalities on number lines. Extract 2.1 shows a sample of the best solutions from one of the students.

Extract 2.1

$$\begin{aligned}
 \text{Q (a)} \quad \frac{1}{a} + \frac{1}{b} &= \frac{1}{c} \\
 \frac{1}{b} &= \frac{1}{c} - \frac{1}{a} \\
 \frac{1}{b} &= \frac{a-c}{ca} \\
 ca &= b(a-c), \\
 ca &= ba - bc \quad (\text{put "b" outside.}) \\
 \frac{ca}{a-c} &= \frac{b(a-c)}{a-c} \quad (\text{Divide by } a-c \text{ each side}) \\
 b &= \frac{ca}{a-c}.
 \end{aligned}$$

$$\begin{aligned}
 \text{Q (b)} \quad C &= \frac{nE}{R+nr} \\
 \text{where by } C &= 1.5 \\
 E &= 3 \\
 R &= 6.3 \\
 r &= 1.1 \\
 n &= ? \\
 1.5 &= \frac{3n}{6.3 + 1.1n}.
 \end{aligned}$$

$$1.5(6.3 + 1.1n) = 3n$$

$$9.45 + 1.65n = 3n$$

$$9.45 = 3n - 1.65n$$

$$9.45 = 1.35n$$

$$n = \frac{9.45}{1.35}$$

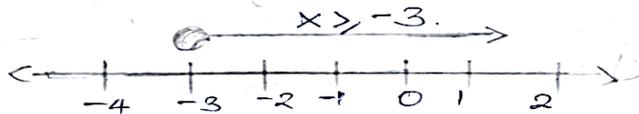
$$n = 7.$$

The Value of $n = 7.$

2(a).

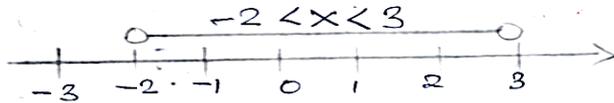
(i)

$$x \geq -3.$$



(ii)

$$-2 < x < 3$$



In Extract 2.1, the student was able to make b the subject, determine the value of n from the given formula; and he/she showed each of the given inequalities on number lines correctly.

Nevertheless, some students (31.2%) performed this question poorly since they scored below 3 out of 10 marks. They were unable to perform the transposition of the given formula, make substitution of values which were provided and represent the inequalities on number lines. In part (a), the students failed to use the algebraic operations properly to make b the subject. For instance, some of the students answered wrongly as follows:

$$\left\langle \frac{1}{a} + \frac{1}{b} = \frac{1}{c}; \frac{1}{a} + 1 \times c = 1 \times b; \right.$$

$$a \times \frac{1}{a} + 1c = 1b \times a; 1 + 1c = 1ba$$

$$\therefore 2c = 1ba''.$$

In part (b), some students failed to interpret the given formula; therefore they made incorrect substitution of values and simplification of equation.

For instance, one of the students wrote the following answer:

$$\left\langle c = \frac{nE}{R + nr} = c = 1.5, E = 3, R = 6.3, r = 1.1 = 15 = 3 = 6.3 = 11; 15 = 3 = 18 = 71 = 8; R + nr = 81; \therefore r = 81'' \right.$$

which is irrelevant. Extract 2.2 shows a sample of poor solution from one of the students.

Extract 2.2

Handwritten student work for Extract 2.2:

Q $\frac{1}{a} + \frac{1}{b} = \frac{1}{c}$ make b the subject of equation.

$$\frac{1}{a} + \frac{1}{c} = \frac{1}{b}$$

Q $\frac{nE}{R + nr}$ Find the value of n when $C = 1.5E =$

$R = 6.3$ and $r = 1.1$

$$\frac{AE}{A} = \frac{A}{A}$$

$$E (1.5E) = 3$$

$$R = 6.3$$

$$E = (1.5E) \quad R = 6.3$$

$$\frac{1.5}{1.5} \quad \frac{6.3}{1.1}$$

$$\cancel{E} = 5 \quad R = 6.3$$

$$5 \times 6.3$$

$$\frac{5}{5} \quad 56.3$$

(a) $x \geq -3$ (b) $-9 < x < 3$

Soln

$x \geq -3$
 $x + 3 \geq -3$
 $x + 3 \geq -6$
 $x + 3 \geq -3$
 $x \geq -3$
 $x - 3 \geq$

(b) $-9 < x < 3$
 $-2 < x < 3$
 $1 < x < 3$
 $2 < x < 3$
 $2 < x < 3$
 $1 < x < 3$
 $x = 1$

In Extract 2.2, the student was unable to make b the subject, determine the value of n from a given formula and represent the given inequalities on number lines.

2.3 Question 3: Geometrical Constructions

The question had two parts; (a) and (b). In part (a), the students were required to find the sum of interior angles of regular pentagon and hence determine the size of each interior angle, while in part (b) they were required to draw a regular hexagon with 5cm each side.

This question was attempted by 643 students, out of which 48.5 percent scored from 0 to 2.5 marks, 31.1 percent scored from 3 to 6 marks and 20.4 percent scored from 6.5 to 10 marks. The general students' performance in this question was average because 51.5 percent of the students scored above 2.5 out of 10 marks. Figure 3 represents a summary of students' performance in this question.

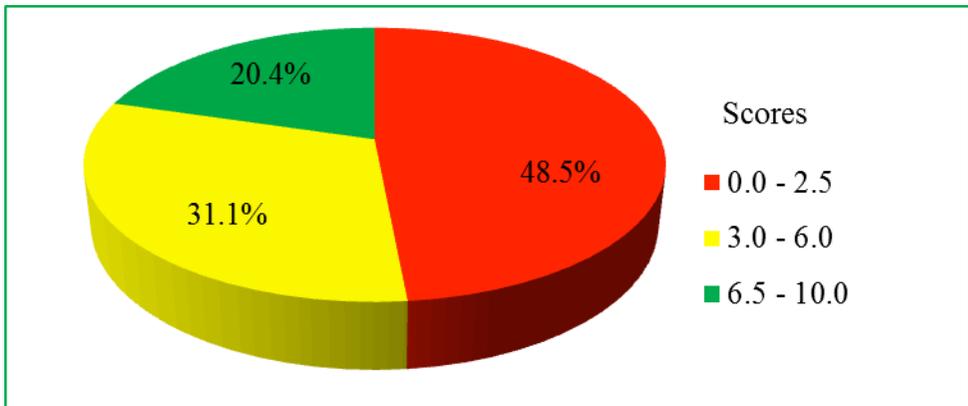


Figure 3: Students' performance in question 3.

The students who performed well were able to answer this question correctly. They had adequate knowledge and skills on constructing the polygons. In part (a), they were able to find the sum of interior angles of a regular pentagon by using the formula: Sum of interior angle = $(n - 2)180^{\circ}$ and hence managed to determine the size of each interior angle by using the formula: Interior angle = $\frac{(n - 2)180^{\circ}}{n}$. In part (b), the students managed to draw correctly a regular hexagon with 5cm each side. The responses show that those students had adequate knowledge on *Geometrical Constructions*. Extract 3.1 indicates a sample answer from one of the students who demonstrated his/her work clearly.

Extract 3.1

3a) Pentagon = 5 sided polygon
 Sum of interior angles = $(n - 2) 180^{\circ}$
 $n = \text{number of sides}$
 Sum of interior angles = $(n - 2) 180^{\circ}$
 $= (5 - 2) 180^{\circ}$
 $= (3) 180^{\circ}$
 $= 3 \times 180^{\circ}$
 $= 540^{\circ}$
 \therefore Sum of interior angles = 540°

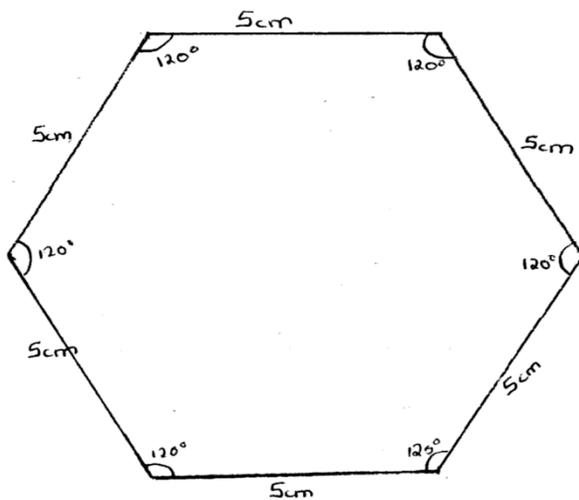
$$\begin{aligned}
 \text{Size of each interior angle} &= \frac{(n-2)180}{n} \text{ or} \\
 &= \frac{\text{Sum of interior angle}}{n} \\
 &= \frac{540^\circ}{5} \\
 &= 108^\circ
 \end{aligned}$$

\therefore The size of each interior angle is 108°

Hexagon = 6 sided polygon

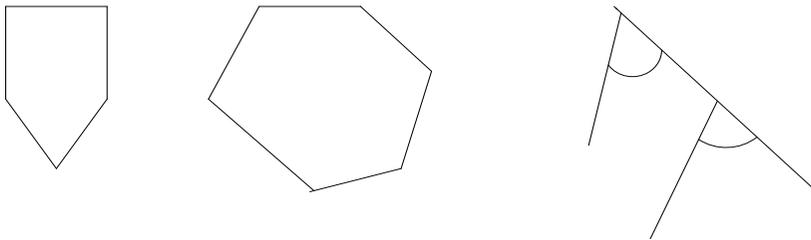
$$\begin{aligned}
 \text{Size of each interior angle} &= \frac{(n-2)180}{n} \\
 &= \frac{(6-2)180}{6} \\
 &= \frac{4 \times 180}{6} \\
 &= \frac{720}{6} \\
 &= 120
 \end{aligned}$$

REGULAR HEXAGON WITH 5CM EACH SIDE



In Extract 3.1, the student was able to find the sum and size of interior angles of regular pentagon and draw a regular hexagon correctly.

However, there were some students (48.5%) who performed poorly in this question. These students had inadequate skills on constructing different polygons by using geometrical instruments such as compasses, protractors and rulers. In addition, they lacked computational skills and also misconceived the formula for interior angles and exterior angles. Therefore, they used the formula for exterior angle to find the interior angle. Furthermore, they failed to draw a regular hexagon of 5cm each side. Others drew irregular pentagon, irregular heptagon and other unrelated geometrical figures as follows:



These responses indicate that the student had inadequate knowledge on *Geometrical Constructions*. Extract 3.2 shows a sample answer from a student who poorly answered this question.

Extract 3.2

Solu

3a)

$$n = \frac{360}{\text{ext}}$$

$$5 = \frac{360}{\text{ext}}$$

$$\frac{5 \text{ ext}}{5} = \frac{360}{5}$$

$$\text{ext} = \frac{360}{5}$$

$$\text{ext} = 72$$

$$\angle \text{ext} + \angle \text{int} = 180$$

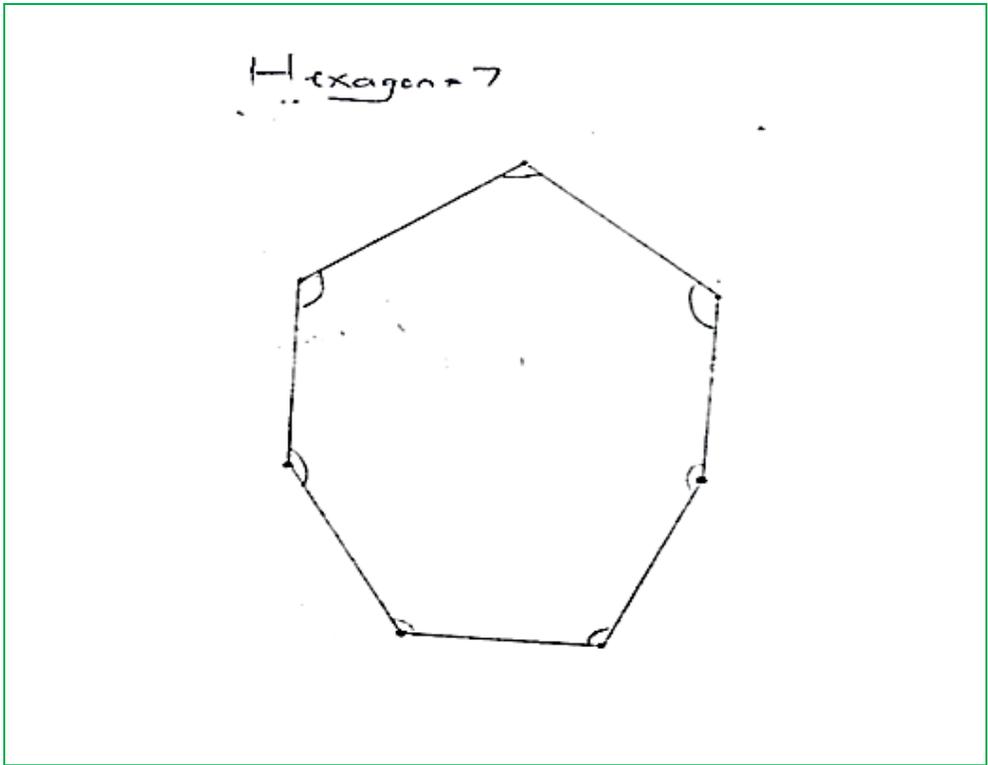
$$72 + \angle \text{int} = 180$$

$$\angle \text{int} = 180 - 72$$

$$\angle \text{int} = 108$$

$$\frac{360}{5} = 72$$

∴ each interior angle is 108 size



In Extract 3.2, the student was unable to find the sum of interior angles of regular pentagon, identify number of sides of the pentagon and draw a regular hexagon.

2.4 Question 4: Locus

The question consisted of two parts; (a) and (b). In part (a), the students were required to find the equation of locus of the points which have equal distance from points A $(-5, 8)$ and B $(6, 7)$. In part (b), they were required to find equation of the locus of point P which its distance from point A $(-1, -3)$ is twice its distance from point B $(2, 4)$.

This question was attempted by 603 students, out of which 67.8 percent scored from 0 to 2.5 marks, 19.8 percent scored from 3 to 6 marks and 12.4 percent scored from 6.5 to 10 marks. The analysis of data reveals that, this question was averagely performed since only 32.2 percent scored above 2.5 out of 10 marks. Figure 4 shows the students' performance in this question.

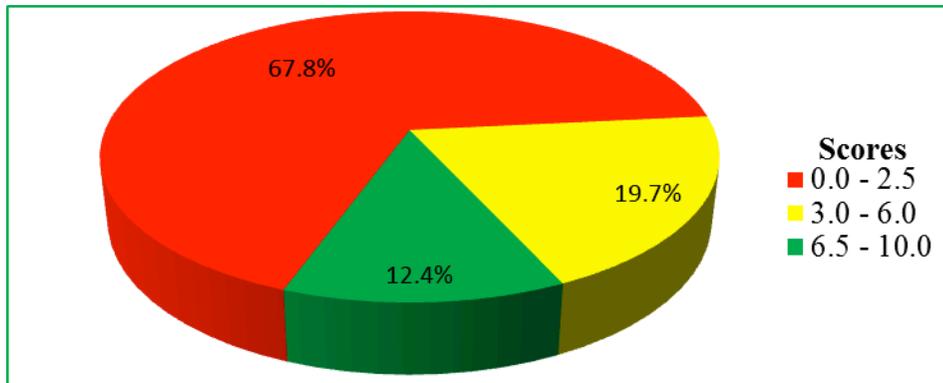


Figure 4: Students' performance in question 4.

The analysis of data shows that some of the students were able to answer this question according to the given instructions. They managed to find equation of locus of the points which have equal distance from the given points. They were able to apply the distance formula to relate the distance of the variable points from point A and B, that is, $AP = PB$. Also, they were able to find the equation of locus of P whose distance from point A is twice its distance from B, that is, $AP = 2PB$. Furthermore, they were able to interpret, formulate and perform algebraic operations to find the correct equations of loci. This implies that, the students had adequate knowledge and computational skills on *Algebra*. A sample of the best solution is shown in Extract 4.1.

Extract 4.1

soln

4(a)

$d_1 = d_2$
 $\overline{PA} = \overline{PB}$
 But distance = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$$\left(\sqrt{(x-(-5))^2 + (y-8)^2} \right)^2 = \left(\sqrt{(x-6)^2 + (y-7)^2} \right)^2$$

$$(x+5)^2 + (y-8)^2 = (x-6)^2 + (y-7)^2$$

$$x^2 + 10x + 25 + y^2 - 16y + 64 = x^2 - 12x + 36 + y^2 - 14y + 49$$

$$x^2 + y^2 + 10x - 16y + 89 = x^2 + y^2 - 12x - 14y + 85$$

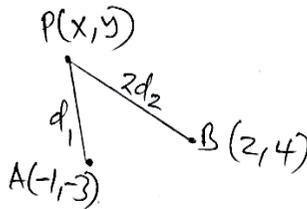
$$x^2 - x^2 + y^2 - y^2 + 10x + 12x - 16y + 14y + 89 - 85 = 0$$

$$\frac{22x - 2y + 4}{2} = 0$$

$$11x - y + 2 = 0$$

∴ The equation is $11x - y + 2 = 0$ for which it is a linear equation

4(b)



$$d_1 = 2d_2$$

$$\overline{PA} = 2\overline{PB}$$

$$\text{dist distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\sqrt{(x-(-1))^2 + (y-(-3))^2} = 2\sqrt{(x-2)^2 + (y-4)^2}$$

$$\left(\sqrt{(x+1)^2 + (y+3)^2} \right)^2 = \left(2\sqrt{(x-2)^2 + (y-4)^2} \right)^2$$

$$(x+1)^2 + (y+3)^2 = 4((x-2)^2 + (y-4)^2)$$

$$x^2 + 2x + 1 + y^2 + 6y + 9 = 4(x^2 - 4x + 4 + y^2 - 8y + 16)$$

$$x^2 + y^2 + 2x + 6y + 10 = 4x^2 + 4y^2 - 16x - 32y + 80$$

$$4x^2 - x^2 + 4y^2 - y^2 - 16x - 2x - 32y - 6y + 80 - 10 = 0$$

$$3x^2 + 3y^2 - 18x - 38y + 70 = 0$$

$$\frac{3x^2}{3} + \frac{3y^2}{3} - \frac{18x}{3} - \frac{38y}{3} + \frac{70}{3} = 0$$

$$x^2 + y^2 - 6x - \frac{38y}{3} + \frac{70}{3} = 0$$

∴ The equation is $x^2 + y^2 - 6x - \frac{38y}{3} + \frac{70}{3} = 0$ for which it is an equation of circle.

In Extract 4.1, the student correctly interpreted and formulated the required equations of loci from the given conditions.

On the contrast, about 67.8 percent of the students scored below 2.5 out of 10 marks in this question. Some of them were not able to write the distance formula correctly as they had confusion between distance and midpoint formulae. Some wrote the incorrect formulae like " $PA = \sqrt{x_2 + x_1} + \sqrt{y_2 + y_1}$ " instead of $PA = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ and others multiplied the coordinates of the given points, that is, $A(-5, 8) = 5 \times 8 = 40$ and $B(6, 7) = 6 \times 7 = 42$. In addition, there were some students who added the coordinates instead of finding the required loci, that is, " $A(-5, 8) = 5 + 8 = 13$ " and " $B(6, 7) = 6 + 7 = 13$ " thus leading to incorrect answers. Others determined the equation of a straight line instead of equation of the locus of the given points. This indicates that students had inadequate knowledge on *Locus* and *Coordinate Geometry*. Extract 4.2 shows a sample of poor solution from one of the students.

Extract 4.2

Solution

$$\begin{aligned}4(a) \text{ midpoint} &= \left(\frac{-5+6}{2}, \frac{8+7}{2} \right) \\ &= \left(\frac{1}{2}, \frac{15}{2} \right)\end{aligned}$$
$$\begin{aligned}\text{Gradient of } \overline{AB} &= \frac{\text{change in } y}{\text{change in } x} \\ &= \frac{8-7}{-5-6} = \frac{1}{-11} \\ &= -\frac{1}{11}\end{aligned}$$

Equation is $-\frac{1}{11} = \frac{y - \frac{15}{2}}{x - \frac{1}{2}}$

$$\begin{aligned}-1\left(x - \frac{1}{2}\right) &= 11\left(y - \frac{15}{2}\right) \\ -x + \frac{1}{2} &= 11y - \frac{165}{2} \\ -x + \frac{1}{2} + \frac{165}{2} &= 11y \\ -x + \frac{166}{2} &= 11y \\ \frac{-x + 83}{11} &= \frac{11y}{11}\end{aligned}$$

(b) $y = -\frac{x}{11} + \frac{83}{11}$

$$y = -\frac{1}{11}x + \frac{83}{11}$$

$y = -\frac{1}{11}x + 7\frac{6}{11}$ required as equation of locus

In Extract 4.2, the student found the equation of straight line through the given points instead of using the given conditions to find the equations of loci.

2.5 Question 5: Coordinate Geometry

This question had two parts; (a) and (b). In part (a), the students were required to find the value of r from the line joining point $P(r,3)$ to the point $Q(2,-3)$ which is perpendicular to the line joining point $R(10,1)$ to point Q . In part (b), they were required to find the equation of a line passing through the point $(1,-3)$ which is parallel to the line $2x + 3y - 4 = 0$.

The analysis of data shows that, 626 students attempted this question, out of which 54.5 percent scored from 0 to 2.5 marks, 20.5 percent scored from 3 to 6 marks and 24.8 percent scored from 6.5 to 10 out of 10 marks. This implies that the performance in this question was average since 45.5 percent of students scored above 2.5 out of 10 marks. Figure 5 illustrates the students' performance in this question.

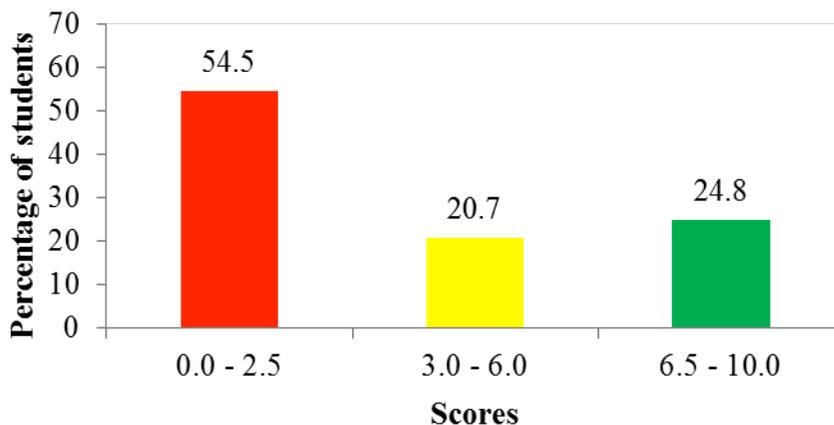


Figure 5: Students' performance in question 5.

The students who did well in this question had adequate knowledge on the tested concepts. They answered correctly all parts of the question. In part (a), they were able to find the value of r using the concepts of slopes of perpendicular lines, that is, $m_1 m_2 = -1$ and the slopes of parallel lines, that is, $m_1 = m_2$ to find the equation of the line through the given point. Extract 5.1 shows an example of the best solution from one of the students.

Extract 5.1

soln

Q. slope of RQ = $\frac{y_2 - y_1}{x_2 - x_1}$

$$m = \frac{-3 - 1}{2 - 10}$$

$$m = \frac{-4}{-8}$$

$$m = \frac{1}{2}$$

slope RQ (m_2) = $\frac{1}{2}$

but when two lines are perpendicular the product of slope one and slope two is equal to -1 .

$$M_1 \times M_2 = -1$$

$$M_1 \times \frac{1}{2} = -1$$

$$\frac{M_1}{2} = -1$$

$$M_1 = -2$$

so we use $M_1 = -2$, point $P(r, 3)$ and $Q(2, 3)$ to find value of r

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$-2 = \frac{-3 - 3}{2 - r}$$

$$\frac{-2}{1} \times \frac{-6}{2 - r}$$

$$-2(2 - r) = -6 \times 1$$

$$5a) \quad -4 + 2r = -6$$

$$2r = -6 + 4$$

$$\frac{2r}{2} = \frac{-2}{2}$$

$$r = -1$$

\therefore The value of r is -1 .

6b) When two lines are parallel slope one is equal to slope two.

$2x + 3y - 4 = 0$ we re-write this equation in $y = mx + c$
slope $y = mx + c$

$$3y = \frac{-2x + 4}{3}$$

$$y = \frac{-2x + 4}{3}$$

$$M_2 = -2/3$$

$$\text{but } M_1 = M_2, \quad M_1 = -2/3$$

we use $M_1 = -2/3$ and point $(1, 3)$ to find equation

$$\text{eqn slope} = \frac{y - y_1}{x - x_1}$$

$$\begin{aligned}
 -\frac{2}{3} &= \frac{y-3}{x-1} \\
 -\frac{2}{3} &\neq \frac{y+3}{x-1} \\
 3(y+3) &= -2(x-1) \\
 3y+9 &= -2x+2 \\
 3y &= -2x+2-9 \\
 3y &= -2x-7 \\
 3y+2x+7 &= 0 \quad \text{or} \quad \frac{3y}{3} = \frac{-2x-7}{3} \\
 \therefore \text{The equation of the line is } &3y+2x+7=0 \\
 \text{or } &3y = \frac{-2x-7}{3}
 \end{aligned}$$

In Extract 5.1, the student managed to obtain the slopes of perpendicular lines to find the required value of r and applied the concepts of slope of parallel lines to find the equation of a line as it was required.

However, there were 341 (54.5%) students who scored low marks. These students did not provide the relevant solution in accordance with the demands of the question.

In part (a), some students used $m_1 = m_2$ instead of $m_1 m_2 = -1$ to find the required equation, which led to incorrect solution. Others applied the wrong formulae for slope, like $m = \frac{x_1 - y_1}{x_2 - y_2}$. In part (b), the students failed to rearrange the equation $2x + 3y - 4 = 0$ in the form $y = mx + c$ which could help them to obtain the slope. For instance, some students rearranged the equation $2x + 3y - 4 = 0$ into $y = \frac{2}{3}x - \frac{4}{3}$ instead of $y = -\frac{2}{3}x + \frac{4}{3}$. Other students rearranged the equation properly as $y = -\frac{2}{3}x + \frac{4}{3}$ but failed to identify the slope (m) of the equation. They wrote $m = \frac{4}{3}$ instead of $m = -\frac{2}{3}$. This indicates that the student had inadequate knowledge as

well as computational skills on *Coordinate Geometry*. Extract 5.2 shows a sample of poor solution from one of the students.

Extract 5.2

5(a) Solution:

P(x_1, y_1) , Q(x_2, y_2) and R(x_3, y_3)

Q and R

$$\frac{x_1 - x_1}{x_2 - y_2} = M$$

$$\frac{2 + 3}{10 - 1} = \frac{5}{9}$$

$$M_1 = \frac{5}{9}$$

P and Q

$$\frac{x_1 - y_1}{x_2 - y_2} = M$$

$$\frac{1 + 3}{2 + 3} = \frac{r - 3}{5}$$

$$M_2 = \frac{r - 3}{5}$$

P and R

$$\frac{x_1 - y_1}{x_2 - y_2} = M$$

$$\frac{r - 3}{10 - 1} = \frac{r - 3}{9}$$

$$M_3 = \frac{r - 1}{3}$$

From $M_1 = M_2 = M_3$

$$\frac{5}{9} = \frac{r - 3}{5} = \frac{r - 1}{5}$$

Value of "r"

$$\frac{r - 3}{5} \neq \frac{r - 1}{5}$$

$$5r - 15 = 5r - 5$$

$$5r - 5r = -5 + 15$$

$$r = -10$$

$\therefore r = -10$

Solution

5(b)

$$2x + 3y - 4 = 0$$

$$2x + 3y = 4$$

$$\frac{3y}{3} = \frac{4 - 2x}{3}$$

$$y = \frac{4}{3} - \frac{2x}{3}$$

$$\therefore M_1 = \underline{\underline{\frac{4}{3}}}$$

But parallel $\neq M_1 = M_2$

$$\frac{4}{3} = M_2$$

$$\therefore M_2 = \underline{\underline{\frac{4}{3}}}$$

From equation

$$\frac{x - x_1}{y - y_1} = M$$

$$\frac{x - 1}{y + 3} = \frac{4}{3}$$

$$3x - 3 = 4y + 12$$

$$3x = 4y + 12 + 3$$

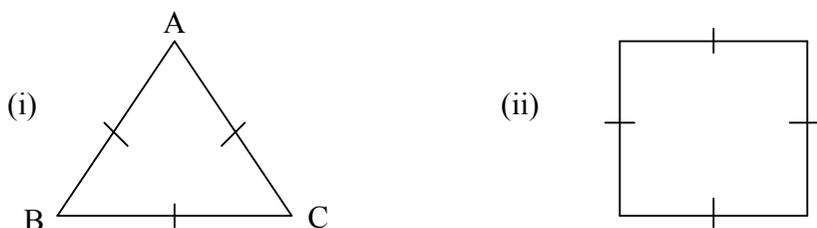
$$\frac{3x}{3} = \frac{4y}{3} + \frac{15}{3}$$

$$x = \underline{\underline{\frac{4y}{3} + 5}}$$

In Extract 5.2, the student used incorrect formula of slope and also had misconception of slopes of perpendicular and parallel lines. He/she was unable to perform arithmetic operations and failed to identify the slope from the standard form of linear equation.

2.6 Question 6: Symmetry

The question had two parts; (a) and (b). In part (a), the students were required to find the number of lines of symmetry for the following figures:



In part (b), they were required to draw the lines of symmetry on the figures in part (a).

The analysis of data shows that, 654 students attempted this question, out of which 34.7 percent scored from 0 to 2.5 marks, 34.4 percent scored from 3 to 6 marks and 30.9 percent scored from 6.5 to 10 out of 10 marks. This implies that the students' performance in this question was good since 65.3 percent scored above 2.5 out of 10 marks. Figure 6 illustrates the students' performance in this question.

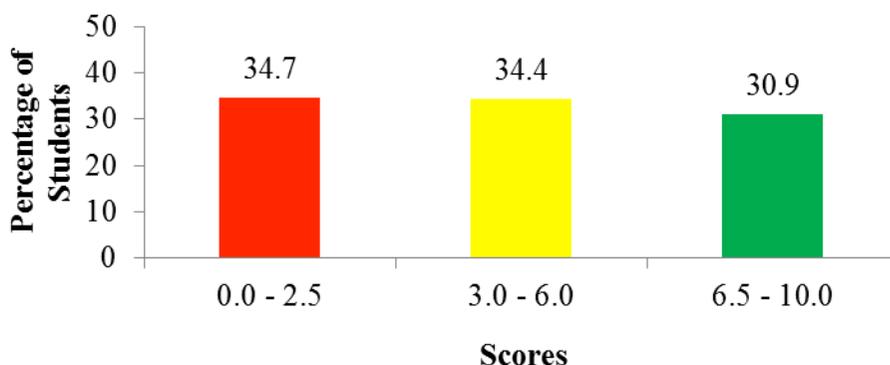


Figure 6: Students' performance in question 6.

Majority of the students (65.3%) scored above 2.5 out of 10 marks in this question. These students were able to recognise the number of lines of symmetry in part (a) and draw them in part (b) as required in the question. They knew that the line of symmetry divides a figure into two halves that are exactly the same. This implies that they had adequate knowledge and skills on *Symmetry*. Extract 6.1 is a sample of the best solution from one of the students.

Extract 6.1

6a) i) In Figure remain (i), There are Three (3) Lines of Symmetry
 ii) In Figure remain (ii), there are Four (4) Lines of Symmetry

6b) i)

1st Line of symmetry
 2nd Line of symmetry
 3rd Line of symmetry

In general showing all Lines of symmetry

b)

1 Line of symmetry
 2nd Line of symmetry
 3rd Line of symmetry
 4 Line of symmetry

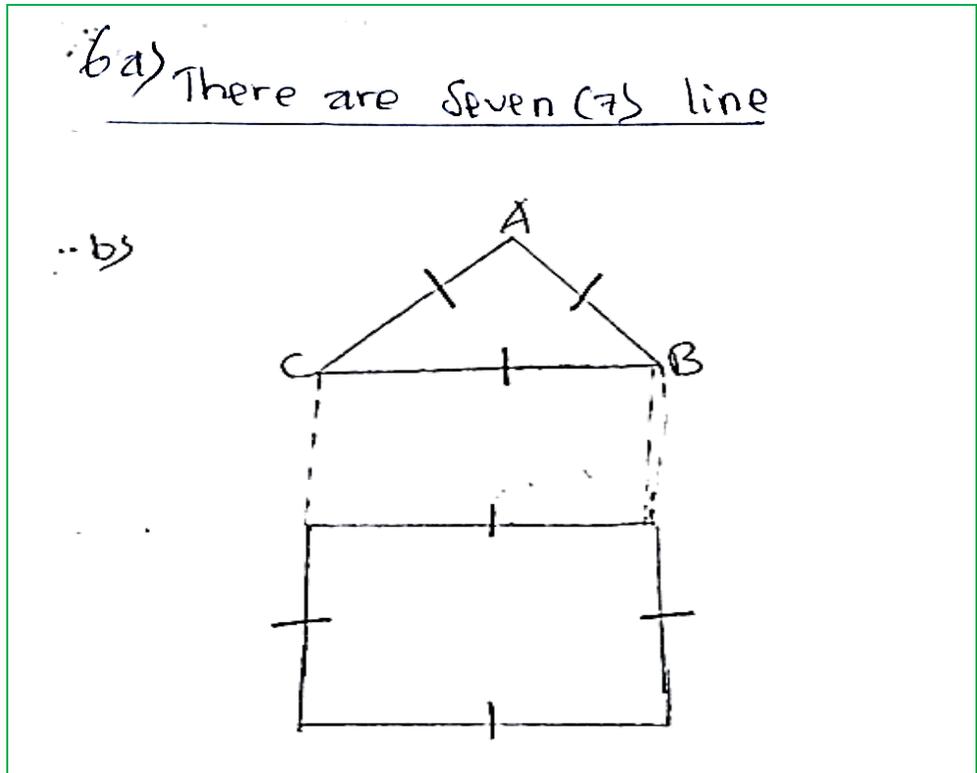
In general showing all Lines of symmetry

In Extract 6.1, the student recognised the number of symmetrical lines and correctly drew them on the given equilateral triangle and square.

Conversely, 34.7 percent of students who attempted this question scored below 3 marks. These students were unable to provide the answers

according to the requirements of the question. For instance, in part (a), some students only counted the number of sides as the number of lines of symmetry. In part (b), the students joined the two figures which were given instead of drawing the required lines of symmetry on them. These responses indicate that they lacked knowledge and skills on *Symmetry*. Extract 6.2 shows a sample of incorrect solution from one of the students.

Extract 6.2



In Extract 6.2, the student was unable to recognise the number of symmetrical lines and failed to draw them on the given equilateral triangle and square.

2.7 Question 7: Logic

This question had two parts; (a) and (b). In part (a), the students were given that, let p be “He is tall” and q be “He is handsome”. Then, they were required to write the following statements in symbolic form:

- (i) *He is tall or he is short and handsome.*
- (ii) *It is not true that he is short or not handsome.*
- (iii) *He is handsome if and only if he is tall.*

(iv) *If he is handsome then he is either tall or short.*

In part (b) (i), they were required to construct a truth table of the compound proposition $(p \vee q) \wedge (\neg p \vee \neg q)$ and in part (b) (ii), the students were required to show whether the logical statement $(p \rightarrow q) \vee (q \rightarrow p)$ is tautology or not.

This question was attempted by 635 students, out of which 39.7 percent scored from 0 to 2.5 marks, 20.1 percent scored from 3 to 6 marks and 40.2 percent scored from 6.5 to 10 out of 10 marks. The performance in this question was average as 60.3 percent of the students scored above 2.5 out of 10 marks. The summary of the students' performance in this question is illustrated in Figure 7.

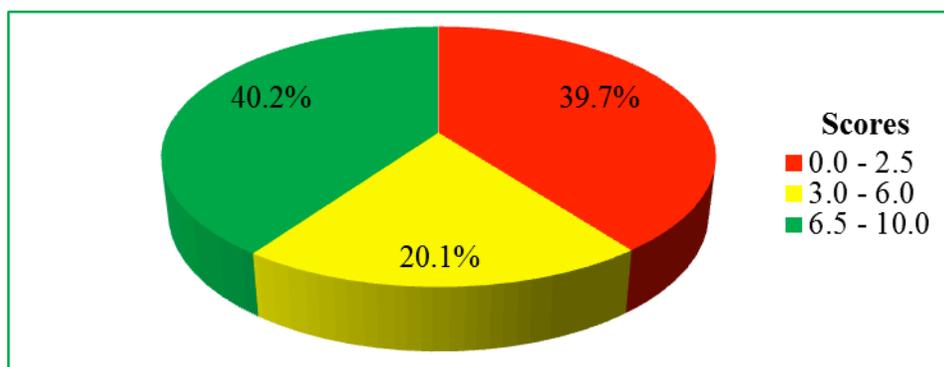


Figure 7: *Students' performance in question 7.*

In part (a) of this question, some students were able to write correctly the logical statements in symbolic form. They identified the logical connectives and formed the compound statements which were required. In part (b), the students managed to construct the truth table of the logical statement $(p \vee q) \wedge (\neg p \vee \neg q)$. Furthermore, they were able to show whether $(p \rightarrow q) \vee (q \rightarrow p)$ is tautology or not. Those students were able to apply the logical connectives and truth values correctly in completing the truth table in part (b) (i). Likewise, in part (b) (ii), they managed to test the validity of the statement $(p \rightarrow q) \vee (q \rightarrow p)$ and obtain the results from the conclusion column. Also, some students used the truth tables or laws of propositions of algebra to test the validity of the given compound proposition. Those students answered well this question due to adequate

knowledge they had on *Logic*. A sample of the best solutions from the students is shown in Extract 7.1.

Extract 7.1

a) Solution

P "He is tall"
 Q "He is handsome"

(i) $P \vee (\sim P \wedge Q)$
 (ii) $\sim(\sim P \vee \sim Q)$
 (iii) $Q \leftrightarrow P$
 (iv) $Q \rightarrow (P \vee \sim P)$

b) (i)

P	Q	$\sim P$	$P \vee Q$	$\sim P \vee Q$	$(P \vee Q) \wedge (\sim P \vee Q)$
T	T	F	T	T	T
T	F	F	T	F	F
F	T	T	T	T	T
F	F	T	F	T	F

(ii)

P	Q	$P \rightarrow Q$	$Q \rightarrow P$	$(P \rightarrow Q) \vee (Q \rightarrow P)$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

$\therefore (P \rightarrow Q) \vee (Q \rightarrow P)$ is tautology because the last column all is true.

In Extract 7.1, the student was able to write logical statements symbolically, construct the truth tables and test the validity of given logical statements.

However, 39.7 percent of the students scored below 3 marks as they were unable to answer the question in accordance with its demands. For instance, in part (a) (i) to (iv), some students copied the questions instead of writing the compound statements required. Other students misconceived the question with the True and False items; For example one of the students answered it as follows:

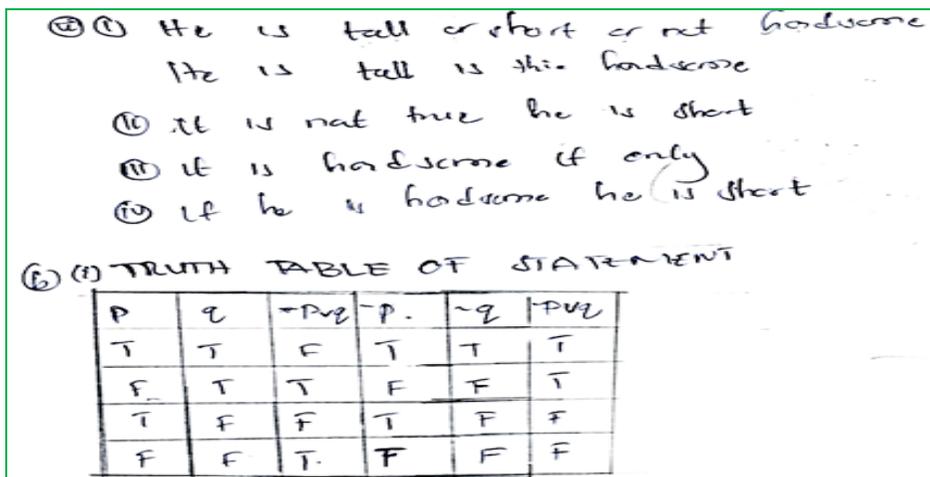
- (i) *He is tall or he is short and handsome. F*
- (ii) *It is not true that he is short or not handsome. T*
- (iii) *He is handsome if and only if he is tall. F*
- (iv) *If he is handsome then he is either tall or short. T*

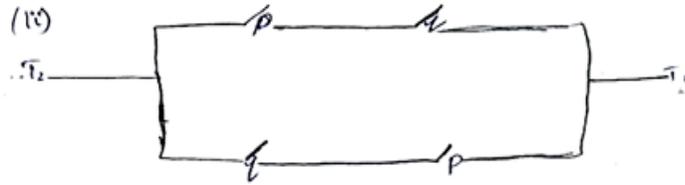
Moreover, in part b (i), they constructed the truth tables which were not related to the given logical statements. For example, some drew the truth tables of two propositions with only two rows instead of four, and incorrectly assigned the truth values to p and q as follows:

p	q	$p \vee q$	$\sim p$	$\sim p \vee q$	$(p \vee q) \wedge (\sim p \vee q)$
T	F	T	F	F	F
F	T	T	T	T	T

In addition, some of them drew an electrical network for the logical statements instead of truth tables in part (b) (ii). The responses indicate that those students had inadequate knowledge and skills on *Logic*. Extract 7.2 shows a sample of a poor solution from one of the students.

Extract 7.2





In Extract 7.2, the student copied the question in part (a), drew an irrelevant truth table and misconceived between testing the validity and drawing an electrical network of a logical statement in part (b).

2.8 Question 8: Variations

This question had two parts; (a) and (b). In part (a), the students were given the variable y which varies jointly as x and z . Also, they were given that, if $y = 10$, $x = 4$ and $z = 5$ then find the value of z when $x = 2$ and $y = 5$. In part (b), it was given that, if 2 students can type 210 pages in 3 days, find the number of students that are needed to type 700 pages in 2 days.

This question was attempted by 645 students, out of which 32.6 percent scored from 0 to 2.5 marks, 39.3 percent scored from 3 to 6 marks and 28.1 percent scored from 6.5 to 10 out of 10 marks. The students' performance in this question was good since 67.4 percent scored from 3 to 10 out of 10 marks. Figure 8 is a summary of the students' performance in this question.

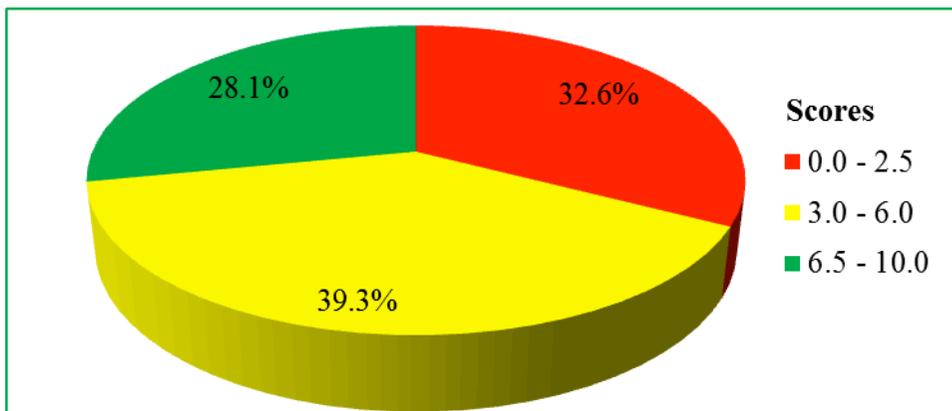


Figure 8: Students' performance in question 8.

The students who scored high marks were able to provide the correct answers in one or two parts of the question. They were able to formulate joint variation equations and define the variables correctly, that is, $y \propto xz$ which implies $y = kxz$, where k is proportionality constant. Others

substituted the values of the variables directly into the formula:

$$\frac{y_1}{x_1 z_1} = \frac{y_2}{x_2 z_2} \text{ in part (a) as follows:}$$

$$\text{“} \frac{10}{4 \times 5} = \frac{5}{2z_2} \rightarrow 20z_2 = 100 \rightarrow z_2 = 5\text{”}. \text{ Therefore, the value of } z \text{ is } 5.$$

$$\text{Similarly in part (b), one of the students wrote, “} \frac{s_1 d_1}{p_1} = \frac{s_2 d_2}{p_2} \rightarrow \frac{2 \times 3}{210} = \frac{2s_2}{700}$$

and $420s_2 = 4200 \rightarrow s_2 = 10$ ”. Therefore, the number of students who can type 700 pages in 2 days is 10. This indicates that the students had adequate knowledge on *Joint Variations*. Extract 8.1 shows a sample of the best solutions from one of the students.

Extract 8.1

Soln

8(a) $y \propto xz$

$$\frac{y}{xz} = \frac{kxz}{xz}$$

$$k = \frac{y}{xz}$$

When $y=10, x=4, z=5, k=?$

$$k = \frac{10}{4 \times 5}$$

$$k = \frac{10}{20}$$

$$k = \frac{1}{2}$$

$z=?$ when $x=2$ and $y=5?$

$$k = \frac{y}{xz}$$

$$\frac{1}{2} = \frac{5}{2z}$$

$$\frac{2z}{2} = \frac{2 \times 5}{2}$$

$$z = 5$$

$\therefore z = 5.$

8(b) let students = S
 pages = P
 days = d

Then $S \propto \frac{P}{d}$

$$S = \frac{KP}{d}$$

$$\frac{KP}{P} = \frac{Sd}{P}$$

$$K = \frac{Sd}{P}$$

When $S=2$, $P=210$ and $d=3$, $K=?$

$$K = \frac{2 \times 3}{210}$$

$$K = \frac{2}{7035}$$

$$K = \frac{1}{35}$$

What is S ? when $P=700$ and $d=2$?

$$S = \frac{KP}{d}$$

$$S = \frac{\frac{1}{35} \times 700}{2}$$

$$S = \frac{1}{35} \times 350$$

$$S = 10$$

\therefore 10 students are needed to type that pages

In Extract 8.1, the student correctly formulated the equations, determined the values of k and the required answers.

Despite the good performance, there were some students who failed to respond according to the requirements of the question. Most of them had misconceptions on the types of variations. They used inverse variation instead of direct variation. For instance, some of the students wrote the following incorrect variation equations:

“ $y \propto \frac{x}{z} \rightarrow y = \frac{kx}{z}$; $x \propto \frac{y}{z} \rightarrow x = \frac{ky}{z}$; $y \propto \frac{1}{x} \frac{1}{z} \rightarrow y = \frac{k}{xz}$ ” instead of $y = kxz$. Further analysis shows that some students managed to formulate the equation but could not correctly substitute the data. This implies that they had inadequate knowledge on direct, inverse and joint variations. Extract 8.2 represents a sample work of a student who failed to answer this question.

Extract 8.2

Soln

② $Y \propto \frac{1}{x} \propto \frac{1}{z}$

Into equation

$$Y = \frac{k}{xz}$$

$$10 = \frac{k}{4 \times 5}$$

$$\frac{10}{1} \times \frac{k}{20}$$

$$k = 200$$

$$Y = \frac{200}{xz}$$

where by $x = 2$, and $y = 5$ then $z = ?$

$$\frac{5}{1} = \frac{200}{2z}$$

$$\frac{10z}{10} = \frac{200}{10}$$

$$z = 20$$

∴ The value of z is 20

(b)

S	P	D
2	210	3
?	700	2

$S \propto \frac{1}{P} \propto \frac{1}{D}$
 into equation
 $S = \frac{k}{PD}$
 $\frac{2}{1} = \frac{k}{210 \times 3}$
 $k = 1260$
 $S = \frac{1260}{PD}$
 where by $P = 700, D = 2$ then $S = ?$
 $S = \frac{1260}{700 \times 2}$
 $S = \frac{126}{140}$
 $\therefore \frac{126}{140}$ students are needed

In Extract 8.2, the student formulated wrong variation equations which led to incorrect responses in parts (a) and (b).

2.9 Question 9: Sets

The question stated that:

In a group of 450 students; 100 play volleyball, 70 play athletics, 200 play drama, 90 play volleyball and participate in drama, 30 play volleyball and athletics, 45 athletics and participate in drama, 220 do not participate in any game. Use the general formula of union of sets to find the number of participants in all games.

In this question students were supposed to have knowledge of general formulae used to answer questions that involve three sets.

The analysis of data reveals that, 612 students attempted this question, out of which 81.7 percent scored from 0 to 2.5 marks, 8 percent scored from 3 to 6 marks and 10.3 percent scored from 6.5 to 10 out of 10 marks. The students' performance in this question was weak as only 18.3 percent scored above 2.5 out of 10 marks. A summary of students' performance in this question is shown in Figure 9.

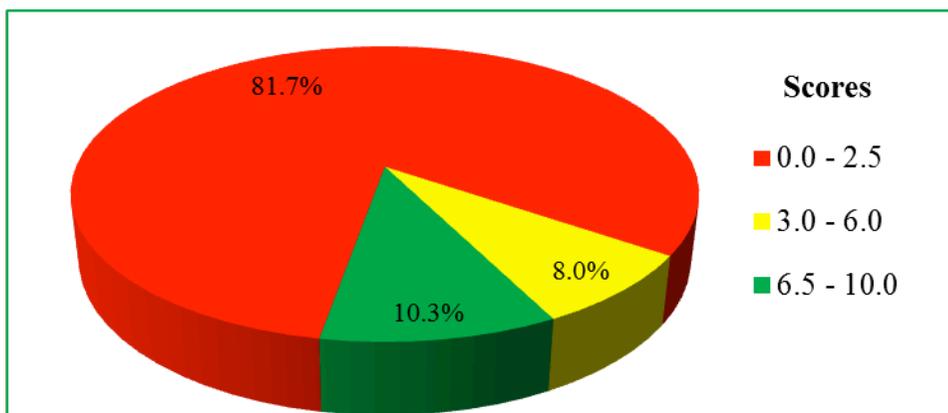


Figure 9: Students' performance in question 9.

The analysis shows that, this is the only question in which students' performance was weak. The reason for the weak performance was mainly associated with failure of students to use the formula for the union of three sets in answering the given question. For instance, some students used the formula for union of two instead of three sets, that is, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$. Others wrote incorrect formulae such as:

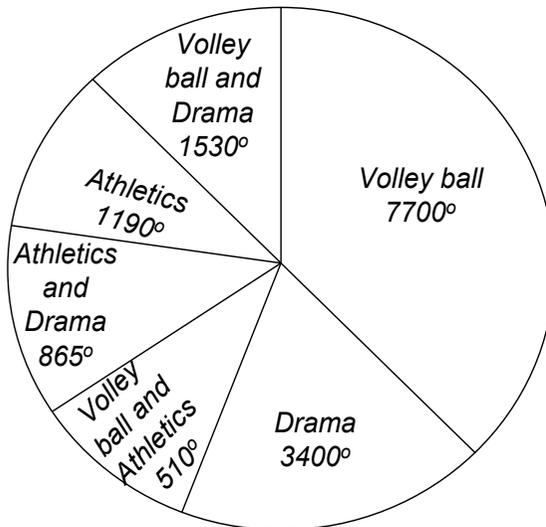
$$= n(A) + n(V) + n(D) - n(A \cap V) + n(A \cap D) + n(V \cap D) - n(A \cap V \cap D) \text{ and}$$

$$n(A \cup V \cup D) = n(A) + n(V) + n(D) - n(A \cap V \cap D) \text{ instead of writing}$$

$$n(A \cup V \cup D)$$

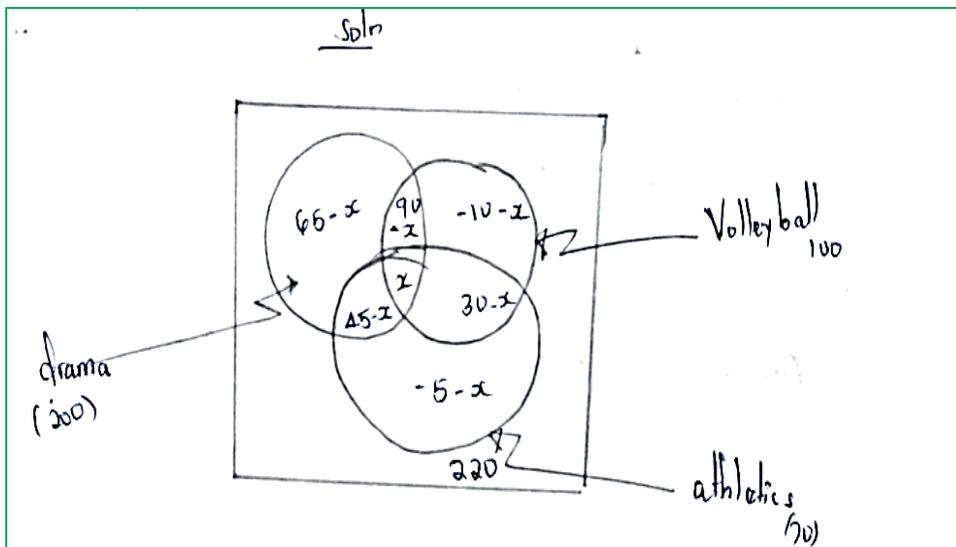
$$= n(A) + n(V) + n(D) - n(A \cap V) - n(A \cap D) - n(V \cap D) + n(A \cap V \cap D).$$

In addition, there were some students who used Venn diagram instead of the required formula to answer this question. Furthermore, other students did not understand the requirements of the question as they provided unrelated responses including converting the given information into degrees and expressed them in pie chart as follows:



In general, the students' poor performance was due to inadequate knowledge on solving word problems involving the union of three sets. Extract 9.1 shows an example of the poor solution from one of the students.

Extract 9.1



$$70 - (45 + 30 - x)$$

$$70 - 75 - x$$

$$-5 - x$$

$$100 - (90 + 30 - x)$$

$$100 - 120 - x$$

$$-20 - x$$

$$200 - (45 + 90 - x)$$

$$200 - 135 - x$$

$$65 - x$$

$$65 - x + 90 - x + x + (-10 - x) + 30 - x + 45 - x + (-5 - x) + 200 = 450$$

$$65 + 90 - 10 + 30 + 45 - 6 + 200 - x - x - x - x = 450$$

$$425 - 4x = 450$$

$$450 - 425 = 4x$$

$$\frac{25}{4} = \frac{4x}{4}$$

$$x = 6.25$$

The students who participate in all games are 6.25.

In Extract 9.1, the student used Venn diagram to answer the question instead of general formula, and ended up with incorrect answer.

Though the question was performed poorly, some students (18.3%) managed to score above 2.5 marks. These students were able to define and use the general formula for the union of three sets correctly. They calculated the correct number of participants in all three games. This shows that they had adequate knowledge and skills on solving problems involving

the union of three sets. A sample of the best solution from one of the students is presented in Extract 9.2.

Extract 9.2.

450 all students
 100 play volleyball
 70 play are athletics
 200 play drama
 90 play volleyball and drama
 30 play volleyball and are athletics
 45 are athletics and drama
 220 not participate in any game

Let V for volleyball
 A for athletics
 D for drama

$$n(V \cup A \cup D) = n(V \setminus A) + n(A) + n(D) - n(V \cap A) - n(A \cap D) - n(V \cap D) + n(V \cap A \cap D)$$

$$450 = 100 + 70 + 200 - 30 - 45 - 90 + 220 + n(V \cap A \cap D)$$

$$450 = 170 + 200 - 30 - 45 - 90 + 220 + n(V \cap A \cap D)$$

$$450 = 370 - 30 - 45 - 90 + 220 + n(V \cap A \cap D)$$

$$450 = 370 + 220 - 30 - 45 - 90 + n(V \cap A \cap D)$$

$$450 = 590 - 30 - 45 - 90 + n(V \cap A \cap D)$$

$$450 = 560 - 45 - 90 + n(V \cap A \cap D)$$

$$450 = 515 - 90 + n(V \cap A \cap D)$$

$$450 = 425 + n(V \cap A \cap D)$$

$$425 - 450 = n(V \cap A \cap D)$$

$$25 = n(V \cap A \cap D)$$

\therefore The number of students who participate in all games is 25

In Extract 9.2, the student was able to interpret the question and correctly apply the formula in answering it.

2.10 Question 10: Algebra

In this question the students were required to solve

$$\begin{cases} xy = 64 \\ 4x - y = 60 \end{cases} \text{ simultaneously by using substitution method.}$$

The statistics shows that, 628 students attempted this question, out of which 53.5 percent scored from 0 to 2.5 marks, 17.7 percent scored from 3 to 6 marks and 28.8 percent scored from 6.5 to 10 out of 10 marks. The students' performance in this question was average since 46.5 percent scored from 3 to 10 marks. Figure 10 is a summary of the students' performance in this question.

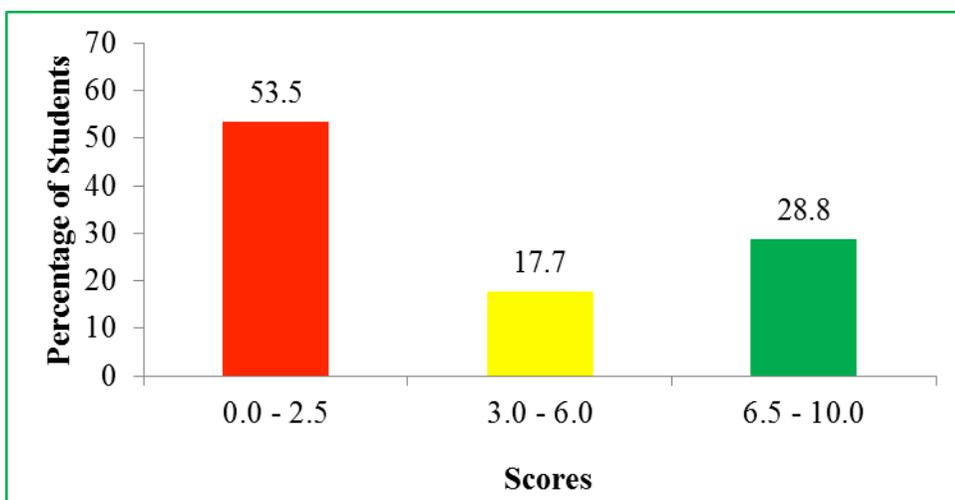


Figure 10: Students' performance in question 10.

The analysis of data shows that, 45.5 percent of the students who scored above 2.5 marks were able to answer the question correctly. For instance, some of them started by making either x or y the subject of the equation $4x - y = 60$, then substituted in the equation $xy = 64$ or vice versa. In this approach the students were able to obtain the quadratic equation $x^2 - 15x - 16 = 0$ or $y^2 + 60y - 256 = 0$. They solved these equations by either completing the square, factorization method or quadratic formula to obtain the required solution. This shows that these students had adequate knowledge on *Algebra* especially in solving simultaneous equations by using substitution method. Extract 10.1 shows a sample of the best solution from one of the students.

Extract 10.1

$$xy = 64 \text{ ----- (i)}$$

$$4x - y = 60 \text{ ----- (ii)}$$

From eqn (i) make x the subject of the formula

$$\frac{xy}{y} = \frac{64}{y}$$

$$x = \frac{64}{y} \text{ ----- (iii)}$$

Substitute eqn (iii) from eqn (ii)

$$4x - y = 60$$

$$4\left(\frac{64}{y}\right) - y = 60$$

$$\frac{256 \times 4}{y} - y = 60 \times y$$

$$256 - y^2 = 60y$$

$$256 - y^2 - 60y = 0$$

$$-y^2 - 60y + 256 = 0$$

$$ax^2 + bx + c = 0$$

$$a = -1, b = -60, c = 256$$

$$= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-60) \pm \sqrt{(-60)^2 - 4 \times 256 \times -1}}{2 \times -1}$$

$$= \frac{60 \pm \sqrt{3600 + 1024}}{-2}$$

$$= \frac{60 \pm \sqrt{3600 + 1024}}{-2}$$

$$= \frac{60 \pm \sqrt{4624}}{-2}$$

$$= \frac{60 \pm 68}{-2}$$

$$y = \frac{60+68}{-2} \quad \text{or} \quad y = \frac{60-68}{-2}$$

$$y = \frac{128}{-2}$$

$$y = \frac{-8}{-2}$$

$$y = -64$$

$$y = 4$$

- Substitute $y = -64$ and $y = 4$ in eqn (iii) To obtain the value of x

$$x = \frac{64}{y}$$

$$x = \frac{64}{-64}$$

$$\text{or} \quad x = \frac{64}{4}$$

$$x = -1$$

$$\text{or} \quad x = 16$$

$$\therefore \underline{x = -1 \text{ or } 16 \text{ and } y = -64 \text{ or } 4}$$

In Extract 10.1, the student managed to show all the necessary steps required to solve the given simultaneous equations by using substitution method.

However, some students were not able to provide the answers according to the requirements of the question. For instance, there were the students who wrongly wrote the equation $xy = 64$ as $x + y = 64$, or $x = 6$ and $y = 4$, which also resulted into incorrect solution. This indicates that those students lacked knowledge on solving simultaneous equations. Extract 10.2 shows a sample of a poor solution from one of the students.

Extract 10.2

$$\begin{cases} xy = 64 & \text{--- eqn (i)} \\ 4x - y = 60 & \text{--- eqn (ii)} \end{cases}$$

From eqn (i)
 $xy = 64$
 $x = 64 - y$
Substitute into eqn (ii)

$$4x - y = 60$$
$$4x(64 - y) - y = 60$$
$$4 \times 64 - y - y = 60$$
$$256 - y - y = 60$$
$$256 - 60 = 2y$$
$$\frac{196}{2} = \frac{2y}{2}$$
$$y = 98$$

From substitute into eqn (ii)

$$4x - y = 60$$
$$4x - 98 = 60$$
$$4x = 60 + 98$$
$$\frac{4x}{4} = \frac{158}{4}$$
$$x = 39.5$$

\therefore The value of $(x, y) = (39.5, 98)$ Ans.

In Extract 10.2, the student failed to make x subject of $xy = 64$ as he/she wrote $x = 64 - y$ instead of $x = \frac{64}{y}$, which led to incorrect solution.

3.0 STUDENTS' PERFORMANCE IN EACH TOPIC

In 042 Additional Mathematics examination, each topic was assessed by one question, except *Algebra* which was assessed by two questions. Among these questions, three (3) had good, six (6) had average and one (1) had weak performance. The general performance of students in this subject was average.

The analysis shows that, the students performed well the questions which were set from the topics of *Numbers* (69.4%), *Variations* (67.4%) and *Symmetry* (65.3%). The good performance of students on these topics was mainly attributed to their adequate knowledge and skills on the topics, correct interpretation of the requirements of questions and ability to use relevant concepts in answering the questions.

Further analysis reveals that, the topics in which the students had average performance were: *Logic* (60.3%), *Algebra* (57.7%), *Geometrical Construction* (51.5%), *Coordinate Geometry* (55.5%) and *Locus* (32.2%). The average performance in these topics was mainly attributed to students' moderate knowledge on the assessed concepts.

Furthermore, there was one topic which had a weak performance and that was *Sets* (18.3%). The reasons for weak performance were mainly due to failure of students to recall the formula for the union of three sets and poor computation skills.

4.0 CONCLUSION AND RECOMMENDATIONS

4.1 Conclusion

The topics which were assessed in 042 Additional Mathematics FTNA 2018 were *Numbers*, *Variations*, *Symmetry*, *Logic*, *Algebra*, *Geometrical Construction*, *Coordinate Geometry*, *Locus* and *Sets*. The assessment covered all topics in Form One and Form Two of Additional Mathematics syllabus. In the examination there were two questions (2 and 10) from the topic of *Algebra* while the remaining topics contributed one question each. So, the performance in the topic of *Algebra*, was average. The performance of students in each topic is shown in the **Appendix**.

The analysis of data shows that, the topic that had a weak performance was *Sets*. The reasons for weak performance were mainly due to failure to identify the requirements of the questions, failure to recall the formula for the union of three sets, misconception between sets and statistics, poor computational skills and application of incorrect concepts.

4.2 Recommendations

In order to improve the performance in Additional Mathematics subject it is recommended that:

- (a) Teachers should provide enough exercises to students in order to give them more experience in applying formulae, concepts and laws in answering questions.

- (b) Students should consistently follow instructions outlined in the examinations and show all necessary steps in arriving to final solution for each question.
- (c) Students should ensure that they have learning materials such as books, journals and mathematical tables in order to enhance learning and understanding in answering Mathematics examination questions.
- (d) The government should conduct research at school level on Additional Mathematics that would help in improving the performance and introduce the subject in more schools in which it is not taught. This would give a wider chance to many students wishing to opt the subject.

APPENDIX

Analysis of Students' Performance in Each Topic

S/N	Topic	No. of Questions	Percentage of Students who Scored an Average of 30% or Above	Remarks
1	Numbers	1	69.4	Good
2	Variations	1	67.4	Good
3	Symmetry	1	65.3	Good
4	Logic	1	60.3	Average
5	Algebra	2	57.7	Average
6	Geometrical Construction	1	51.5	Average
7	Coordinate Geometry	1	45.5	Average
8	Locus	1	32.2	Average
9	Sets	1	18.3	Weak

