THE UNITED REPUBLIC OF TANZANIA
MINISTRY OF EDUCATION, SCIENCE AND TECHNOLOGY
NATIONAL EXAMINATIONS COUNCIL OF TANZANIA

STUDENT'S ITEM RESPONSE ANALYSIS REPORT ON THE FORM TWO NATIONAL ASSESSMENT (FTNA) 2020

BASIC MATHEMATICS

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041 BASIC MATHEMATICS

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FOREWORD

The National Examinations Council of Tanzania is grateful to issue this report on Students' Item Response Analysis (SIRA) in 041 Basic Mathematics subject for the Form Two National Assessment (FTNA) 2020.

The purpose of preparing this report is to provide feedback mainly to students, teachers, policy makers and the public on how the students responded to each assessment item. The feedback entails the findings from students' responses and recommendations that seem to be valuable for improving the performance in the future National Assessments.

The data analysis shows that the students performed averagely in question 1 only out of all ten questions. The performance was weak in the other remaining questions, among them question 4 had the weakest performance. The core factors for weak performance include the students' inability to: formulate the correct mathematical expressions and equations, recall and apply correct formulae, rules, theorems, properties and concepts to solve problems in different topics; perform metric unit conversion and proper manipulations and failure to interpret plane figures related to congruence and similarity, regular polygons and frequency distribution table when solving related problems.

The National Examinations Council of Tanzania is humbled and honored to express its sincere thanks to all the examiners and examination officers for preparing this report.

Dr. Charles E. Msonde

EXECUTIVE SECRETARY

1.0 INTRODUCTION

This report is based on the analysis of the Students' Item responses for the students who sat for the 041 Basic mathematics assessment in FTNA 2020. The analysis mainly addresses the areas in which the students faced difficulties and the areas in which they performed well when answering the assessment items.

The number of students who sat for the assessment in FTNA 2020 was 600,751 out of which 95,743 (15.94%) students passed. In FTNA 2019, 570,591 students sat for the assessment, out of which 120,310 (21.09%) students passed. Therefore, the performance has decreased by 5.15 percent.

The assessment paper consisted of ten (10) compulsory questions carrying 10 marks each. Section 2.0 of this report summarizes the analysis of the students' performance in each question. The national assessment results are based on the score intervals 75 – 100, 65 – 74, 45 – 64, 30 – 44 and 0 – 29 which are equivalent to *excellent*, *very good*, *good*, *satisfactory* and *fail*, respectively. For the purpose of this report, the students' performance in each question is considered *good*, *average* or *weak* if the percentage of students who scored at least 30 out of 100 percent is 65 – 100, 30 – 64 or 0 – 29, respectively.

In presenting the data, red, yellow and green colours were used in both Figures and Appendix to represent good, average and weak performance, respectively. However, section 3.0 gives the conclusion and recommendations that will help students, teachers and government to improve the performance of the future Basic mathematics assessments.

2.0 ANALYSIS OF THE STUDENT'S PERFORMANCE IN EACH QUESTION

This section addresses the tasks and requirements of each item of the questions. The second issue presented is the statistical or data analysis basing on students' performance. The description of data and charts was done using a criterion of the score intervals of 10 – 6.5, 6.4 – 3.0 and 2.5 – 0 out of 10 allotted marks for each question representing *good*, *average* and *weak* performances, respectively. Thirdly, the section entails the students' item response analysis in regard of strengths and weaknesses they had when answering the assessment items. The extracts for best and worst responses for each question are presented as representative samples to support the findings noted from students' responses.

2.1 Question 1: Numbers and Approximations

The question consisted of parts (a) and (b). In part (a), students were required to write each of the given numbers 18, 24 and 36 as a product of prime factors and hence find their greatest common factor. In part (b), they were supposed to write 0.009765 correct to (i) three decimal places (ii) three significant figures and state the place value of 9 in the given number.

This question was attempted by 587,898 (97.8%) students. The analysis shows that 341,247 (58.0%) students scored from 3 to 10 marks, among them 10,132 (1.7%) scored full marks. This shows that the students' performance in this question was average. In contrast, out of 246,651 (42.0%) students who scored below the average, 196,337 (33.4%) scored 0 mark. The students' performance in this question is summarized in Figure 1.

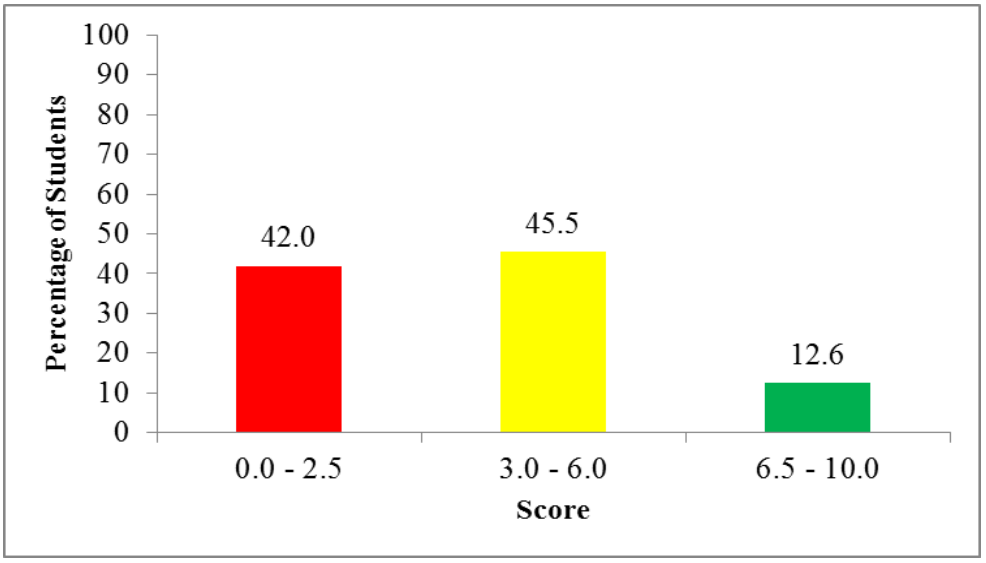
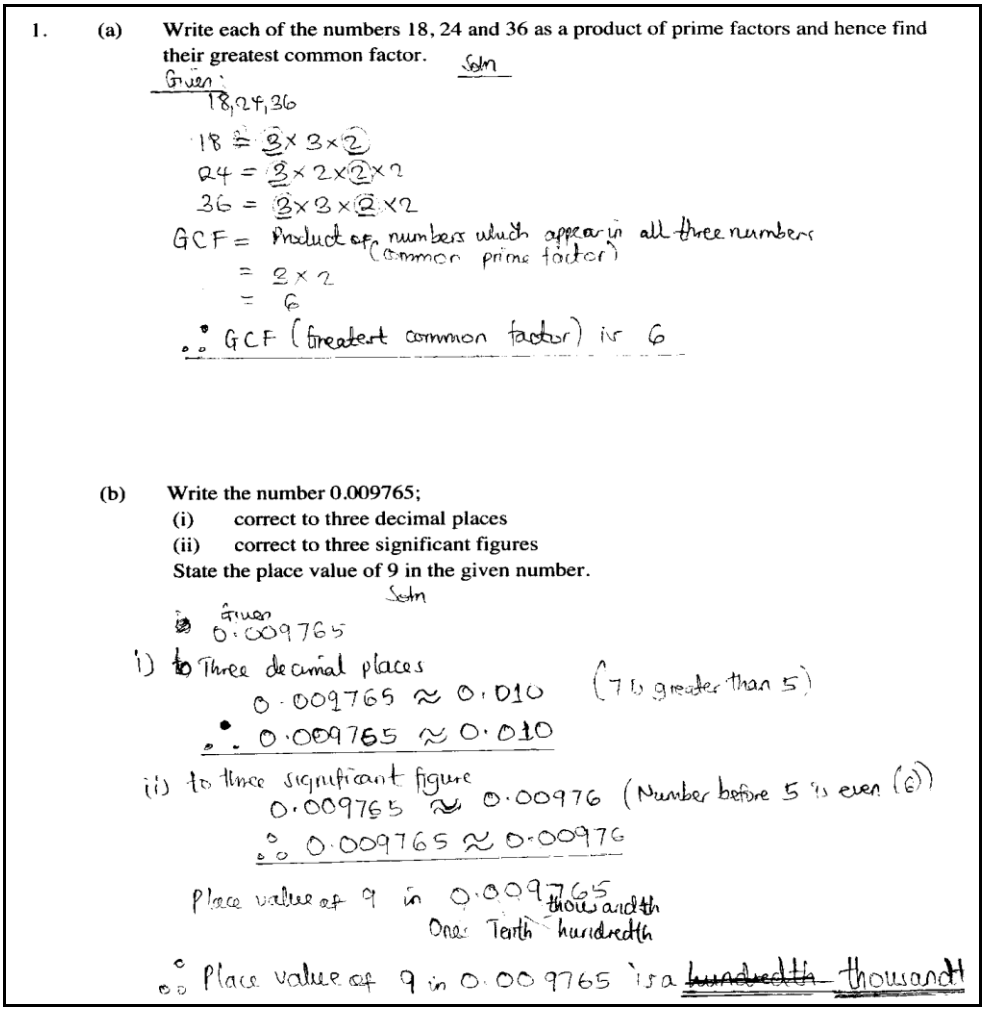


Figure 1: *Students' performance in question 1*

The findings from students' response reveal that the ability to score high or full marks was contributed by the following factors: in part (a), the students were able to write 18, 24 and 36 as a product of prime factors and got the Greatest Common Factor (GCF), that is $18 = 2 \times 3 \times 3$, $24 = 2 \times 2 \times 2 \times 3$ and $36 = 2 \times 2 \times 3 \times 3$. By using these results they were able to get the GCF of the given numbers, that is $GCF = 2 \times 3 = 6$. In part (b), the students who answered the question correctly had sufficient knowledge to write the number 0.009765 correct to (i) three decimal places as 0.010 and (ii) three significant figures as 0.00976 and were able to state that the place value of 9 is thousandth. This shows that the students had adequate knowledge of the factors of a number, the place of a decimal, determination of a significant figure and the place value of a number. Extract 1.1 is a sample response of a student who answered this question correctly.


1. (a) Write each of the numbers 18, 24 and 36 as a product of prime factors and hence find their greatest common factor.

Soln

Given:
18, 24, 36

18 = 3 × 3 × 2
24 = 3 × 2 × 2 × 2
36 = 3 × 3 × 2 × 2

GCF = Product of numbers which appear in all three numbers (common prime factor)
= 2 × 2
= 6

∴ GCF (greatest common factor) is 6

(b) Write the number 0.009765;
(i) correct to three decimal places
(ii) correct to three significant figures
State the place value of 9 in the given number.

Soln

Given
0.009765

i) to Three decimal places
0.009765 ≈ 0.010 (7 is greater than 5)
∴ 0.009765 ≈ 0.010

ii) to three significant figure
0.009765 ≈ 0.00976 (Number before 5 is even (6))
∴ 0.009765 ≈ 0.00976

Place value of 9 in 0.009765
thousandth
One Tenth hundredth

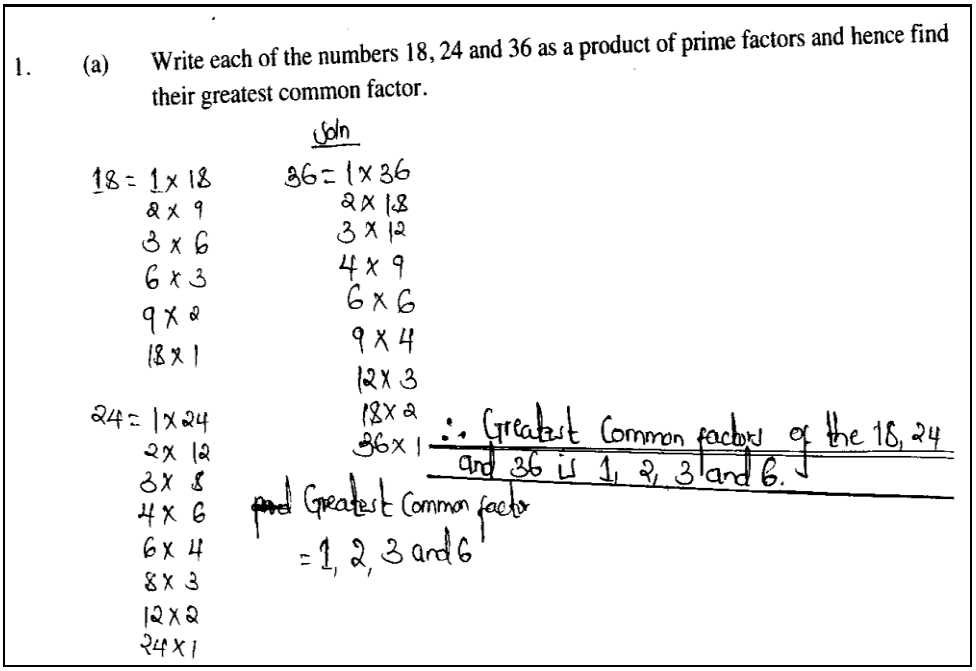
∴ Place value of 9 in 0.009765 is a ~~hundredth~~ thousandth

Extract 1.1: A sample of a correct response from one of the students in question 1.

In Extract 1.1, the student was able to apply the method of prime factorization to correctly determine the GCF of the number given in part (a). The student also applied the knowledge of approximations and got the required answers in part (b).

On the other side of the analysis, students failed to answer the question correctly due to the following reasons: some students calculated the Lowest Common Multiple (LCM) in answering part (a) contrary to the given instructions. Others managed to get the prime factors of 18, 24 and 36 but were unable to determine common factors that could be multiplied to give 6 as the required GCF.

In part (b)(i), the students failed to identify the third decimal place (occupied by digit 9 in the numeral 0.009765) and the rules of rounding off a number. For example, some students wrote incorrect approximates like 0.009, 0.0976, 0.0010 and 0.001 instead of 0.010 as required. This indicates that the students lacked knowledge of the concept of a decimal place. In part (b)(ii), they failed to identify the significant figures from the given number, which is an important step before rounding off. For example, several students considered the zeros in 0.009765 being significant figures and wrote incorrect approximates like 0.009 and 0.010 instead of writing it correctly as 0.00977. These students failed to realize that the zeros before the non-zero digits are not significant figures. Furthermore, the students failed to state correctly the place value of 9 in the numeral 0.009765. They stated incorrect place values of 9 as hundredth and thousands instead of thousandth due to inability to apply the rules for identifying the place value of a decimal. Moreover, some students simply shifted the decimal point to the right three times in the number 0.009765 and wrote the results like 9.000, 9.765, 9000 and 10,000 contrary to the procedures for specifying the place value of a decimal and a whole number. Extract 1.2 illustrates a sample of a student's answer who lacked knowledge of the application of factors to find GCF as well as approximating the given decimal into given number of decimal places and significant figures.


1. (a) Write each of the numbers 18, 24 and 36 as a product of prime factors and hence find their greatest common factor.

John

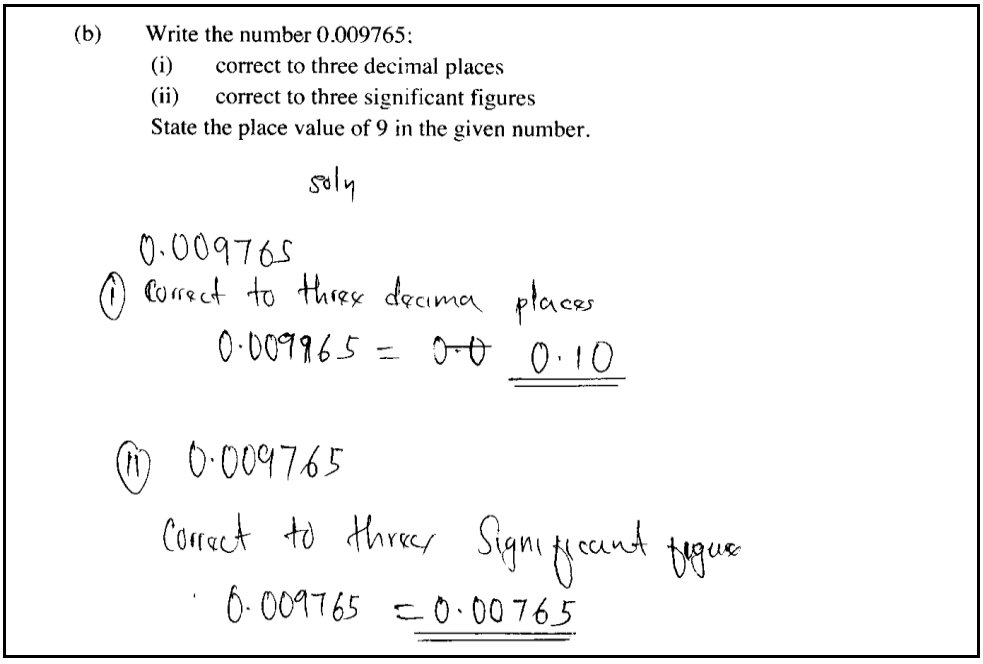
18 = 1 x 18
2 x 9
3 x 6
6 x 3
9 x 2
18 x 1

24 = 1 x 24
2 x 12
3 x 8
4 x 6
6 x 4
8 x 3
12 x 2
24 x 1

36 = 1 x 36
2 x 18
3 x 12
4 x 9
6 x 6
9 x 4
12 x 3
18 x 2
36 x 1

∴ Greatest Common factors of the 18, 24 and 36 is 1, 2, 3 and 6.

and Greatest Common factor
= 1, 2, 3 and 6



Extract 1.2: A sample of an incorrect response from one of the students in question 1.

In Extract 1.2, the student listed the common factors of 18, 24 and 36 but did not state the greatest common factor from the list. The student was also unable to write 0.009765 to three decimal places and three significant figures.

2.2 Question 2: Fractions, Decimals and Percentages

The question had parts (a) and (b). In part (a), students were required to find the value of the expression $\frac{5}{2}-\left(3\frac{3}{5}\div 1\frac{1}{5}-\frac{4}{5}\right)$. In part (b)(i), it was given that the price of a shirt costing shs. 15,000 was reduced by 15% in a sales promotion and students were required to find the new price of the shirt. In part (b)(ii), they were required to change $0.5\dot{6}$ into a fraction in its simplest form.

A total of 589,942 (98.1%) students attempted this question, among them 148,996 (25.3%) students scored at least 3 out of 10 marks, and 19175 (3.2%) students scored full marks. However, 440,946 (74.7%) students scored below 3 marks and among them, a total of 323,467 (53.8%) scored 0

mark. This shows that the students' performance in this question was weak as presented in Figure 2.

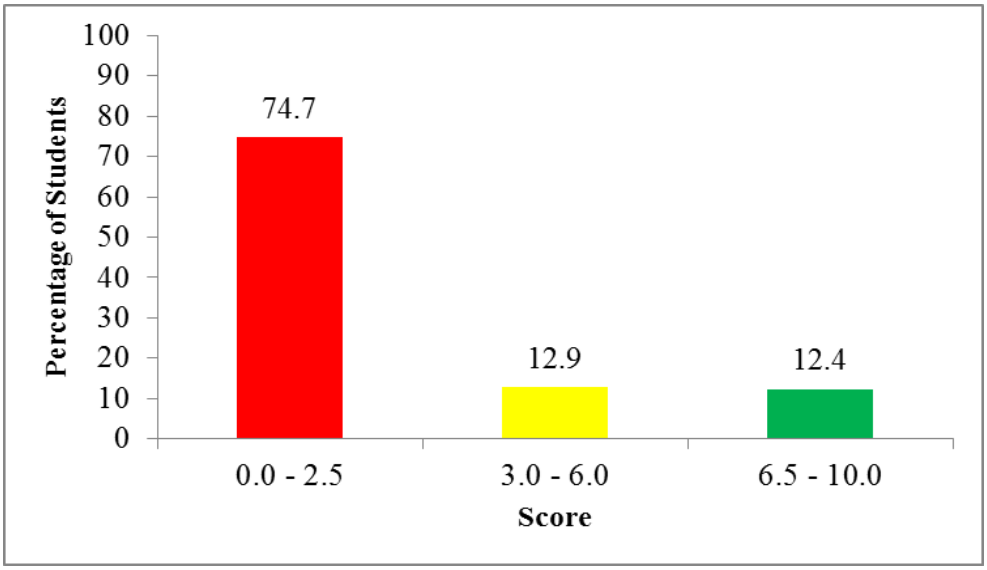


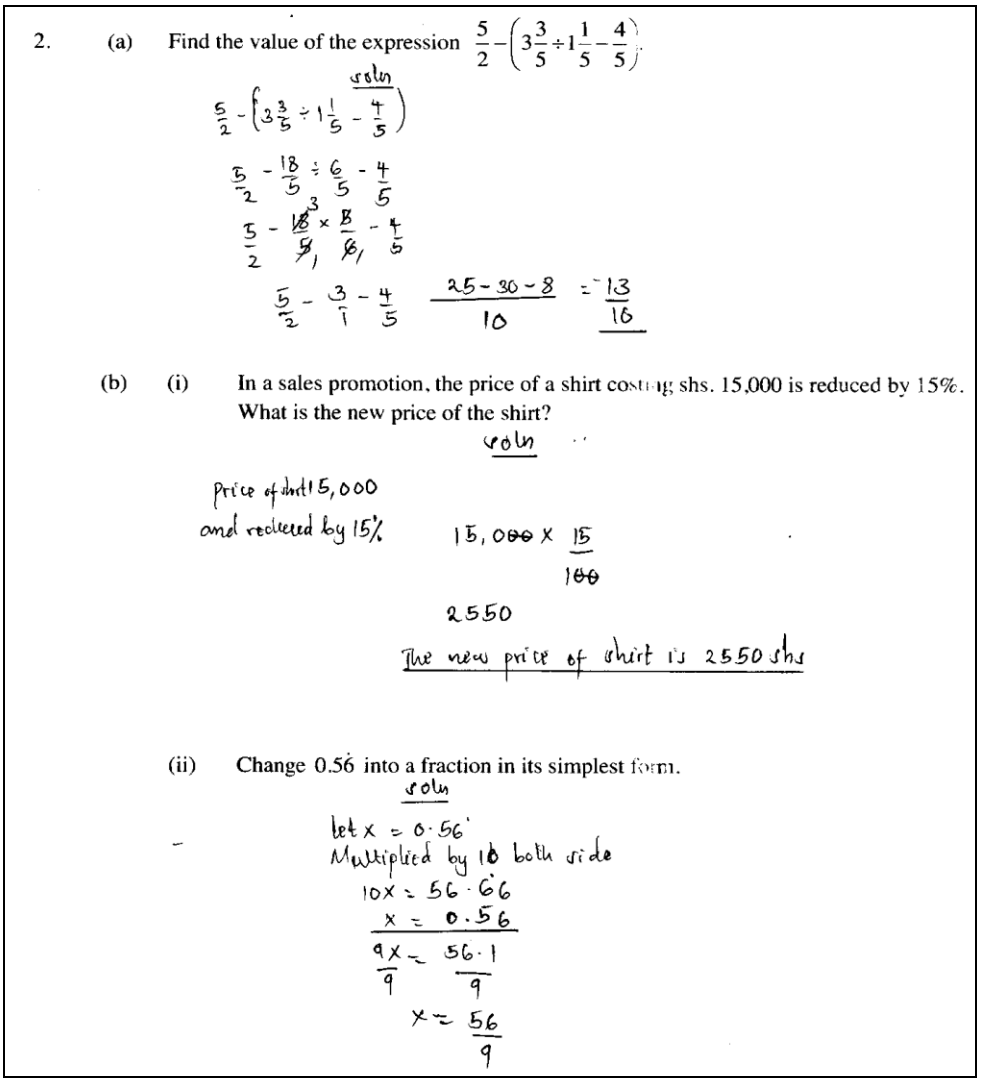
Figure 2: *Students' performance in question 2*

The students failed to perform correctly the tasks instructed in question 2 due to various factors. In part (a), most of students did not apply the BODMAS rule when evaluating the expression $\frac{5}{2}-\left(3\frac{3}{5}\div 1\frac{1}{5}-\frac{4}{5}\right)$. For example in opening the brackets, some students incorrectly wrote $\frac{5}{2}-3\frac{3}{5}\div 1\frac{1}{5}-\frac{4}{5}$. Instead, they were supposed to evaluate $3\frac{3}{5}\div 1\frac{1}{5}$, then subtract $\frac{4}{5}$ to get $\frac{11}{5}$ and lastly take $\frac{5}{2}-\frac{11}{5}$ to obtain $\frac{3}{10}$ which is the required answer. Others failed to know the operation to be given the first priority while others faced difficulty in dividing and subtracting the fractions.

In part (b)(i), the students calculated 15% of 15,000 and considered it a new price of the shirt. Others calculated 15% of 15,000 correctly but due to computational errors failed to subtract the obtained value from 15,000 to get the new price. Others divided the cost of the shirt by the reduced percentage, that is $\frac{15{,}000}{15}=1{,}000$ which is an incorrect step. Also, some

students added the reduced amount to the original price, that is $\left(15{,}000+\left(\frac{15}{100}\times 15{,}000\right)\right)$. They were supposed to write $\left(15{,}000-\left(\frac{15}{100}\times 15{,}000\right)\right)$ that would give the new price of the shirt. Those students lacked knowledge and skill of the application of the concept of profit and loss in finding the amount of a quantity after a percentage reduction.

In part (b)(ii) the students lacked knowledge of applying the procedures of converting repeating decimals into fractions. For instance, some of them wrote $\frac{56}{100}$ while others multiplied both sides of the equation $x=(0.5666...)$ by 100 instead of 10 to get $10x=(5.666...)$ which was an important step in arriving at the required result. Others managed to covert the repeating decimal into fraction but failed to write it in its simplest form. Extract 2.1 is an example of a student's incorrect response.

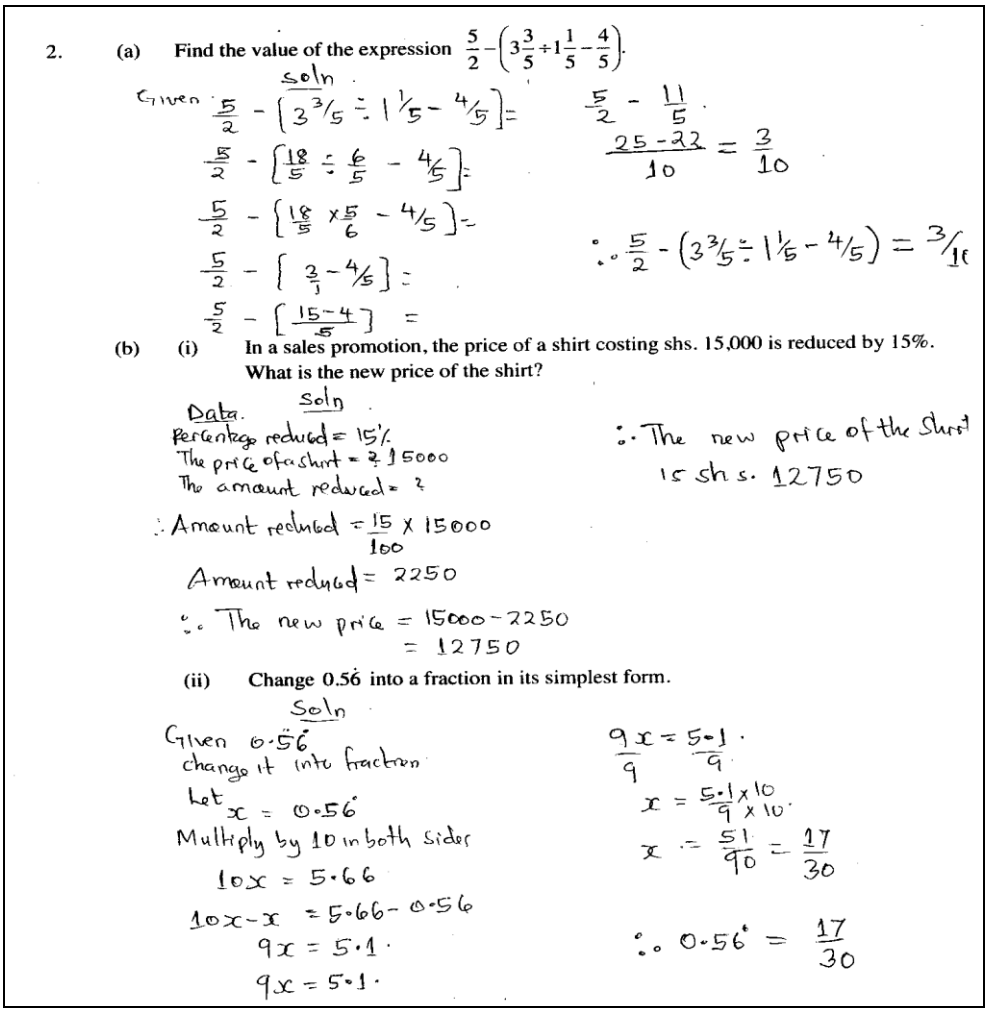


Extract 2.1: A sample of an incorrect response from one of the students in question 2.

The response in Extract 2.1 shows that the student lacked knowledge of solving expressions involving mixed operations. The student regarded the reduced percentage as the new price of the shirt. He/she was also unable to multiply and subtract correctly the recurring decimals to get a simple fraction.

Despite the weak performance, there were students who managed to answer this question correctly.

In part (a), they managed to find the value of the expression $\frac{5}{2}-\left(3\frac{3}{5}\div 1\frac{1}{5}-\frac{4}{5}\right)$ by converting the mixed fraction into improper fraction, then divided the fractions correctly, which led them to get the required value of the expression as $\frac{3}{10}$ or 0.3. In part (b)(i), they were able to calculate the 15% of shs.15,000 to get shs. 2,250 and subtracted it from shs.15,000 to get shs. 12,750 as the new price. In part (b)(ii), they were able to convert the repeating decimal into fraction and expressed it in its simplest form. Extract 2.2 is a sample solution of a student who answered this question correctly.


2. (a) Find the value of the expression $\frac{5}{2}-\left(3\frac{3}{5}\div 1\frac{1}{5}-\frac{4}{5}\right)$.

Soln

Given $\frac{5}{2}-\left[3\frac{3}{5}\div 1\frac{1}{5}-\frac{4}{5}\right]=$

$\frac{5}{2}-\left[\frac{18}{5}\div\frac{6}{5}-\frac{4}{5}\right]=$

$\frac{5}{2}-\left[\frac{18}{5}\times\frac{5}{6}-\frac{4}{5}\right]=$

$\frac{5}{2}-\left[\frac{3}{1}-\frac{4}{5}\right]=$

$\frac{5}{2}-\left[\frac{15-4}{5}\right]=$

$\frac{5}{2}-\frac{11}{5}$

$\frac{25-22}{10}=\frac{3}{10}$

$\therefore \frac{5}{2}-\left(3\frac{3}{5}\div 1\frac{1}{5}-\frac{4}{5}\right)=\frac{3}{10}$

(b) (i) In a sales promotion, the price of a shirt costing shs. 15,000 is reduced by 15%. What is the new price of the shirt?

Soln

Data.

Percentage reduced = 15%

The price of a shirt = shs 15000

The amount reduced = ?

$\therefore$ Amount reduced $=\frac{15}{100}\times 15000$

Amount reduced $= 2250$

$\therefore$ The new price $= 15000-2250$

$= 12750$

$\therefore$ The new price of the shirt is shs. 12750

(ii) Change $0.5\dot{6}$ into a fraction in its simplest form.

Soln

Given $0.5\dot{6}$

change it into fraction

Let $x = 0.5\dot{6}$

Multiply by 10 in both sides

$10x = 5.66$

$10x-x = 5.66-0.56$

$9x = 5.1$

$9x = 5.1$

$\frac{9x}{9}=\frac{5.1}{9}$

$x=\frac{5.1\times 10}{9\times 10}$

$x=\frac{51}{90}=\frac{17}{30}$

$\therefore 0.5\dot{6}=\frac{17}{30}$

Extract 2.2: A sample of a correct response of one of the students in question 2.

In Extract 2.2, the student calculated $3\frac{3}{5} \div 1\frac{1}{5}$, then subtracted $\frac{4}{5}$ from the result to get $\frac{11}{5}$ and lastly calculated $\left(\frac{5}{2} - \frac{11}{5}\right)$. The student also managed to subtract the reduced price (shs. 2,250) from the original amount (shs. 15,000) and got new price. He/she was also able to establish the equation $x = 0.5\dot{6}$ and multiplied both sides by 10 to get $10x = 5.6\dot{6}$, then he/she subtracted, that is $(10x - x) = \left(5.6\dot{6} - 0.5\dot{6}\right)$ to get $9x = 5.1$ that was an important step towards the answer.

2.3 Question 3: Units and Ratio, Profit and Loss

This question comprised parts (a) and (b). Part (a) stated that "a lorry carries 7.2 tonnes of sand from the mining area to the construction site. On the way, 230 kg of sand either fall off or blow away." Students were required to find the mass (in tonnes) of sand that would remain by the end of the journey. In part (b), the students were required to find the buying price of an article that was sold for shs.160,000 at profit of 25%.

The data analysis shows that a total of 558,304 (92.9%) students attempted this question. It also shows that 69,790 (12.5%) students scored from 3 to 10 marks and among them, 11,393 (2.0%) scored full marks. However, 488,514 (87.5%) students scored from 0 to 2.5 marks and among them, 452,243 (81.0%) scored 0 mark. Generally, the performance in this question was weak. The summary of performance is presented in Figure 3.

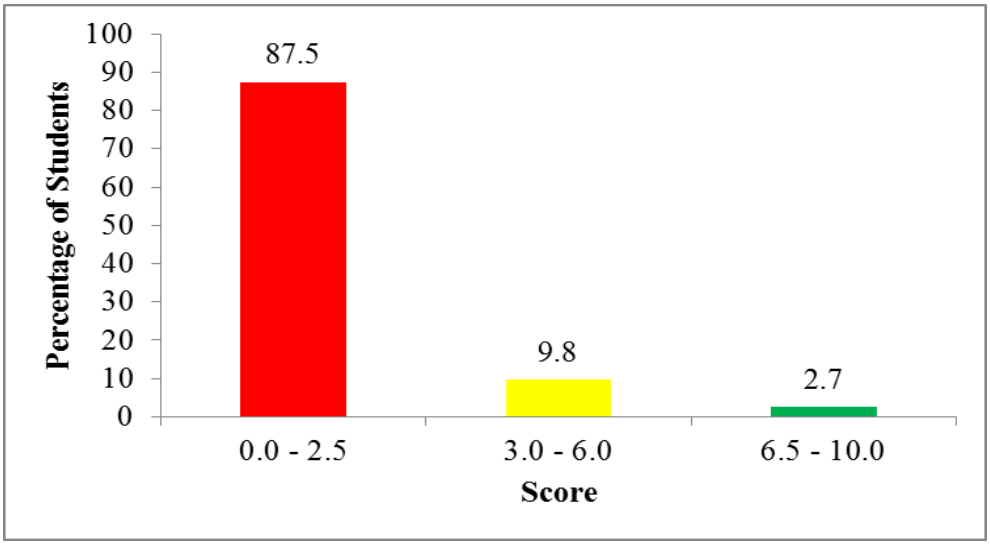
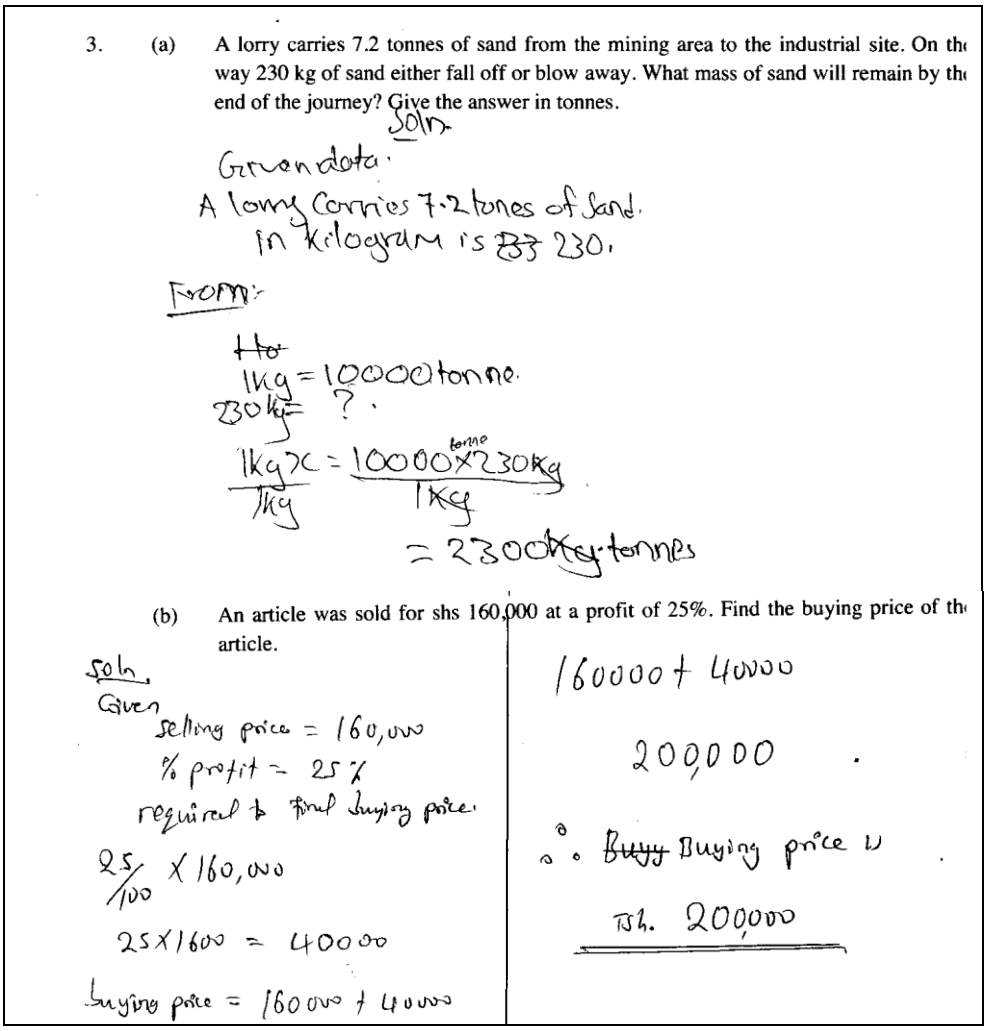


Figure 3: *Students' performance in question 3*

The analysis of students' responses shows that failure to perform well was contributed by the following reasons: in part (a), the students failed to convert 230kg into tonnes using the conversion $1\,\text{tonne} = 1000\,\text{kg}$. For instance, some of them wrote a wrong conversion like $1\,\text{tonne} = 10\,\text{kg}$ or $1\,\text{tonne} = 100\,\text{kg}$. Others got the answer in kilograms but failed to convert it back into tonnes as required in the question. Also, some students added the amount fell off or blown away to the initial amount of sand. Others multiplied 230kg by 7.2 and converted the final answer into tonnes. These students lacked knowledge of unit conversion especially the metric unit of mass.

In part (b), most students failed to recall and apply the correct formula to find the buying price of the article. Incorrect formulae that were commonly noted during the analysis of students' responses were $\text{Profit} = \dfrac{\text{Buying price}}{\text{Selling price}} \times 100\%$ and $\text{Buying price} = \dfrac{\text{Selling price - profit}}{\text{Buying price}} \times 100$. Other students calculated 25% of shs. 160,000 and subtracted it from shs. 160,000 to get shs. 120,000. In addition, some of students equated 25% of an unknown value x to 160,000 and solved for x which is a wrong approach. Extract 3.1 shows the response of a student who performed poorly in this question.



Extract 3.1: A sample of an incorrect response from one of the students in question 3.

In Extract 3.1, the student wrote an incorrect conversion standard, that is $1\,\text{kg} = 10{,}000\,\text{tonnes}$. In part (b), the student multiplied the percentage profit by the given selling price, that is $\frac{25}{100} \times 160{,}000$ and then added 160,000. The students lacked knowledge of application of the concepts of profit and loss in solving real life related problems.

On the other hand, there were students who managed to score full marks. In part (a), the students were able to convert 230 kg into tonnes to get 0.23 tonnes using the conversion $1\,\text{tonne} = 1000\,\text{kg}$ and hence subtracted

it from 7.2 tonnes to get 6.97 tonnes as the remaining mass of sand by the end of the journey. In answering part (b), they recalled and applied the correct formula, $\text{Percentage profit} = \frac{\text{Profit}}{\text{Cost}} \times 100$ and "profit = selling price – buying price" and evaluate the buying price. A sample response of a student who answered the question correctly is shown in Extract 3.2.

3. (a) A lorry carries 7.2 tonnes of sand from the mining area to the industrial site. On the way 230 kg of sand either fall off or blow away. What mass of sand will remain by the end of the journey? Give the answer in tonnes.

soln.

Given

Total tonnes of sand = 7.2 tonnes.

We change into kg.

1 tonne = 1000 kg.

7.2 t = ?

$\frac{7.2\text{ t} \times 1000\text{ kg}}{1\text{ t}}$

= 7200 kg.

Mass remaining = 7200 kg
− 230 kg
6970 kg.

Mass remaining was 6970 kg.

We change into tonnes.

1 t = 1000 kg.

? = 6970 kg

$\frac{6970\text{ kg} \times 1\text{ t}}{1000\text{ kg}}$

= 6.97 tonnes.

∴ Mass remaining at the end of the journey was 6.97 tonnes.

(b) An article was sold for shs 160,000 at a profit of 25%. Find the buying price of the article.

soln

Given

Percentage profit = 25 %.

Selling price = 160,000/=

Buying price = ?

From,-

$\text{Percentage Profit} = \frac{S.P - B.P}{B.P} \times 100\%$

$25\% = \frac{160{,}000 - B.P \times 100}{B.P}$

$25 = \frac{16{,}000{,}000 - 100\,B.P}{B.P}$

$25\,B.P = 16{,}000{,}000 - 100\,B.P$

$100\,B.P + 25\,B.P = 16{,}000{,}000$

$\frac{125\,B.P}{125} = \frac{16{,}000{,}000}{125}$

B.P = 128,000/=

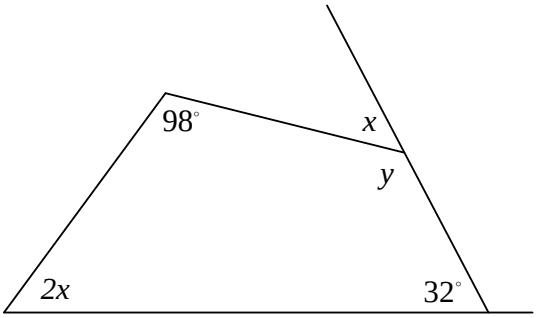
Buying price = shs. 128,000/=

Extract 3.2: A sample of a correct response of one of the students in question 3.

In Extract 3.2, the student was able to convert 7.2 tonnes into 7,200 kg and lastly subtracted to get 6,970 kg which is equivalent to 6.97 tonnes remained. He/she also applied the correct formula and performed correct computations and got the buying price of an article, which is shs. 128,000.

2.4 Question 4: Geometry, Perimeters and Areas

This question had parts (a) and (b). In part (a), the students were required to calculate the values of x and y in the following figure.



Part (b) stated that, a metal wire is bent to form a semi-circle with a radius of 14 cm. The students were required to find: (i) the total length of the metal wire and (ii) the area bounded by the metal wire.

This question was attempted by 548,946 (91.3%) students. The analysis shows that 15,316 (2.8%) students scored from 3 to 10 marks, among them 789 (0.1%) students scored full marks. Contrarily, a total of 533,630 (97.2%) students scored below the average performance and among them, 484,285 (88.2%) students scored 0 mark. This shows that the students' performance in this question was very weak. The summary of students' performance in this question is presented in Figure 4.

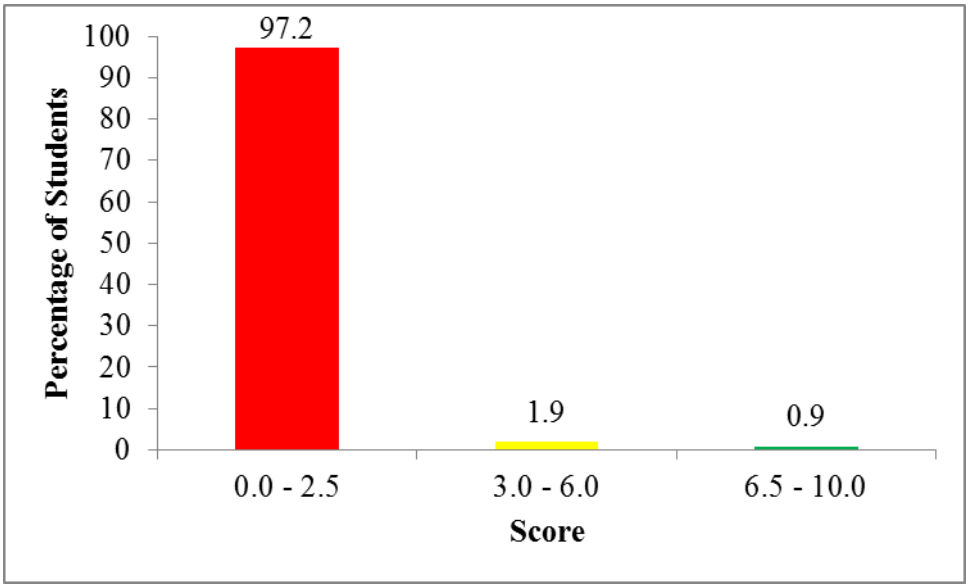


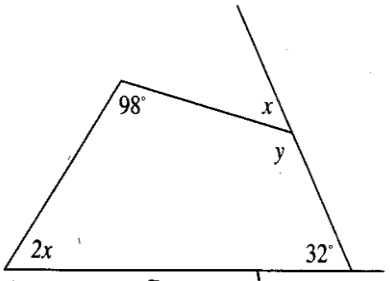
Figure 4: *Students' performance in question 4*

The students who answered part (a) incorrectly encountered various challenges including the following; they failed to realize that x and y are supplementary angles and 98°, y, 32° and $2x$ are angles of a quadrilateral whose sum of its degree measure is 360°. In most cases they wrote wrong steps like $98^\circ + y + 32^\circ + 2x = 180^\circ$, $98^\circ + y + 32^\circ + 2x = 520^\circ$ and $98^\circ + y = 180^\circ$ instead of formulating the correct equation $98^\circ + y + 32^\circ + 2x = 360^\circ$ that could lead to the required answers. Such students lacked knowledge of the sum of interior angles of a quadrilateral. Others failed to recognize that for a straight line, $y = 180^\circ - x$ which was required to be substituted in the equation $98^\circ + y + 32^\circ + 2x = 360^\circ$. As a result, they were unable to get the correct values of x and y.

In part (b), the students failed to use the correct formula to find the total length and area of a semi-circle resulting from bending a metal wire. They wrote incorrect formulae like $\text{circumfere nce} = 2\pi r$, $\text{circumfere nce} = \frac{2\pi r^2}{4}$, $\text{circumfere nce} = \frac{1}{2}\pi r$ and $\text{circumfere nce} = \pi r$ instead of applying the correct formula $\text{circumfere nce} = \pi r + 2r$ when finding the total length (that is, the circumference) of resulted semi-circle. Moreover, there were some

students who wrote the total length of the metal wire as $\text{circumference} = \pi r$ by considering the arc instead of a semicircle. Others had wrong interpretation of the given word problem as they considered it a quarter of a circle and a square. Also, when finding the area bounded by the metal wire some students applied incorrect formulae $\text{area} = \frac{1}{4}\pi r^2$, $\text{area} = \pi r^2$ and $\text{area} = 2\pi r^2$ instead of $\text{area} = \frac{1}{2}\pi r^2$ which is the correct formula for finding the area of a semi-circle. This indicates that the students lacked adequate knowledge of the application of circumference and area of semi-circle in solving real life related problems. Extract 4.1 shows a response from a student with inadequate knowledge of the tested concept.

4. (a) Find the values of x and y in the following figure.



$2x + 98 = 180$ and $32 + y = 180$

$2x = 180 - 98$ $\quad y = 180 - 32$

$\frac{2x}{2} = \frac{82}{2}$ $\quad y = 148°$

$\therefore x = 41$

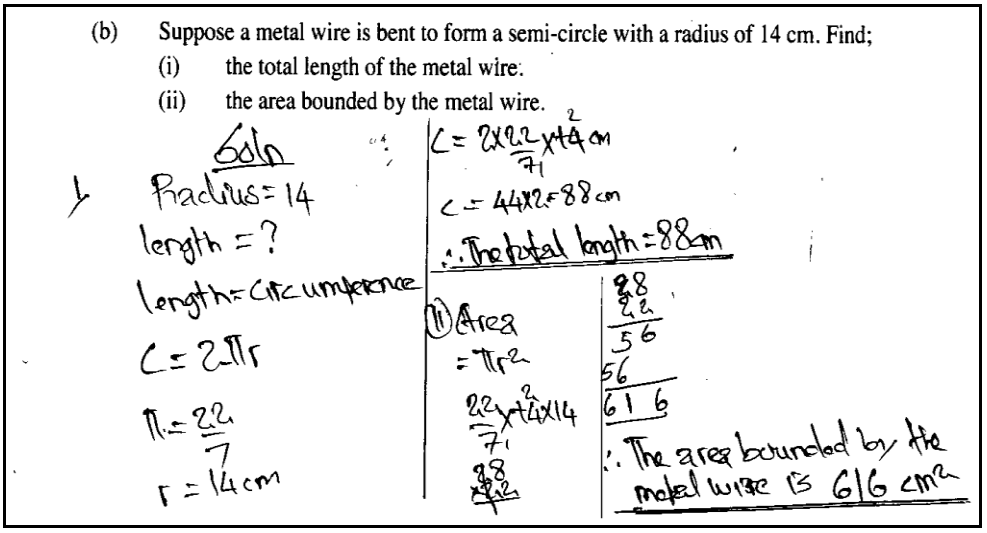
$y = 148°$

$148 + x = 180°$

$x = 180 - 148$

$x = 32$

$\therefore$ The value of $x = 32$ and $y = 32$ 148°

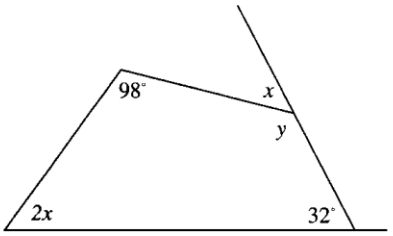


Extract 4.1: An incorrect response from one of the students in question 4.

In Extract 4.1, the student wrote incorrect equations $32^\circ + y = 180^\circ$, $2x + 98^\circ = 180^\circ$ and yet failed to know that sum of supplementary angles x and y is 180°. Also the student was unable to apply the correct mathematical formulae to obtain the required total length and area of a metal wire.

In contrast to weak performance, there were students who answered this question correctly. Those students were able to do the following; in part (a), they were able to identify that angle x is a supplement of the angle y, that is $x + y = 180^\circ$. Also, they recognized that 98°, y, 32° and $2x$ as the angles forming a quadrilateral whose sum is 360°. This was a sufficient condition to get the correct answers. In part (b), the students were able to interpret correctly the given problem as a semi-circle. So, they were able to find the perimeter and the area of a semi-circle resulted from a bent wire. Extract 4.2 shows a response of a student who answered this question correctly.

4. (a) Find the values of x and y in the following figure.



Soln.

Given: Quadrilateral = 360°.

$2x + 98^\circ + y + 32^\circ = 360^\circ$

$2x + y + 130^\circ = 360^\circ$

$2x + y = 360^\circ - 130^\circ$

$2x + y = 230^\circ$

$2x + y = 230^\circ$ ... 1st Equation

Straight line = 180°

$x + y = 180^\circ$.

$x + y = 180$ ... 2nd Equation.

$\begin{cases} 2x + y = 230^\circ \\ x + y = 180^\circ \end{cases}$

$\therefore \begin{cases} 2x + y = 230^\circ \\ x + y = 180^\circ \end{cases}$

$x = 230^\circ - 180^\circ$

$x = 50^\circ$.

But: If $x = 50^\circ$, $y = ?$

Take eqn ii)

$x + y = 180^\circ$.

$50^\circ + y = 180^\circ$

$y = 180^\circ - 50^\circ$

$y = 130^\circ$

$\therefore$ Value of $x = 50^\circ$ and $y = 130^\circ$

(b) Suppose a metal wire is bent to form a semi-circle with a radius of 14 cm. Find;

(i) the total length of the metal wire.

(ii) the area bounded by the metal wire.

Soln.

14cm 14cm.

i) Total length = Perimeter/Circumference

Circumference of Semi Circle = $\frac{\pi d}{2} + d$

diameter (d) = radius x 2

= 14 x 2

= 28 cm

$\frac{\pi d}{2} + d$.

$\frac{\frac{22}{7} \times 28}{2} + 28$

$\frac{22 \times 4}{2} + 28$

44 + 28

72cm.

Total length = 72 cm

ii) Area of Semicle Circle.

$= \frac{\pi r^2}{2}$

$\frac{\frac{22}{7} \times 14 \times 14}{2}$

$\frac{22 \times 2 \times 14}{2}$

$22 \times 2 \times 7$

44×7

$308\ cm^2$

Area bounded

$= 308\ cm^2$

Extract 4.2: A sample of a correct response from one of the students in question 4.

In Extract 4.2, the student was able to use the concepts of supplementary angles and sum of interior angles of a quadrilateral to find the values of x and y. The student also managed to find the total length and the area of a semi-circle resulted from bending a metal wire.

2.5 Question 5: Algebra and Quadratic Equations

This question was composed of two parts, (a) and (b). In part (a), the students were required to find two numbers whose sum is 127 and difference is 7. In part (b), they were instructed to solve the equation $x^2 - 10x + 13 = 0$ by completing the square leaving the answer in surd form.

This question was attempted by 560,943 (93.3%) students. A total of 72,288 (12.9%) students scored from 3 to 10 marks, among them 8,673 (1.5%) students scored full marks. However, 488,655 (87.1%) students scored less than 3 marks and among them, 484,285 (88.2%) scored 0 mark. This shows that the students' performance in this question was weak. Figure 5 presents a summary of students' performance in this question.

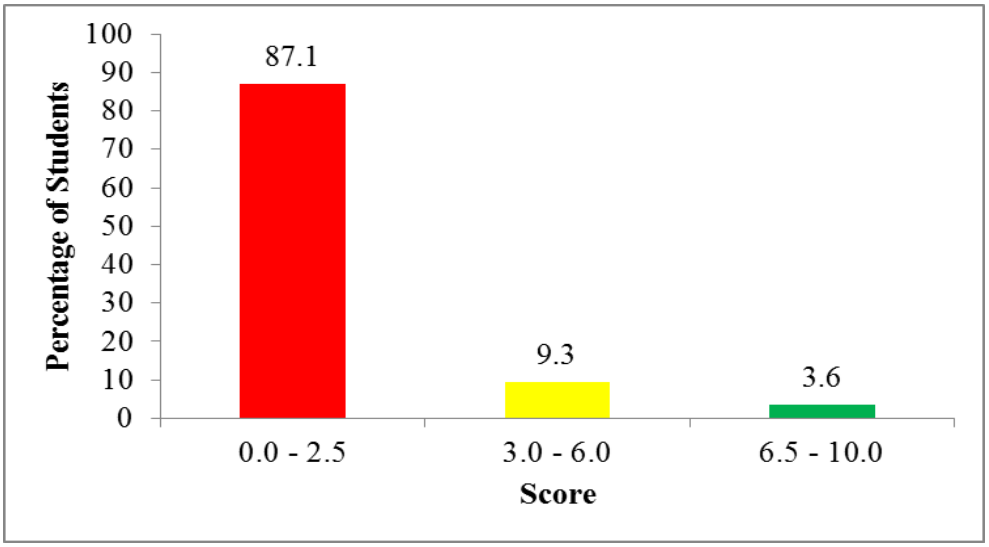


Figure 5: *Students' performance in question 5*

The analysis of the students' responses reveals the factors for weak performance as follows: in part (a), most students failed to formulate correctly the required simultaneous linear equations from the given word problem. Others managed to formulate the equations $x + y = 127$ and $x - y = 7$ but failed to solve them simultaneously. Few of them divided and multiplied 127 by 7 indicating lack of knowledge in solving word problems related to simultaneous equations. Others regarded that the two unknown numbers are consecutive numbers and they let them be x and x+1. In part

(b), most of the students used quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ to solve the given quadratic equation contrary to the instruction given. Others had a concept of using the method of completing the square but were unable to solve it correctly. Other students wrote a wrong form of the given equation as $x^2 - 10x = 13$ instead of $x^2 - 10x = -13$. Subsequently, they came up with incorrect steps like $x^2 - 10x + \left(\frac{-10}{2}\right)^2 = 13 + \left(\frac{-10}{2}\right)^2$ instead of correctly writing it in the form $\left(x - \frac{10}{2}\right)^2 - \left(\frac{10}{2}\right)^2 = -13$. This shows that the students lacked knowledge of solving quadratic equations by using the method of completing the square. Extract 5.1 represents a sample solution of one of the students who failed to answer this question correctly.

5. (a) The sum of two numbers is 127. If the difference between the numbers is 7, find the numbers.

Solution

$2x + x = 127$

$\frac{3x}{3} = \frac{127}{3}$

$x = 42\frac{1}{3}$

$127 - 7 = 120$

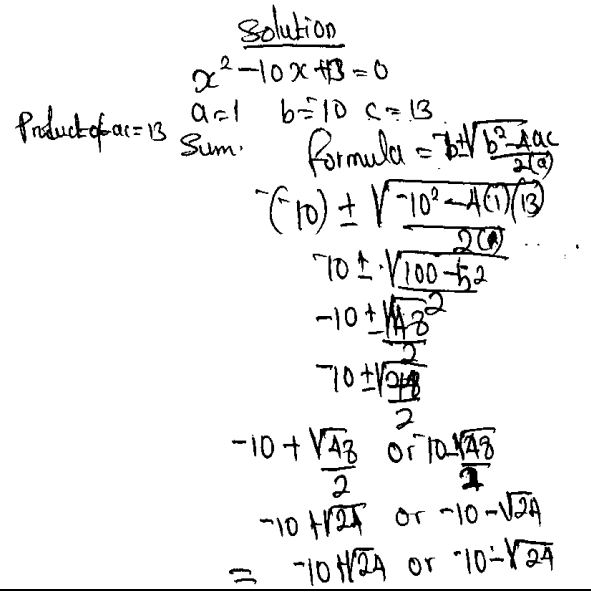
$\frac{2x}{2} = \frac{120}{2}$

$x = 60 + 7$

$x = 67$

Those number of 127 are 67

(b) Solve the equation $x^2 - 10x + 13 = 0$ by completing the square. Leave the answer in surd form.



Extract 5.1: An incorrect response from one of the students in question 5.

In part (a) of Extract 5.1, the student formulated incorrect equation $2x + x = 127$ for two unknown numbers whose sum is 127 instead of $x + y = 127$. Yet, he/she wrote $127 - 7$ as the difference between the numbers instead of $x - y = 127$, where $x > y$. In part (b), the student applied the quadratic formula instead of using the method instructed.

In spite of weak performance, some students were able to answer the question correctly due to the following reason: in part (a), they were able to formulate and solve linear equations from the given problem. The students had adequate knowledge of solving simultaneous equations with two unknowns. In part (b), they were able to use the method of completing the square and undertake all necessary steps to solve the equation $x^2 - 10x + 13 = 0$. Extract 5.2 shows a correct response from a student who answered the question correctly.

5. (a) The sum of two numbers is 127. If the difference between the numbers is 7, find the numbers.

Soln.

Given:-

sum = 127

difference = 7

let larger no. be x

smaller no. be y

$x+y=127$ - - -(i)

$x-y=7$ - - -(ii)

combine:-

$$-\begin{cases} x+y=127 \\ x-y=7 \end{cases}$$

$2y=120$

$\frac{2y}{2}=\frac{120}{2}$

$y=60$

From $x-y=7$

but $y=60$

$x-60=7$

$x=7+60$

$x=67$

∴ The two numbers are 67 and 60.

(b) Solve the equation $x^2-10x+13=0$ by completing the square. Leave the answer in surd form.

Soln.

Given:-

$x^2-10x+13=0$

$x^2-10x+13=0$

$x^2-10x=-13$

$b=-10$

$x^2-10x+\left(-10\frac{1}{2}\right)^2=-13+\left(-10\times\frac{1}{2}\right)^2$

$x^2-10x+(-5)^2=-13+(-5)^2$

$x^2-10x+(-5)^2=-13+25$

$(x-5)^2=12$

Find square root.

$\pm\sqrt{(x-5)^2}=\pm\sqrt{12}$

$x-5=\pm\sqrt{12}$

$x=5\pm\sqrt{12}$

$x=5\pm\sqrt{12}$

∴ $x=5\pm2\sqrt{3}$

Extract 5.2: A sample of a correct response from one of the students in question 5.

In Extract 5.2, the student managed to get 67 and 60 as the required numbers in part 1(a). He/she also managed to complete the square, a step that was necessary to get the answer $x-5=\pm\sqrt{12}$ or more simply $x-5=\pm 2\sqrt{3}$ in surd form.

2.6 Question 6: Coordinate Geometry and Geometrical Transformations

This question comprised parts (a) and (b). In part (a), students were required to find (i) the equation of a line passing through the point P(-1, 4) with a gradient of 10 and (ii) the values of a and b if the line of the equation obtained in part (a)(i) passes through the points $(a,0)$ and $(0,b)$. In part (b), they were required to find the image of the point P(4,1) when it is (i) reflected in the x-axis, (ii) reflected in the line $y=x$ and (ii) translated by the point $T(3,5)$.

A total of 497,928 (82.8%) students attempted this question. From the analysis done, 63,303 (12.7%) students scored from 3 to 10 marks, among them 4,416 (0.9%) students scored full marks. In contrast, a total of 434,625 (87.3%) students scored from 0 to 2.5 marks and among them, 373,845 (75.1%) students scored 0 mark. This shows that the students' performance was weak. Figure 6 gives a summary of students' performance in this question.

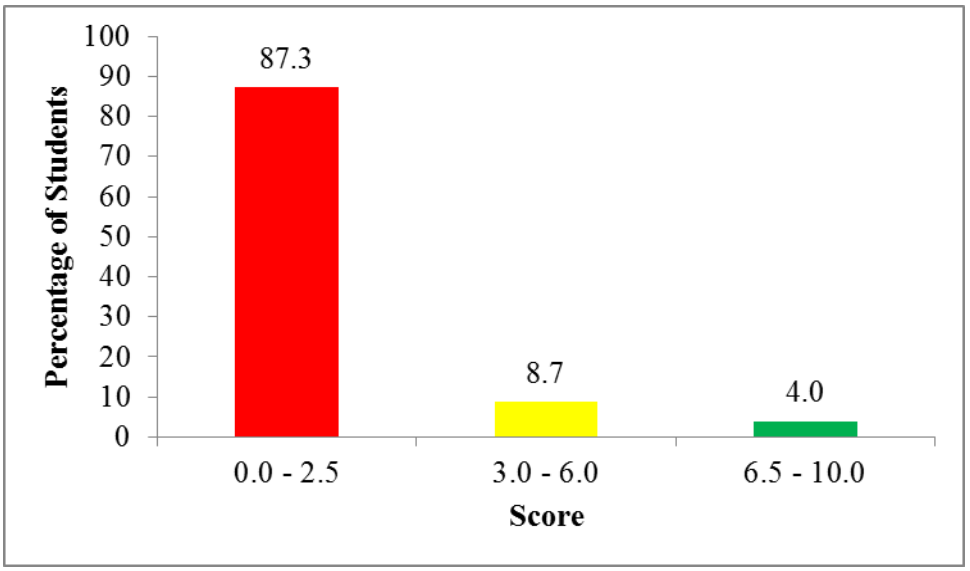
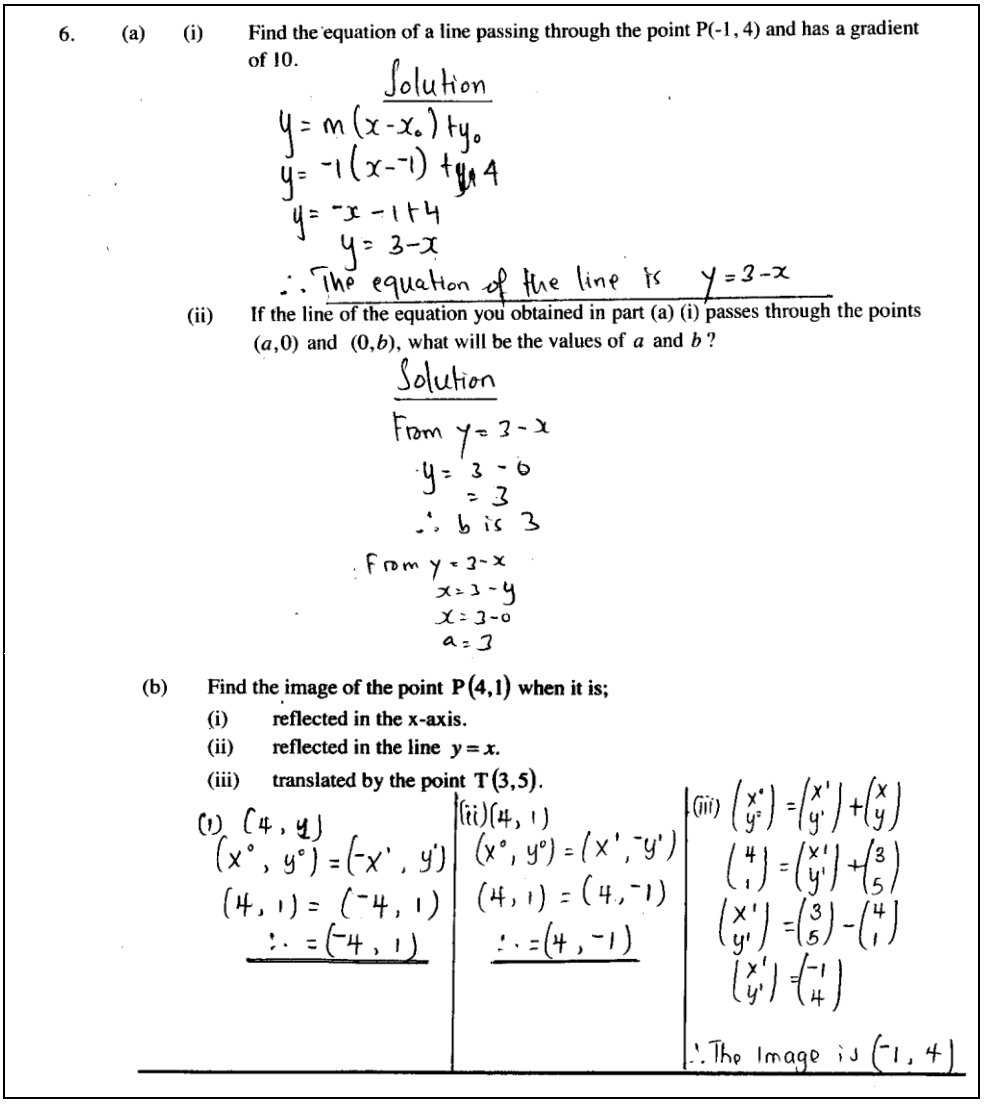


Figure 6: *Students' performance in question 6*

The weak performance was contributed by the following factors: in part (a)(i), there were some students who failed to recall the formula for gradient. Instead of writing $\text{gradient} = \frac{\text{change in y}}{\text{change in x}}$, they wrote incorrect formulae like $\text{gradient} = \frac{\text{change in x}}{\text{change in y}}$ or $\text{gradient} = \frac{\text{y} + \text{y}_1}{\text{x} + \text{y}_1}$ when finding the equation of a line. Others lacked the understanding of the concept of coordinates of a point. For example, a certain student considered the letter "P" a number as he/she wrote $\text{p} + \text{-1} + 4 = 10$. In part (a)(ii), they gave incorrect responses because they failed to get the correct answer in part (a)(i) and others failed to substitute correctly the points $(a, 0)$ and $(0, b)$ in the equation $y = 10x + 14$ they obtained in part (a)(i) to get the correct values of *a* and *b*.

In part (b)(i), some students had inadequate knowledge of reflection about the *x*-axis. When finding the image of $P(4,1)$, they changed the sign of *x*-coordinate instead of *y*-coordinate and subsequently got an incorrect point $P'(–4,–1)$ instead of $P'(4,–1)$ as the required image of $P(4,1)$. In part b(ii), they failed to recall that, when a point is reflected about the line $y = x$, the x-coordinate interchanges position with the y-coordinate. For example, some of them wrote the image of $P(4,1)$ as $P'(–4,–1)$ instead of $P'(1,4)$. In part (b(iii), the majority of students failed to recall that the translation by a point $T(a,b)$ takes a given point $P(x, y)$ to $P'(x + a, y + b)$, instead, they wrote $P'(x – a, y – b)$ and consequently obtained an incorrect answer $P'(3, 0)$. Extract 6.1 illustrates a sample response of a student who lacked adequate knowledge of coordinate geometry and geometrical transformation.

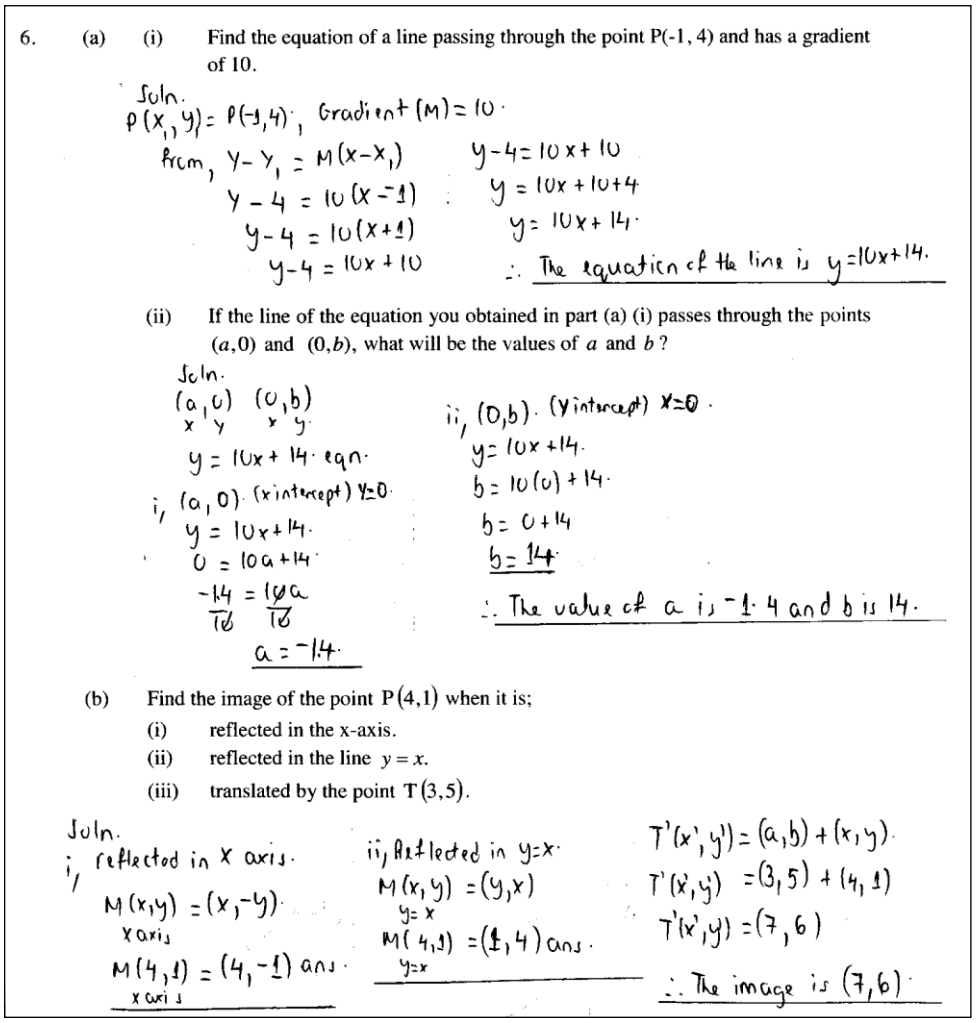


Extract 6.1: An incorrect response from one of the students in question 6.

In Extract 6.1, the student failed to substitute the given point and gradient in the equation $y = mx + y$, as a result they incorrectly wrote $y = -(x - (-1)) + 4$ instead of $4 = 10(-1) + y$ to get the y-intercept. Also, the student failed to use the points $(a, 0)$, $(0, b)$ and the equation they were supposed to obtain from part (a)(i) in order to get the values of *a* and *b*. In part (b), he/she failed to recall and apply the properties of reflection and translation to get the correct images of the point $P(4,1)$.

Despite the poor performance, there were students who answered correctly this question. In part (a)(i), they applied the correct formula $m = \frac{y_- y_1}{x - x_1}$ and the point $P(-1,4)$ such that $x_1 = -1$ and $y_1 = 4$ and $m = 10$ to get the required equation. Others were able to recall $y = mx + c$ as the general equation of the line passing through the given points and correctly substituted the values $m = 10$, $x = -1$ and $y = 4$ to obtain $c = 14$ and hence the equation $y = 10x + 14$. In part (a)(ii), they were able to substitute the points $(a,0)$, $(0,b)$ in the equation $y = 10x + 14$ obtained from part (a)(i) and solved for a and b.

In part (b)(i), the students were able to recall that reflection about the *x*-axis changes the sign of the y-coordinate and therefore reflection of the point P(4,1) in the x-axis gives the image $P'(4,-1)$. In part (b)(ii), they were able to recall that reflection of a point about the line $y = x$ takes a point (a,b) to (b,a) and for that reason, the required image should be the point $P'(1,4)$. Lastly, in part (b)(iii), the students were able to translate the point $P(4,1)$ by using the point $T(3,5)$ indicating that they were familiar with the properties of reflection and translation of a point on the *xy*-plane. Extract 6.2 is a sample solution from the script of a student who correctly attempted this question.



Extract 6.2: A sample of a correct response from one of the students in question 6.

In part (a)(i) of Extract 6.2, the student was able to substitute the given information in the standard equation of a line to get $y-4=10(x-(-1))$ which is a necessary step to obtain $y=10x+14$. In part (a)(ii), he/she correctly substituted $(a,0)$, $(0,b)$ in $y=10x+14$ to get $0=10a+14$ and $b=10(0)+14$ and then solved for a and b, respectively. In part (b), he/she applied the properties of reflection and translation to get the images of the given point.

2.7 Question 7: Exponents and Radicals

The question consisted of parts (a) and (b). In part (a), students were required to find the values of *x* and *y* given that $(3^{x+3})(5^{2-y}) = \left(\frac{1}{3}\right)^{5}\left(\frac{1}{5}\right)$. In part b(i), they were asked to find the value of 0.0000234×120 in standard notation, correct to three significant figures and in part b(ii), they were required to rationalize the denominator of the expression $\frac{\sqrt{2}}{\sqrt{3}+\sqrt{2}}$.

This question was attempted by 575,454 (95.7%) students. The analysis of data shows that 116,884 (20.3%) students scored from 3 to 10 marks, among them 8181 (1.4%) students scored full marks. Conversely, a total of 458,570 (79.7%) students scored less than 3 marks and among them, 377,795 (65.7%) students scored 0 mark. The students' performance in this question was weak. The summary of performance is presented in Figure 7.

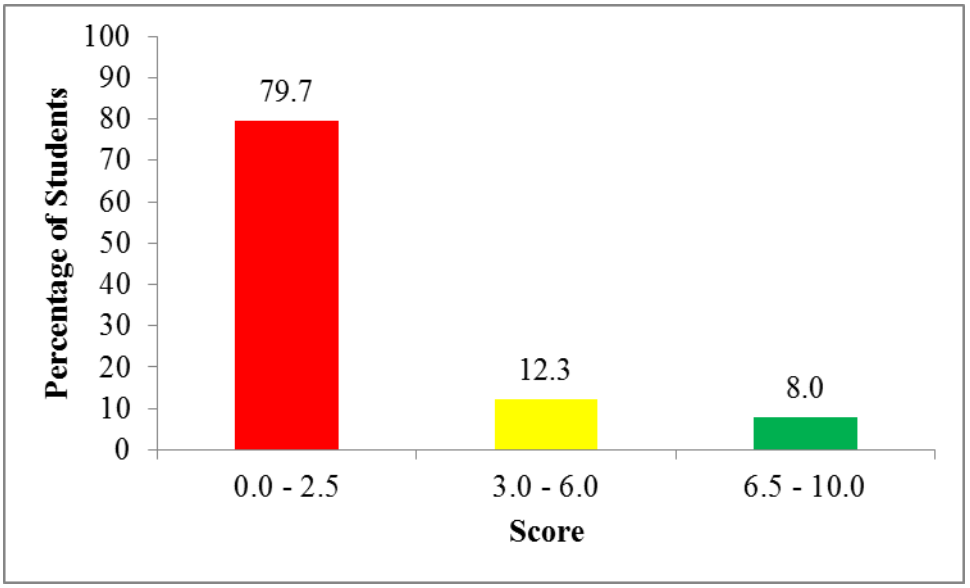


Figure 7: *Students' performance in question 7*

The students' response analysis reveals the following reasons for weak performance in this question: in part (a), the students failed to compare the

exponents of the base of the equation $(3^{x+3})(5^{2-y}) = \left(\frac{1}{3}\right)^5\left(\frac{1}{5}\right)$. For example, they wrote the equations $3(x+3) = \frac{5}{3}$ and $5(2-y) = \frac{1}{5}$ as they failed to differentiate between an exponent and a base. Others managed to write the given equation in the form $\left(3^{x+3}\right)\left(5^{2-y}\right) = \left(3^{-1}\right)^5\left(5^{-1}\right)$ but failed to compare the exponents of the same base. Moreover, there were some students who equated the sum of all exponents to zero, that is $x+3+2-y=0$. This indicates that they lacked knowledge of the application of the laws of exponents in solving exponential equations.

In part (b)(i), some students managed to find the product of 0.0000234 and 120 as 0.002808 but failed to express correctly the resulting answer in standard form and to three significant figures. For example, there were incorrect answers like 2.80×10^{-3}, 2.81×10^{3} and 2.808×10^{-1} instead of 2.81×10^{-3}. In part (b)(ii), most of the students failed to multiply correctly the numerator and denominator of the expression $\frac{\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ by the rationalizing factor $\sqrt{3}-\sqrt{2}$. So, they ended by writing incorrect answers such as $\frac{\sqrt{3}-\sqrt{2}}{3-2}$ and $\sqrt{6}-\sqrt{2}$. Others used either $\sqrt{2}$ or $\sqrt{3}$ as a rationalizing factor, that is $\frac{\sqrt{2}}{\sqrt{3}+\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}$ and $\frac{\sqrt{2}}{\sqrt{3}+\sqrt{2}}\times\frac{\sqrt{3}}{\sqrt{3}}$ which are wrong steps. Extract 7.1 represents a sample response of a student who lacked adequate knowledge of standard notation and approximation of a number.

7. (a) If $(3^{x+3})(5^{2-y}) = \left(\frac{1}{3}\right)^5\left(\frac{1}{5}\right)$, find the values of x and y.

Solution

$(x+3)(2-y) = (5)(1)$

$\begin{cases} x+3=5 \\ 2-y=1 \end{cases}$

$x = 5-3$

$-y = 1-2$

$-y = -1$

$y = 1$ $\therefore x = 2$ and $y = 1$.

(b) (i) Find the value of 0.0000234×120 in standard notation, correct to three significant figures.

Soln

0.0000234×120

2.34×10^{-5}

0.00002808

2.808×10^{-5}

(ii) Rationalize the denominator of the expression $\frac{\sqrt{2}}{\sqrt{3}+\sqrt{2}}$.

Soln

$\frac{\sqrt{2}}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

$\frac{\sqrt{6}+\sqrt{4}}{\sqrt{9}+\sqrt{4}}$

$\frac{\sqrt{6}+2}{3+2} = \frac{2\sqrt{6}+6}{6}$

$= \frac{2\sqrt{6}+6}{6}$

Extract 7.1: An incorrect response from one of the students in question 7.

Extract 7.1 shows that the student failed to get the correct equations involving the exponents of the same base in part (a). The student wrote $x+3=5$ and $2-y=1$ instead $x+3=-5$ and $2-y=-1$. In part (b)(i), the student wrote the incorrect form of standard notation, that is 2.808×10^{-5}. In part (b)(ii), he/she used an incorrect fraction, that is $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$.

On the other hand, the analysis shows that, some of the students answered this question correctly. Those students were able to apply the laws of exponents correctly in part (a). For example, they expressed $\left(\frac{1}{3}\right)^5$ as $\left(3^{-1}\right)^5 = 3^{-5}$ and $\frac{1}{5} = 5^{-1}$, then on comparing the exponents of the same base, they got $x = -8$ and $y = 3$. In part b(i) they were able to express the product of 0.0000234 and 120 in standard notation correct to three significant figures, that is $0.0000234 \times 120 \approx 2.81 \times 10^{-3}$. In part b(ii), they also managed to rationalize correctly the denominator of the expression $\frac{\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ and got $\sqrt{6}-2$. Extract 7.2 shows a sample response of a student who answered this question correctly.

7. (a) If $\left(3^{x+3}\right)\left(5^{2-y}\right) = \left(\frac{1}{3}\right)^5\left(\frac{1}{5}\right)$, find the values of x and y.

Soln:

$\left(3^{x+3}\right)\left(5^{2-y}\right) = \left(3^{-1}\right)^5\left(5^{-1}\right)$

$\left(3^{x+3}\right)\left(5^{2-y}\right) = \left(3^{-5}\right)\left(5^{-1}\right)$

$3^{x+3} \times 5^{2-y} = 3^{-5} \times 5^{-1}$

$3^{x+3} = 3^{-5}$ $\qquad$ $2-y = -1$

$x+3 = -5$ $\qquad$ $-y = -1-2$

$x = -5-3$ $\qquad$ $\frac{-y}{-1} = \frac{-3}{-1}$

$x = -8$ $\qquad$ $y = 3$

∴ The values of x is -8 and y is 3

(b) (i) Find the value of 0.0000234×120 in standard notation, correct to three significant figures.

Soln:

0.0000234×120

$2.34 \times 10^{-5} \times 1.2 \times 10^{2}$

$2.34 \times 1.2 \times 10^{-5} \times 10^{2}$

2.808×10^{-3}

$\approx 2.81 \times 10^{-3}$

$\therefore 0.0000234 \times 120 = 2.81 \times 10^{-3}$

(ii) Rationalize the denominator of the expression $\dfrac{\sqrt{2}}{\sqrt{3}+\sqrt{2}}$.

Soln:

$$\frac{\sqrt{2}}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}$$

$$\frac{\sqrt{2}(\sqrt{3}-\sqrt{2})}{\sqrt{3}(\sqrt{3}-\sqrt{2})+\sqrt{2}(\sqrt{3}-\sqrt{2})}$$

$$\frac{\sqrt{6}-2}{3-\sqrt{6}+\sqrt{6}-2}$$

$$\frac{\sqrt{6}-2}{3-2} = \frac{\sqrt{6}-2}{1} = \sqrt{6}-2$$

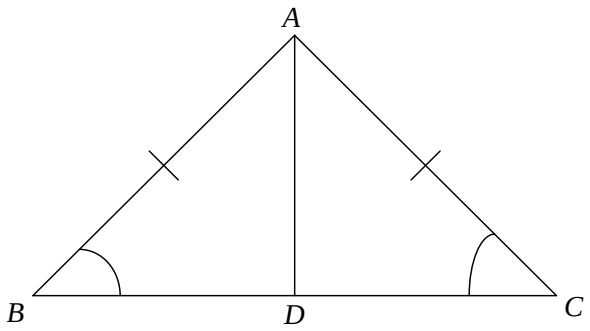
$$\therefore \frac{\sqrt{2}}{\sqrt{3}+\sqrt{2}} = \sqrt{6}-2$$

Extract 7.2: A sample of a correct response from one of the students in question 7.

In Extract 7.2, the student compared the corresponding exponents of base 3 and 5 correctly and eventually solved for x and y. Again, the student expressed 0.0000234×120 in standard form $A \times 10^{n}$ where $1 \leq A < 10$ and n is an integer and then wrote the answer to 3 significant figures. He/she also managed to rationalize the denominator of $\dfrac{\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ correctly.

2.8 Question 8: Congruence and Similarity

This question consisted of two parts (a) and (b). In part (a), students were instructed to use the triangle ABC to prove that the angles ABD and ACD are equal if $\overline{AB} = \overline{AC}$.



In part (b), the similar rectangular metal sheets ABCD and WXYZ were given such that $\overline{AB} = 2\,cm$, $\overline{BC} = 4\,cm$ and $\overline{WX} = 2.5\,cm$. The students were required to calculate the length $\overline{XY}$.

The analysis shows that this question was attempted by 514,615 (85.6%) students. A total of 100,839 (19.6%) students scored from 3 to 10 marks, among them 8,887 (1.7%) students scored 10 marks. However, a total of 413,776 (80.4%) students scored from 0 to 2.5 marks and out of these, 290,041 (56.4%) students scored 0 mark. The students' performance in this question was weak as witnessed in Figure 8.

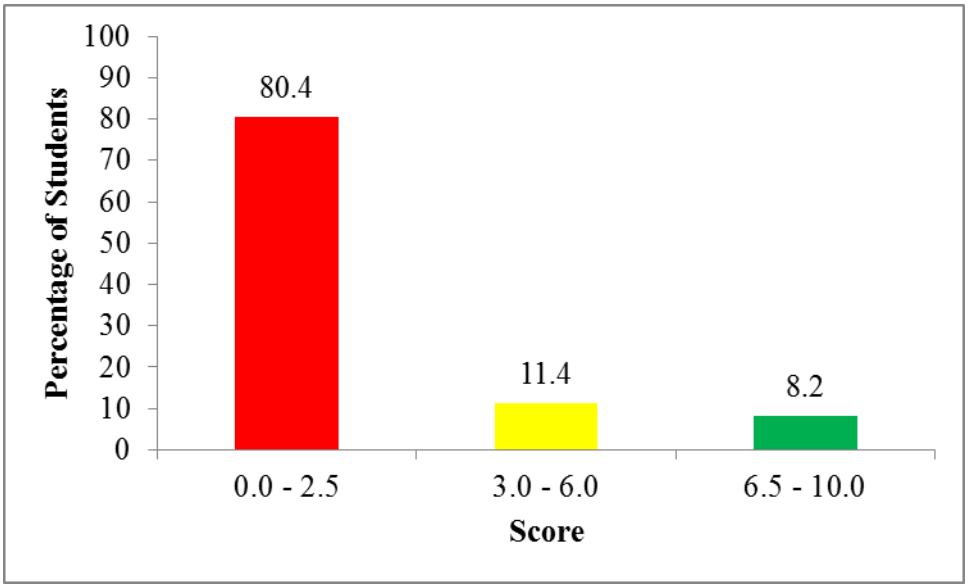
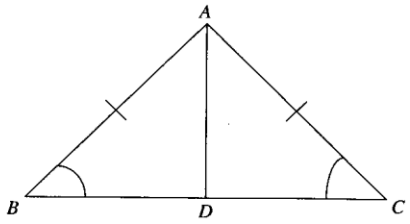


Figure 8: *Students' performance in question 8*

The majority of students failed to answer part (a) correctly by using the correct theorem. For example, some of them used inappropriate conditions like *Side, Side, Side (SSS)* and *Right Angle Hypotenuse Side (RHS)* instead of *Side, Angle, Side (SAS)* which could complete the proof of the congruence of triangles. Other students stated the proving statements without any supporting reasons. For instance, some of them just wrote $\overline{AB} = \overline{AC}$, $\overline{AD}$ is common and then concluded that angle ABD = angle ACD. In other cases, some students stated that $\overline{AD}$ is the perpendicular to $\overline{BC}$, $\overline{BD} = \overline{DC}$, $\overline{AB} = \overline{AC}$, therefore angle ABD = angle ACD. Some students also confused between the concept of congruence and that of similarity in proving that the angles *ABD* and *ACD* are equal. For instance, they wrote $\overline{AB} = \overline{AC}$, $\overline{BC}$ is common and $\Delta ABD \approx \Delta ABD$ by SSS condition, hence concluded that angle ABD = angle ACD. In Part (b), those who failed to perform the question correctly faced a problem of transforming the given information into geometrical form making it difficult to establish the ratio of proportional sides. Together with this weakness, it was noted that some students wrote $\frac{2 \times 2.5}{4}$ equals to 1.25 cm as length of $\overline{XY}$. Others multiplied the given sides, then divided the answer by 4 and got the length of $\overline{XY}$ as 5 cm. In some cases, some students applied the formula for the perimeter of a metal sheet ABCD, that is $P = 2(l + w)$, then substituted $l = 4cm$ and $w = 2cm$ to get $P = 12cm$. Lastly, they compared the two rectangular metal sheets ABCD and WXYZ when finding the length of $\overline{XY}$. Furthermore, some students considered that because ABCD and WXYZ are similar, they equated the area of ABCD to that of WXYZ and solved for the length of $\overline{XY}$ to get 3.5cm which was incorrect. In some rare cases a few students simply multiplied the given dimensions 2 cm, 4 cm and 2.5 cm to get an incorrect length of $\overline{XY}$. Extract 8.1 illustrates a sample response of a student who lacked knowledge of congruence and similarity.

8. (a) If the following figure, $\overline{AB} = \overline{AC}$, prove that the angles ABD and ACD are also equal.



Soln

Statement	Reason	
$\overline{AB} = \overline{AC}$	Parallel sides	S
$\overline{BD} = \overline{CD}$	Parallel sides	S
$B\hat{A}D = C\hat{A}D$		
$A\hat{B}D = A\hat{C}D$	Corresponding angle	A

$\therefore \triangle ABD \equiv ACD$ by SAS theorem

(b) If the rectangular metal sheets $ABCD$ and $WXYZ$ are similar, calculate the length of $\overline{XY}$ when $\overline{AB} = 2\,cm$, $\overline{BC} = 4\,cm$ and $\overline{WX} = 2.5\,cm$.

Soln

$\frac{XY}{AB} = \frac{BC}{WX}$

$\frac{XY}{2cm} = \frac{4cm}{2.5cm}$

$2.5cm\,XY = 4cm \times 2cm$

$\frac{2.5cm\,XY}{cm} = \frac{8cm^2}{cm}$

$\frac{2.5XY}{2.5cm} = \frac{8cm^2}{2.5cm}$

$XY = \frac{8cm \times 10}{25\ 1}$

$XY = \frac{80cm}{25}$

$XY = 3\,cm$

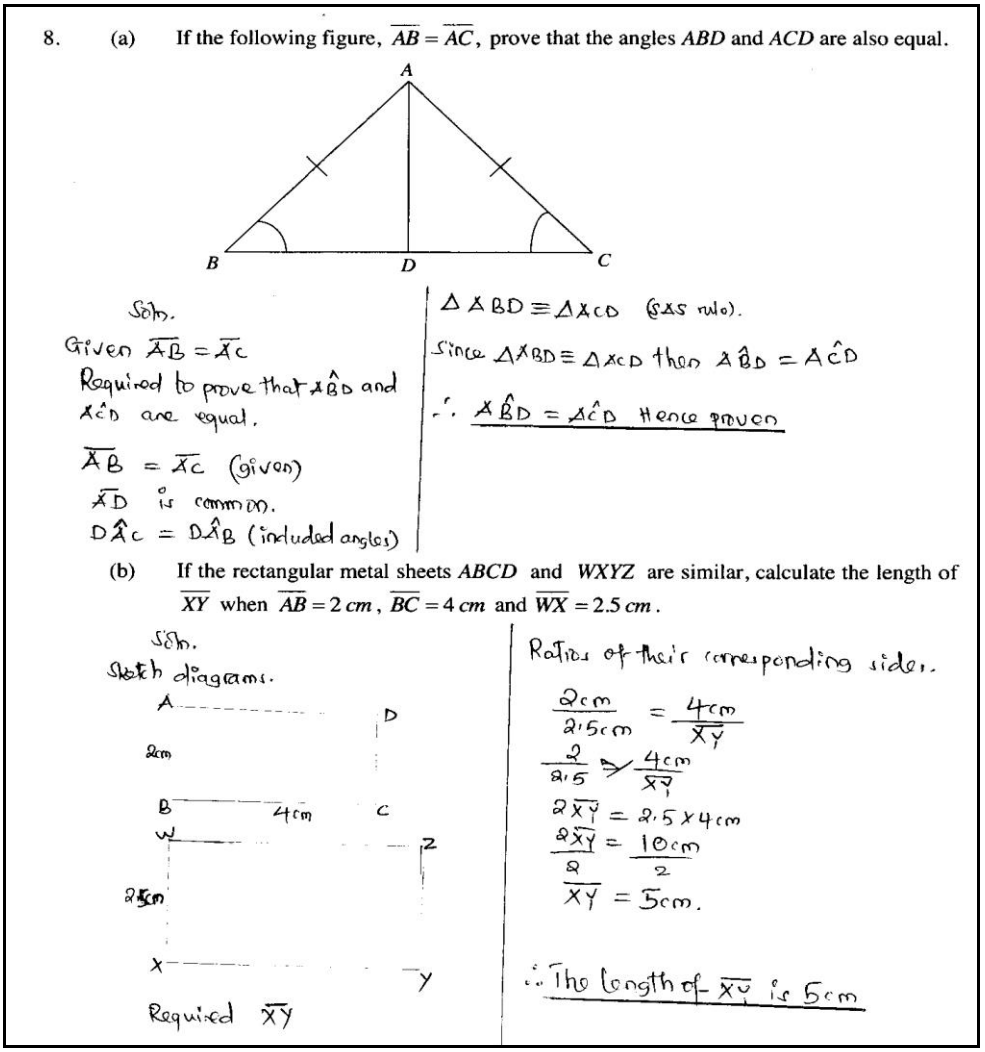
$\therefore$ The length of $\overline{XY}$ is 3 cm

Extract 8.1: An incorrect response from one of the students in question 8.

Extract 8.1 shows that the student failed to distinguish between equal sides and parallel sides. He/she also failed to identify the corresponding sides when finding the required length from the two similar rectangles.

Even though most of the students had poor performance in this question, some of them demonstrated adequate knowledge of congruence. In part (a), the students followed all necessary steps used to prove the congruence of triangles and showed that angle ABD = angle ACD. In part (b), they illustrated well the metal sheets ABCD and WXYZ geometrically and

wrote the ratio of the proportional sides, that is $\frac{\overline{AB}}{\overline{WX}} = \frac{\overline{AD}}{\overline{WZ}} = \frac{\overline{BC}}{\overline{XY}} = \frac{\overline{DC}}{\overline{ZY}}$. From these ratios they were able to get the length of $\overline{XY}$. Extract 8.2 illustrates a sample response of a student who correctly answered this question.



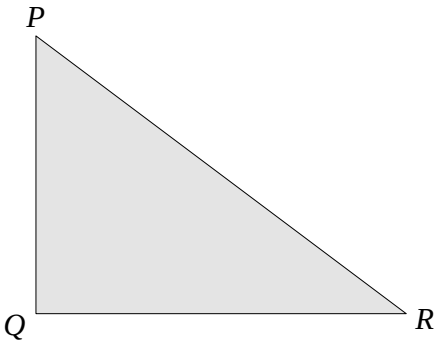
Extract 8.2: A sample of a correct response from one of the students in question 8.

In Extract 8.2, the student was able to prove that, if $\overline{AB} = \overline{AC}$ then $\overline{AD}$ is a bisector of a triangle BAC making angle BAD equal to angle CAD, $\overline{AD}$ is common, ΔABD and ΔACD are congruent by SAS, therefore the angles

ABD and ACD are equal. He/she was also able to use the relation $\frac{2}{2.5}=\frac{4}{\overline{XY}}$ that was an important step to get $\overline{XY}=5\,\text{cm}$.

2.9 Question 9: Pythagoras Theorem and Trigonometry

This question had parts (a) and (b). In part (a), students were required to find $\overline{PR}$ correct to two decimal places by using figure PQR that represents a triangular floor such that $\overline{PQ}=\overline{QR}=2\text{cm}$ and the angle PQR is 90°.



In part (b), they were asked to find (i) $\cos\theta$ (ii) $\tan\theta$ without using mathematical table given that $\sin\theta=\frac{\sqrt{3}}{2}$ where θ is an acute angle.

The data analysis shows that, the question was attempted by 509,855 (84.8%) students. Out of 88,348 (17.3%) students who scored from 3 to 10 marks, 9,305 (1.8%) students scored full marks. On the other side, a total of 421,507 (82.7%) students scored less than 3 marks and among them, 344,091 (67.5%) students scored 0 mark. The students' performance in this question was weak as summarized in Figure 9.

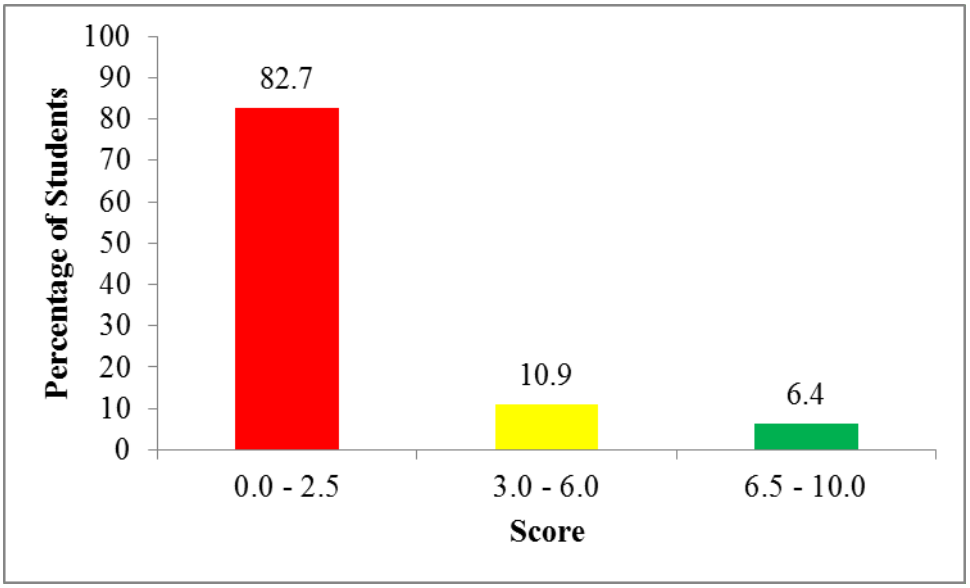
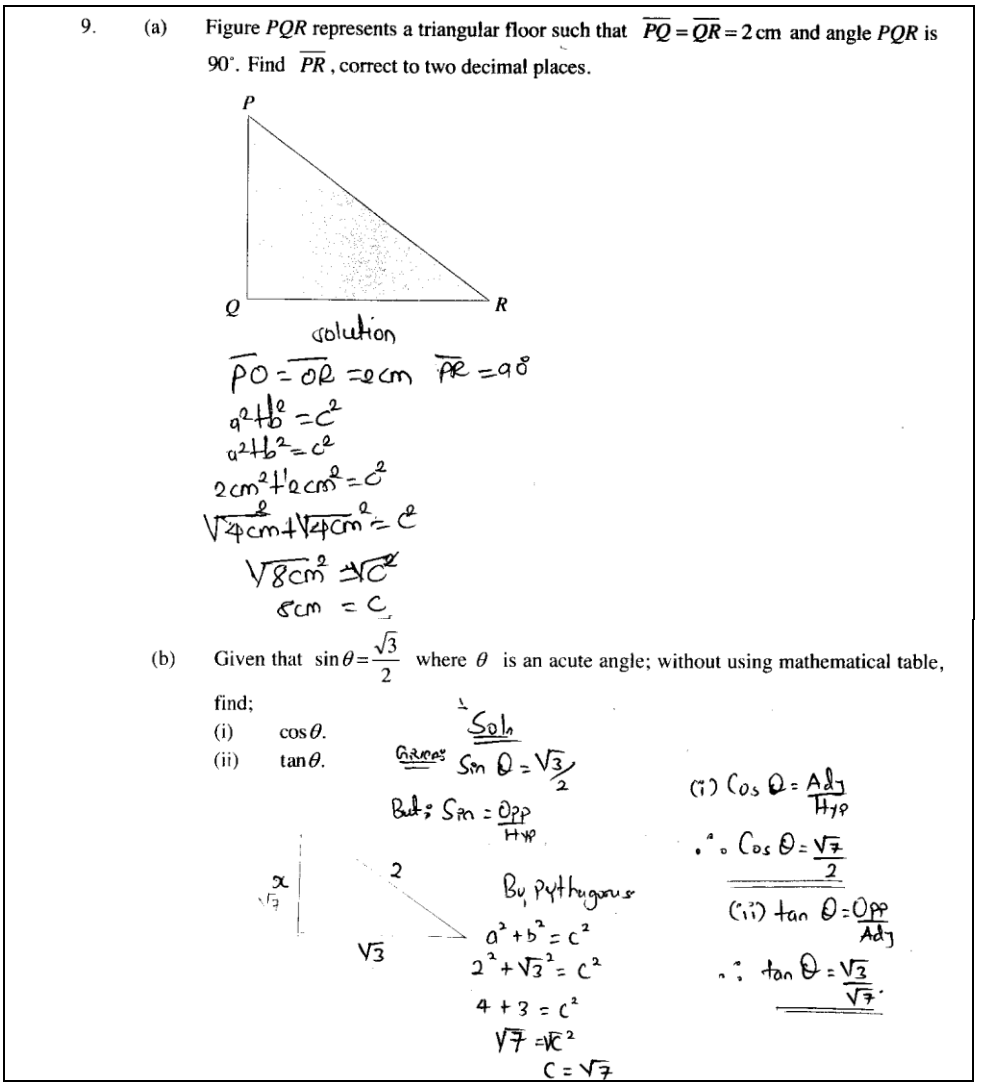


Figure 9: *Students' performance in question 9*

The students' response analysis show that the majority of students failed to answer this question due to the following reasons: in part (a), the students were unable to establish the relation $\overline{PR}^2 = \overline{PQ}^2 + \overline{QR}^2$ which is a formula adhering to Pythagoras theorem and hence failed to get correct answer. For example they wrote incorrect formulae like $PQ = \overline{QR} + \overline{PR}$ and $\overline{PQ}^2 = \overline{PR}^2 + \overline{QR}^2$. In other cases some students were able to recall the Pythagoras theorem and apply it correctly but failed to evaluate $\sqrt{8}$ or $2\sqrt{2}$ by using mathematical tables. Contrary to the given instructions, others calculated the angle PQR instead of finding the length of $\overline{PR}$.

In part (b), the students failed to recognize the hypotenuse and opposite length by using the relation $\sin\theta = \frac{\sqrt{3}}{2}$. Among them, there were those who incorrectly wrote $\sqrt{3}$ as the opposite length, 2 as the adjacent length and hence $2^2 + \sqrt{3}^2 = \sqrt{7}$ as the hypotenuse length. Consequently, they got incorrect answers like $\cos\theta = \frac{2}{\sqrt{7}}$ instead of $\cos\theta = \frac{1}{2}$; and $\tan\theta = \frac{\sqrt{3}}{2}$ instead of $\tan\theta = \sqrt{3}$. Those students lacked knowledge of trigonometric

ratios. Extract 9.2 is a sample response of a student who failed to answer this question correctly.

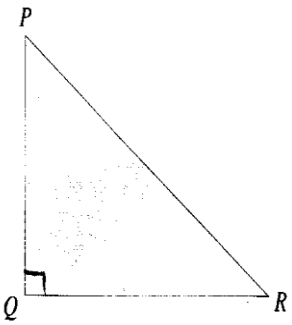


Extract 9.1: An incorrect response from one of the students in question 9.

In Extract 9.1, the student failed to evaluate $\sqrt{8}$ correct to two decimal places by using mathematical table. He/she also failed to determine the correct ratios of $\cos\theta$ and $\tan\theta$, given that $\sin\theta = \frac{\sqrt{3}}{2}$ where θ is an acute angle.

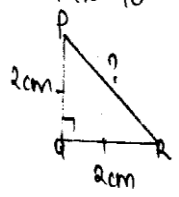
Though the performance was weak, there were students who answered this question correctly. They applied the Pythagoras theorem $\overline{PR}^2 = \overline{PQ}^2 + \overline{QR}^2$ and substituted the correct values of $\overline{PQ}$ and $\overline{QR}$ in the formula to get $\overline{PR} = 2.83\,\text{cm}$. In part (b), the students were able to recall the definition of $\sin\theta = \frac{\text{opposite side}}{\text{hypothenuse side}}$, identify $\sqrt{3}$ as the length of the opposite side and 2 as the hypotenuse side and then apply Pythagoras theorem to get the length of the adjacent side, and consequently the required answers. Extract 9.1 is a sample of a student's correct response in this question.

9. (a) Figure PQR represents a triangular floor such that $\overline{PQ} = \overline{QR} = 2\,\text{cm}$ and angle PQR is 90°. Find $\overline{PR}$, correct to two decimal places.



Soln.

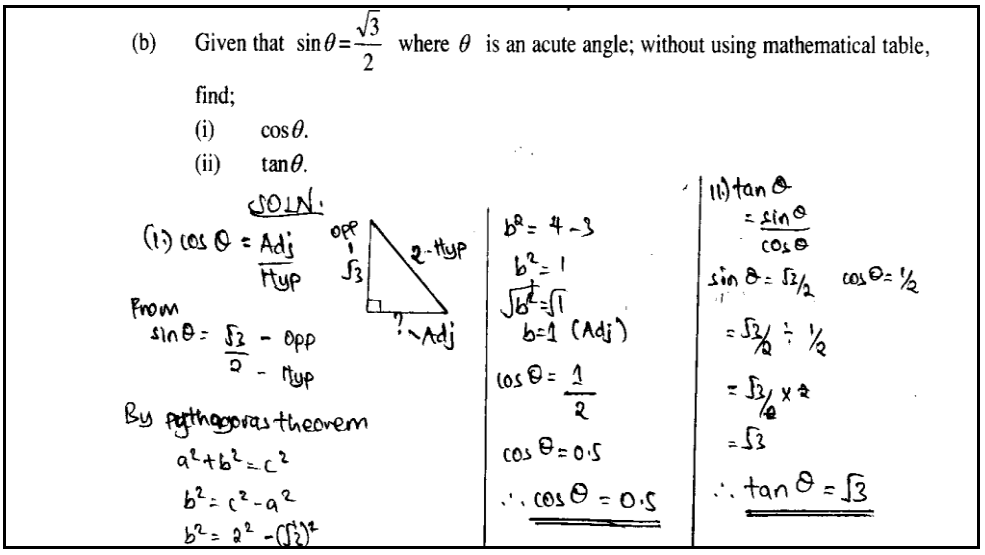
Given
$\overline{PQ} = \overline{QR} = 2\text{cm}$
$P\hat{Q}R = 90^\circ$



By pythagoras theorem
$\overline{PR} = x$
$x^2 = 2^2 + 2^2$
$x^2 = 4 + 4$
$x^2 = 8$
$\sqrt{x^2} = \sqrt{8}$
$x = 2.828\,\text{cm}$

correct to two decimal places
2.828
2.82
+ 1
2.83

∴ The length of $\overline{PR}$ is
$\approx 2.83\,\text{cm}$.



Extract 9.2: A sample of a correct response from one of the students in question 9.

In Extract 9.2, the student was able to formulate $\overline{PR}^2 = 2^2 + 2^2$ which is a core step to get the length of $\overline{PR}$. He/she was also able to determine correctly the lengths of the sides in a right angled triangle and hence obtained the ratios for $\cos\theta$ and $\tan\theta$.

2.10 Question 10: Sets and Statistics

This question consisted of parts (a) and (b). Part (a) states that: "a certain village, 300 people were interviewed about their food preference. It was found that, 200 people like banana, 120 people like rice and 60 people like both banana and rice". The students were required to find the number of people who like neither banana nor rice by using formula.

In part (b), it was given that, the masses of a group of students from Kilimani secondary school were recorded as shown in the following table:

Mass in kilograms	31 – 40	41 – 50	51 – 60	61 – 70	71 – 80
Frequency	2	5	3	9	1

The students were required to: (i) find the number of students in the group, (ii) state the class interval that has the largest number of students and

(iii) prepare a table showing the class boundaries and the corresponding cumulative frequencies.

A total of 576,074 (95.8%) students attempted this question. From the analysis done, 85,326 (14.8%) students scored from 3 to 10 marks, among them 2,778 (0.5%) students scored all 10 marks. Contrarily, a total of 490,748 (85.2%) students scored from 0 to 2.5 marks and among them, 259,322 (45.0%) students scored 0 mark. This shows that, the students' performance in this question was weak. Figure 10 presents a summary of students' performance in this question.

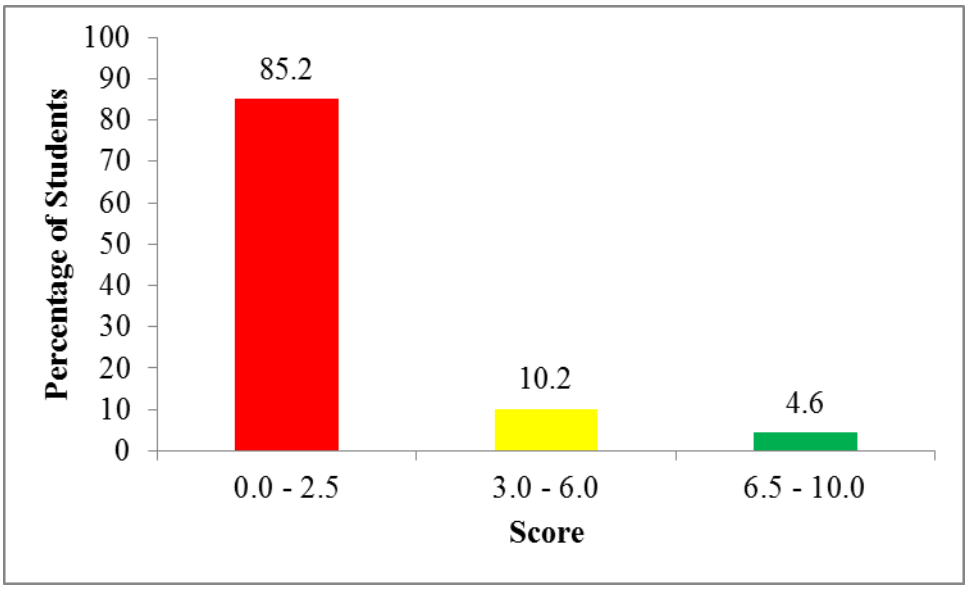


Figure 10: *Students' performance in question 10*

The analysis of responses shows that the majority of students failed to answer this question correctly due to the following reasons. In part (a), most of them failed to recall and apply the correct formula for the union of two sets. There were some students who recalled and applied the formula $n(B \cup R) = n(B) + n(R) - n(B \cap R)$ as required but failed to apply the concept of complement of a set, that is $n(\bigcup) = n(B \cup R) + n(B \cup R)'$. Others used Venn diagrams instead of using formula to find the number of people who disliked banana and rice.

In part (b)(i), most of students failed to add correctly the given frequencies to get the required number of students. In (b)(ii), some of them computed the class mark that is $\frac{61+70}{2}=65.5$ and regarded it as the class interval with the largest number of students while others wrote wrong classes like $31-40$, $41-50$ and $51-60$ instead of $61-70$. In part (b)(iii), they failed to prepare the class boundaries and the corresponding cumulative frequencies correctly. Some of them wrote the class intervals as the class boundaries. In some cases, some students added 0.5 to both the lower and upper limits of each class instead of adding 0.5 to the upper limit and subtracting 0.5 from the lower limit to get the class boundaries. In addition, there were a few samples where students prepared the class size, class mark and others wrote the upper real limits as the class boundaries when preparing the table. Contrarily, a few of students drew histograms and cumulative frequency curves using the cumulative frequency and the class mark. Extract 10.1 shows a sample of an incorrect response from one of the students.

10 (a) In a certain village, 300 people were interviewed about their food preference. It was found that, 200 people like banana, 120 people like rice and 60 people like both banana and rice. By using formula, find the number of people who like neither banana nor rice.

Soln

Number of village = 300

B = 200

R = ~~160~~ 120

B, R = 60

A∩B' = 120 − 60 = 60 people like rice

= 200 − 60 = 140 people like banana

140 − 60 = 80 people like both banana

The people like banana neither and rice

The people neither like banana nor rice is

80

(b) The masses of a group of students from Kilimani secondary school were recorded as shown in the following table:

Mass in kilograms	31 – 40	41 – 50	51 – 60	61 – 70	71 – 80
Frequency	2	5	3	9	1

(i) How many students are there in the group?

2+5+3+9+1 = 21. ∴ there are 21 students.

(ii) State the class interval that has the largest number of students.

61+70 = 131 $\frac{131}{2}$ = 65.5 ∴ The class interval is 65.5

(iii) Prepare a table showing the class boundaries and the corresponding cumulative frequencies.

Class mark	Frequency	Class Interval
31 − 40	2	35.5
41 − 50	5	45.5
51 − 60	3	55.5
61 − 70	9	65.5
71 − 80	1	75.5

Extract 10.1: An incorrect response from one of the students in question 10.

In Extract 10.1, the student failed to recall the correct formula for union of two sets in part (a). He/she also failed to add correctly the given frequencies to get the total number of students in the group in part (b)(i). Again, the student failed to state the class interval with the highest number of students.

Instead, he/she wrote the class mark, that is $\frac{61+70}{2} = 65.5$ for the interval $61-70$ in part (b)(ii). Also, in part (b)(iii), he/she prepared the class marks instead of preparing the class boundaries and corresponding cumulative frequencies as it was instructed.

Nevertheless, there were students who answered this question correctly. In part (a), they were able to get the correct number of people who like neither banana nor rice, that is $n(B \cup R)'$, where B is the set of the number of people who like banana and R is the set of the number of people who like rice by applying the correct formula $n(B \cup R)' = n(\psi) - n(B \cup R)$, where $n(B \cup R) = n(B) + n(R) - n(B \cap R)$ and ψ represents the set of the number of people who were interviewed. In part (b), they were able to (i) add the given frequencies to get the total number of the students in the group; (ii) state the class interval with largest number of students and (iii) prepare a table showing the class boundaries and the corresponding cumulative frequencies as shown in Extract 10.2.

10 (a) In a certain village, 300 people were interviewed about their food preference. It was found that, 200 people like banana, 120 people like rice and 60 people like both banana and rice. By using formula, find the number of people who like neither banana nor rice.

Soln

$\mathcal{U} = 300$ people.

$n(B) = 200$ people.

$n(R) = 120$ people.

$n(B \cap R) = 60$ people.

$n(B \cup R) = ?$

From

$n(B \cup R) = n(B) + n(R) - n(B \cap R)$

$n(B \cup R) = 200 + 120 - 60$

$n(B \cup R) = 260$ people.

$\mathcal{U} = n(B \cup R) + n(B \cup R)'$

$\mathcal{U} = 260 + n(B \cup R)'$

$300 = 260 + n(B \cup R)'$

$300 - 260 = n(B \cup R)'$

$40 = n(B \cup R)'$

∴ 40 people like neither Banana nor Rice.

(b) The masses of a group of students from Kilimani secondary school were recorded as shown in the following table:

Mass in kilograms	31 – 40	41 – 50	51 – 60	61 – 70	71 – 80
Frequency	2	5	3	9	1

(i) How many students are there in the group?

2+3+5+9+1 = 20 students ∴ There are 20 students.

(ii) State the class interval that has the largest number of students.

Class interval of 61 - 70

(iii) Prepare a table showing the class boundaries and the corresponding cumulative frequencies.

Class interval	Frequency	Cummulative frequency	Class boundaries
31 - 40	2	2	30·5 - 40·5
41 - 50	5	7	40·5 - 50·5
51 - 60	3	10	50·5 - 60·5
61 - 70	9	19	60·5 - 70·5
71 - 80	1	1	70·5 - 80·5

Extract 10.2: A sample of a correct response from one of the students in question 10.

In Extract 10.2, the student was able to get the correct number of people who like neither banana nor rice by applying the correct formulae. In part (b), he/she was able to (i) add the frequencies 2, 5, 3, 9 and 1 to get 20 as the total number of the students in the group; (ii) write 61-70 as the class interval with largest number of students and (iii) prepare the class boundaries and the corresponding cumulative frequencies.

3.0 ANALYSIS OF STUDENTS' PERFORMANCE IN EACH TOPIC

The 041 Basic Mathematics assessment paper had ten (10) questions that were set from the following topics: *Geometry*, *Perimeters and Areas*, *Fractions*, *Decimals and Percentages*, *Exponents and Radicals*, *Similarity*, *Congruence*, *Pythagoras Theorem*, *Trigonometry*, *Sets*, *Statistics*, *Algebra*, *Quadratic Equations*, *Coordinate Geometry*, *Geometrical Transformations*, *Units*, *Ratio*, *Profit and Loss*, *Numbers* and *Approximations*.

The analysis of data shows that, among the topics from which the questions were set, none of them had good performance. Only two topics of *Numbers* and *approximations (58.0%)* had the average performance. The remaining topics had poor performance. Those topics include: *Fractions, Decimals and Percentages* (25.3%); *Exponents and Radicals* (20.3%); *Similarity* and *Congruence* (19.6%); *Pythagoras Theorem* and *Trigonometry* (17.3%); *Sets* and *Statistics* (14.8%); *Algebra* and *Quadratic Equations* (12.9%); *Coordinate Geometry* and *Geometrical Transformations* (12.7%); *Units* and *Ratio, Profit and Loss* (12.5%); and *Geometry, Perimeters and Areas* (2.8%). The summary of the performance in each topic is shown in the Appendix.

4.0 CONCLUSION AND RECOMMENDATIONS

4.1 Conclusion

The analysis of students' performance in each question indicated that out of 10 questions which were assessed, only question 1 that was set from the topics of *Numbers* and *Approximations* was averagely performed. The remaining nine questions had weak performance. Question 4 that was set from the topics of *Geometry, Perimeters* and *Areas* has the lowest performance. The analysis also shows that question 6 that was set from the topics of *Coordinate Geometry* and *Geometrical Transformations* was the mostly omitted question. Generally, the performance in FTNA 2020 in 041 Basic Mathematics was weak as only 15.94 percent of the students passed.

4.2 Recommendations

In order to improve the students' performance in Basic Mathematics subject in future, the teachers should:

(a) teach students how to calculate the circumference and area of a circle, semi-circle and a quarter of a circle by using real circular objects.

(b) use geometrical figures to demonstrate to students the properties of interior and exterior angles in polygons.

(c) guide students in their groups to discuss how to change metric units, especially the units of mass by using real-life examples.

(d) guide students to discuss the concepts of percentage profit and percentage loss by using formula and real life examples like money and worksheets.

(e) lead students' discussions in small groups on the meaning of gradient and formulation of linear equation when two coordinate points are given by using teaching and learning resources such as graph paper and ruler.

(f) use participatory methods in teaching the properties of refection in the xy-plane, drawing various reflections and use their properties to solve problems with the aid of tools like plane mirrors, graphs, pictures or identical shapes, mathematical sets and manila paper.

(g) use real objects related to right angled triangles to teach students about the application of Pythagoras theorem in solving related problems.

(h) apply simple strategies to guide students on how to formulate trigonometric ratios of sine, cosine and tangent and solve related problems in groups without using mathematical tables or calculators.

(i) lead students through pair-share thinking to discuss the basic operations of sets such as set complement, union and intersection of two sets and use Venn diagrams and formula to calculate the number of real objects in a set.

(j) instruct students on how to present data or statistical information in a frequency distribution table and interpret the information presented in the frequency distribution table using real data.

(k) guide students to discuss the conditions of congruent triangles and similar figures as well as using such conditions to solve problems related to real-life situations with the aid of various teaching and learning resources like paper-cuttings, pictures and geometric figures.

(l) guide students on how to solve word problems involving a pair of simultaneous linear equations in real life settings by applying different methods and examples.

(m) guide students on how to solve the quadratic equations using various methods, especially "Completing the Square" method.

(n) guide students in their small groups on how to use mathematical tables, number charts and the laws of exponents to solve related problems and simplify radicals.

(o) guide students in their groups to evaluate the expressions involving simple fractions, mixed numbers and improper fractions by using the BODMAS rule and present their findings in the class for evaluation and consolidation.

(p) guide students in solving word problems that involve percentages and decimals of real quantities, and the important steps to convert repeating decimals into simple fractions through examples and exercises.

APPENDIX

ANALYSIS OF STUDENTS' PERFORMANCE PER TOPIC IN BASIC MATHEMATICS – FTNA 2020

S/N	Topics	Question Number	Percentage of Students who scored 30 marks or more	Remarks
1	Numbers and approximations	1	58.0	Average
2	Fractions, Decimals and Percentages	2	25.3	Weak
3	Exponents and Radicals	7	20.3	Weak
4	Similarity and Congruence	8	19.6	Weak
5	Pythagoras Theorem and Trigonometry	9	17.3	Weak
6	Sets and Statistics	10	14.8	Weak
7	Algebra and Quadratic Equations	5	12.9	Weak
8	Coordinate Geometry and Geometrical Transformations	6	12.7	Weak
9	Units and Ratios, Profit and Loss	3	12.5	Weak
10	Geometry, Perimeters and Areas	4	2.8	Weak