THE UNITED REPUBLIC OF TANZANIA
MINISTRY OF EDUCATION, SCIENCE AND TECHNOLOGY
NATIONAL EXAMINATIONS COUNCIL OF TANZANIA

STUDENTS' ITEM RESPONSE ANALYSIS REPORT ON THE FORM TWO NATIONAL ASSESSMENT (FTNA) 2022

BASIC MATHEMATICS

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041 BASIC MATHEMATICS

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FOREWORD

This report presents Students' Item Response Analysis (SIRA) on Form Two Basic Mathematics National Assessment which was conducted in November 2022. The report aims to provide feedback to all educational stakeholders on the factors that contributed to the students' performance in Basic Mathematics.

The Form Two National Assessment (FTNA) is a formative evaluation which intends to monitor students' learning in order to provide feedback that teachers, students and other educational stakeholders can use to improve teaching and learning. This analysis shows justification for the students' performance in the Basic Mathematics subject. The students who scored low marks faced various challenges including failure to: formulate mathematical equations and expressions, interpret the information, sketch correct diagrams and apply appropriate laws, formulae, signs, properties, conditions and theorems in solving problems.

This report will help students to identify strengths and weaknesses for them to improve learning before sitting for their Certificate of Secondary Education Examination (CSEE). It will help teachers to identify the challenging areas and take appropriate measures during teaching and learning.

The National Examinations Council of Tanzania (NECTA) expects that the feedback provided in this report will be useful to teachers, students and other education stakeholders in improving teaching and learning of the Basic Mathematics subject. Consequently, students will acquire knowledge, skills and competences indicated in the syllabus for better performance in future assessments and examinations.

The Council appreciates the contribution of all those who prepared this report.

Dr. Said Ally Mohamed
EXECUTIVE SECRETARY

1.0 INTRODUCTION

This report presents both statistical and descriptive analyses of the students who sat for the Form Two National Assessment (FTNA) 2022 in the Basic Mathematics subject. The report also provides the reasons for the success and failure based on their responses.

The analysis shows that, the number of students who sat for the assessment (FTNA) in 2022 was 633,185 out of whom 104,841 students, equivalent to 16.56 per cent, passed. In FTNA 2021, a total of 601,721 students sat for the assessment, out of whom 117,433, that was equivalent to 19.52 per cent passed. Comparatively, the performance dropped by 2.96 per cent.

The assessment paper consisted of 10 compulsory questions, each carrying 10 marks. The analysis of the students' performance on each question is summarized in this report. The analysis was based on the national assessment score intervals: 100 – 75, 74 – 65, 64 – 45, 44 – 30 and 29 – 0 to mean *Excellent*, V*ery Good*, *Good*, *Satisfactory* and *Fail* respectively. The students' performance on each question is considered *Good*, *Average* or *Weak* under the condition that the percentage of students who passed falls in the intervals: 100 – 65, 64 – 30 or 29 – 0 respectively.

The Figures and Appendix given in this report have been presented by using three types of colours, that is green, yellow and red to mean good, average and weak performance respectively. Lastly, the report contains some recommendations which are deemed useful to both students and teachers in improving performance in future assessments in the Basic Mathematics subject.

2.0 ANALYSIS OF THE STUDENTS' PERFORMANCE ON EACH QUESTION

This section describes what was expected in each question, the statistical analysis based on students' performance, and statistical figures showing the summary of students' performance based on the score intervals: 0.0 to 2.5, 3.0 to 6.0 and 6.5 to 10.0 which were rated *Weak*, *Average*, and *Good* performances, respectively. The section further provides a descriptive analysis based on reasons for the students' success and failure to respond to each assessment item. The analysis includes the provision of the samples of correct and incorrect responses to support the addressed strengths and weaknesses shown by different students when attempting the questions.

2.1 Question 1: Numbers, Decimals and Percentages

The question comprised parts (a) and (b). In part (a), the students were given the information that "Mwajuma deposited Tsh $360,000$ in her bank account. The bank charges Tsh $1,000$ for every withdrawal". The students were assessed whether they could calculate the amount of money that remained in Mwajuma's account if: (i) she withdrew Tsh $106,000$ and (ii) she made a further withdrawal of Tsh $50,000$ from the remaining amount.

In part (b), students were required to convert the repeating decimal $2.4\dot{3}$ into a mixed fraction.

The question was attempted by 634,775 (100%) students, out of whom 524,058 (82.6%) students scored 0 to 2.5 marks. This shows that the students' performance on this question was weak. It was further noted that, only 12,468 (2.0%) students scored full marks while a total of 388,214 (61.2%) scored zero marks. Figure 1 summarizes the students' performance on question 1.

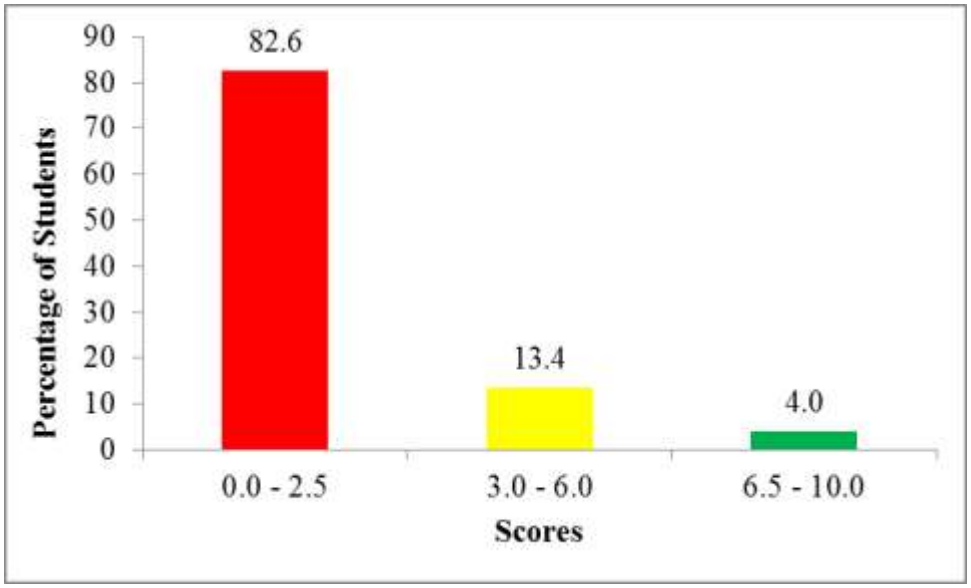
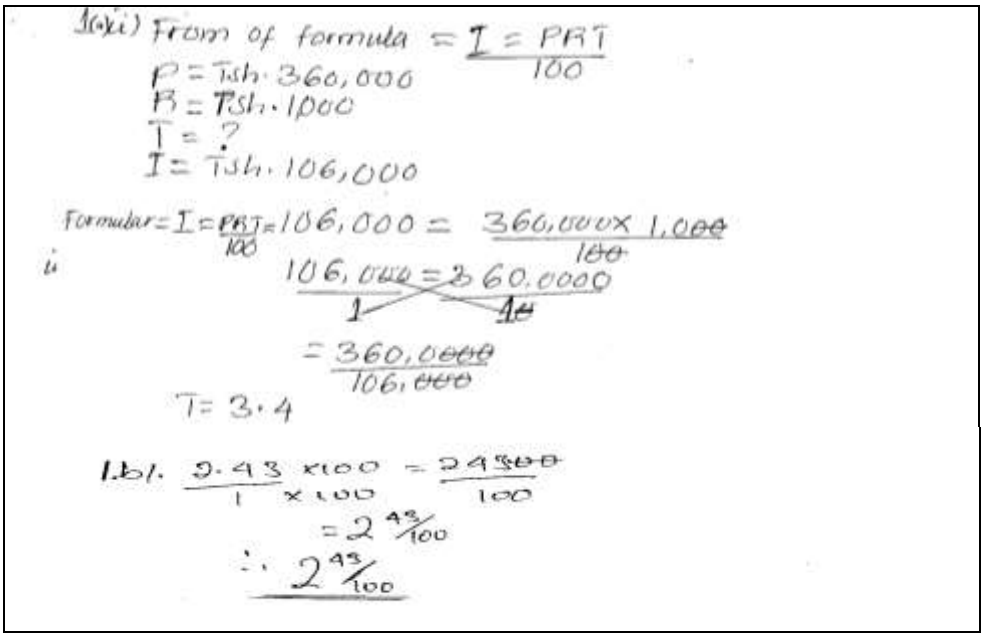


Figure 1: *Students' performance on question 1*

The response analysis shows that the students who failed to answer this question lacked knowledge and skills in the concepts of whole numbers and decimals. In part (a) (i), the students were unable to interpret the word problem in order to formulate a mathematical equation. They considered $\text{Tsh } 360,000$ as the principal, $\text{Tsh } 1,000$ as rate, $\text{Tsh } 106,000$ as interest and time as an unknown variable. Hence, they applied the formula for finding simple interest, that is, $I = \frac{PRT}{100}$ and obtained $I = \text{Tsh } 0.0029$ after substitution of the values, which was incorrect. Other students added bank charges to the money deposited instead of subtracting. In part (b), the students lacked competence in the procedures of converting repeating decimals into fractions. For instance, some of the students wrote $\frac{243}{1000}$ while others multiplied both sides of the equation $x = 2.4333...$ by 1000 instead of 10 or 100, which was an important step in arriving at the required result. Extract 1.1 provides a sample of responses of a student who answered the question incorrectly.



Extract 1.1: A sample of the student's responses to question 1

Extract 1.1 shows that, in part (a), the student interpreted the given information wrongly. He/she did not understand the question. In part (b), the student multiplied the numerator and the denominator by 100 and simplified to get $2\frac{43}{100}$, which was an incorrect answer.

On the other hand, the students who answered this question correctly were able to apply the knowledge and skills gained after learning the basic concepts of numbers, decimals, and percentages. In part (a), they performed correctly the addition and subtraction of whole numbers. Also, they translated the given word problem into mathematical statements as they took into consideration that $\text{Tsh}\,360,000$ was the initially deposited amount. They correctly summed the amount of bank charges and withdrawals to get $\text{Tsh}\,107,000$. This amount of money was subtracted from the deposited amount $\text{Tsh}\,360,000$ to get the amount that remained. That is, $\text{Tsh}\,360,000 - \text{Tsh}\,107,000 = \text{Tsh}\,253,000$. They were able to get the correct answer after applying the correct formula. That is,

"Amount left = Money deposited – (Money withdrawn + Service charges)".

Further analysis shows that the students who performed well, clearly understood the question. They used the Amount Remaining (AR) in the account after the first withdrawal to compute the new balance in Mwajuma's account after the second withdrawal of $\text{Tsh}\,50,000$. They applied the formula "*AR = amount after 1st withdrawal subtracting the sum of the 2nd withdrawal and service charges"*. In part (b), the students were able to convert the repeating decimal into a mixed fraction. They multiplied either by 10 or by 100 on both sides of the equation $x = 2.4\dot{3}$ in order to convert the repeating decimal into an improper fraction. They correctly simplified the resulting improper fraction to obtain a mixed fraction. For instance, the students used $x = 2.4\dot{3}$ as equation (i) and multiplied both sides of the equation by 10 to get $10x = 24.3\dot{3}$ as equation (ii). They were able to subtract the two equations to get $9x = 21.9$ such that $x = 2\frac{13}{30}$ as required. Extract 1.2 provides a sample of a response of a student who answered the question correctly.

1a(i) Money deposited → Tsh 360,000
Bank charges → Tsh 1,000
Withdrawal → Tsh 106,000
Amount remained → ?

Amount remained = Money deposited - [Withdrawal + Bank charges]
= Tsh 360.000 - [Tsh 106,000 + Tsh 1,000]
= Tsh 360.000 - Tsh 107,000
= Tsh 253,000

∴ The amount of money remained was Tsh 253,000.

1a(ii) Remained amount → Tsh 253,000
Bank charges → Tsh 1000
Withdrawal → Tsh 50,000
Amount remained → ?
Amount remained = Remained amount - [Withdrawal + Bank charges]
= Tsh 253,000 - [Tsh 50,000 + Tsh 1,000]
= Tsh 253,000 - Tsh 51,000
= Tsh 202,000

∴ The amount of money remained was Tsh 202,000.

1b Let x represent $2.4\dot{3}$

$x = 2.4\dot{3}$ ... i/
$10x = 24.\dot{3}$ ... ii/
$100x = 243.\dot{3}$ ... iii/

Take eqn (iii) - eqn(ii)

$100x - 10x = 243.\dot{3} - 24.\dot{3}$
$90x = 219$
$\frac{90x}{90} = \frac{219}{90}$
$x = 73/30$
$x = 2\,13/30$

∴ $2.4\dot{3}$ written as a mixed fraction is $2\,13/30$.

Extract 1.2: A sample of the student's responses to question 1

Extract 1.2 shows that in part (a), the student was able to interpret the word problem and perform correct mathematical computations. In part (b), the

student correctly converted the repeating decimal $2.4\dot{3}$ into a mixed fraction.

2.2 Question 2: Units and Approximations

The question consisted of two parts, (a) and (b). In part (a), the students were assessed whether or not they were able to add the metric units of mass and round off the measurements to the nearest whole number, one decimal place and 3 significant figures. This item assessed the students' competence after they have learnt the concepts of units and approximations. In part (a)(i), the students were given the information that, the masses of a dog, cat and goat which were $30.7\ \text{kg}$, $13.44\ \text{kg}$ and $18.26\ \text{kg}$, respectively. The item required the students to find the total mass of the three animals, giving the answer correct to the nearest whole number. In (a)(ii), the students were required to round off the masses of a dog, cat, and goat correctly to the nearest ones, one decimal place and 3 significant figures respectively. In part (b)(i), the students were required to add the measurements of length. That is, to add $8\ \text{km}\ 799\ \text{m}\ 400\ \text{mm}$ and $5\ \text{km}\ 300\ \text{m}\ 609\ \text{mm}$. In part (b)(ii), the students were required to convert the answer obtained in (b)(i) into metres.

The analysis shows that the question was attempted by a total of 634,775 (100%) students and among them 413,546 (65.1%) scored 3 to 10 marks indicating that the students' performance on this question was good. Further analysis revealed that 33,356 (5.3%) students scored zero whereas 11,598 (1.8%) students scored all 10 marks. The summary of the students' performance on question 2 is shown in Figure 2.

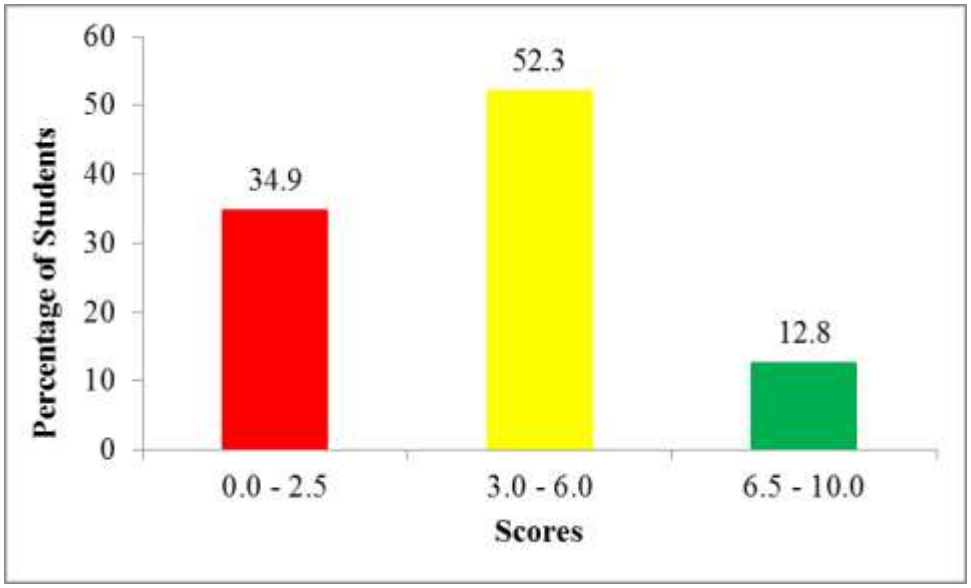


Figure 2: *Students' performance on question 2*

The analysis of students' responses shows that, the students who scored full marks in this question applied properly the knowledge and skills gained after learning the concepts of units and approximations. In part (a)(i), the students added all the measurements as decimals and correctly rounded off the results to the nearest whole number. That is, $30.7\,\text{kg} + 13.44\,\text{kg} + 18.26\,\text{kg} = 62.4\,\text{kg}$, which was approximately equal to $62\ \text{kg}$. In (a)(ii), the students performed correct approximation of the masses of the three animals. They rounded off the mass of a dog; 30.7 to the nearest ones as 31kg, the mass of a cat; $13.44\,\text{kg}$ to one decimal place as $13.4\,\text{kg}$, and the mass of a goat; $18.26\,\text{kg}$ in three significant figures as $18.3\,\text{kg}$. In part (b)(i), the students correctly added the measurements involving metric units of length. They added the units to get $13\,\text{km}\ 1099\,\text{m}\ 1009\,\text{mm}$ and made the necessary conversions to obtain $14\ \text{km}\ 100\ \text{m}\ 9\ \text{mm}$. Finally, those students were able to convert $14\ \text{km}\ 100\ \text{m}\ 9\ \text{mm}$ into metres, that is $1400.009\ \text{m}$ which was the correct answer. Extract 2.1 shows a sample of a response of a student who answered the question correctly.

2. a) (i) Solution.

30·7kg + 13·44kg + 18·26kg.

= 62·40kg.

The answer should be to the nearest whole number

62·40

= 62·00

⇒ 62kg

∴ The total mass of dog, cat and goat to the nearest whole number =62kg.

(ii) Mass of dog to the nearest ones.

30·7kg

⇒ 30·7 kg = 31·0kg

⇒ 31 kg

∴ Mass of dog to the nearest ones = 31kg.

Mass of cat to one decimal place

13·44kg

⇒ 13·44 kg = 13·40 kg

⇒ 13·4 kg

∴ The mass of cat to one decimal place.

⇒ 13·4kg.

Mass of goat to three sig figures.

18·26kg

⇒ 18·26 kg = 18·30 kg

⇒ 18·3kg

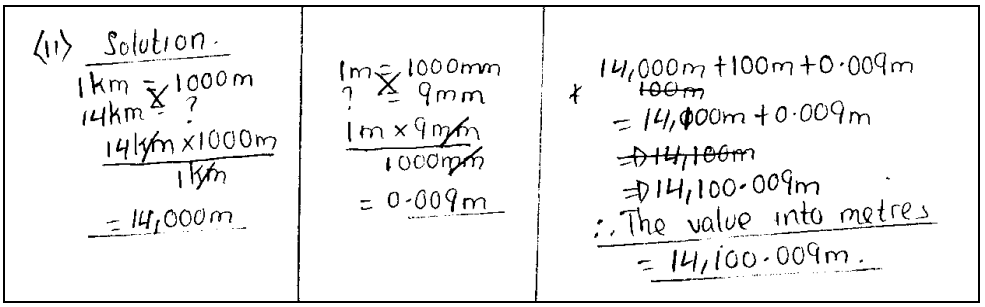
∴ The mass of goat to three sig figure

⇒ 18·3kg.

b. (i) Solution

	km	m	mm
	8	799	400
+	5	300	609
	13	1099	1009
	14	100	9

∴ 14km, 100m, 9mm



Extract 2.1: A sample of the student's responses to question 2

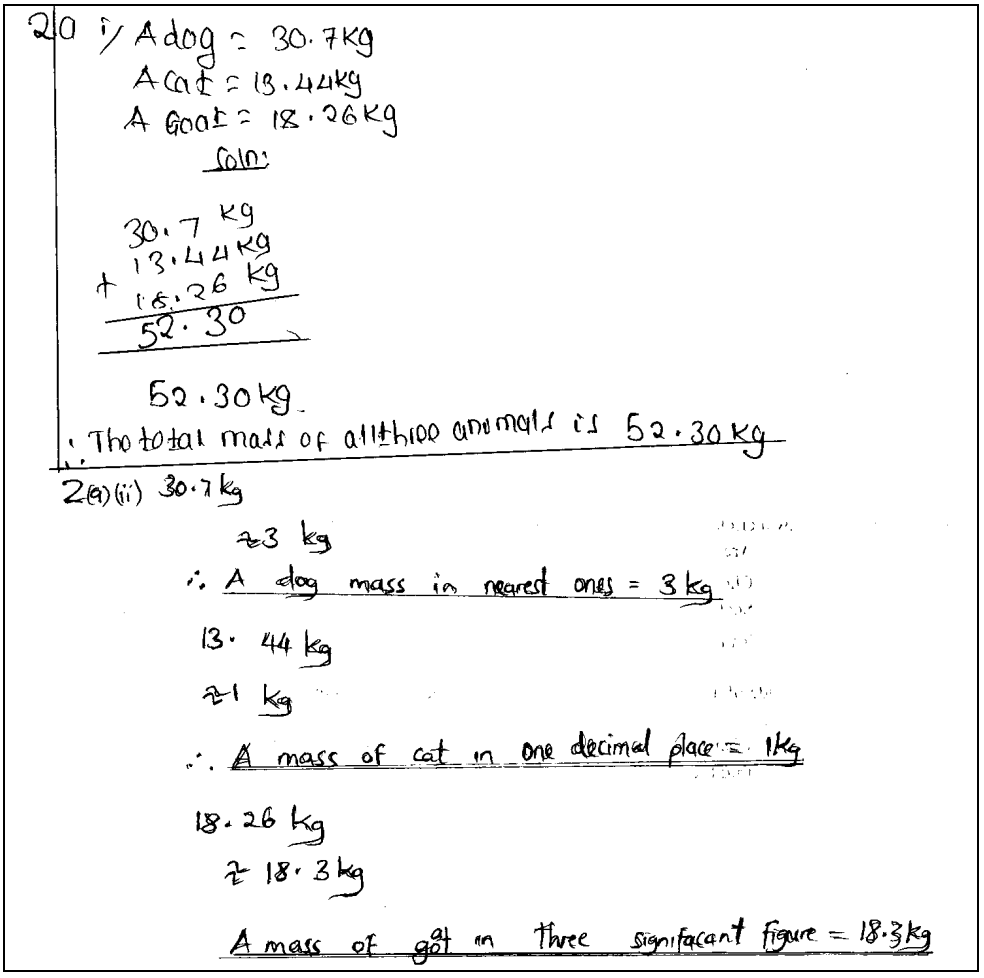
Extract 2.1 shows that, in part (a), the student correctly added the masses and rounded off the results to the nearest whole number. He/she was also able to round off the given masses of a dog, cat, and goat to the nearest ones, one decimal place and 3 significant figures respectively. In part (b), the student added correctly the units of length and converted the results into metres.

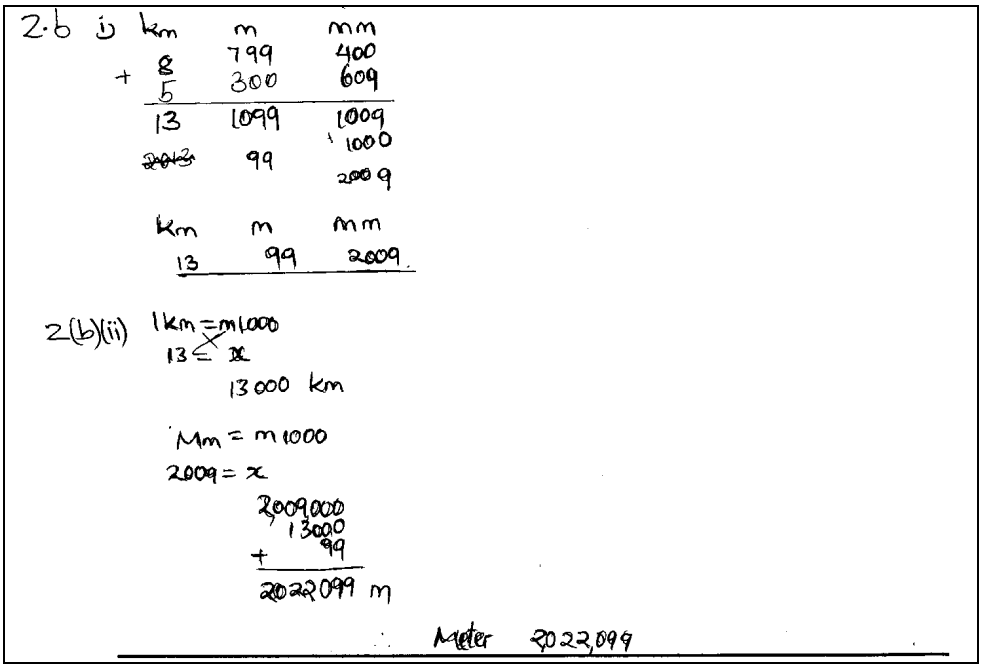
On the other hand, the analysis shows that, the students who performed poorly on this question lacked competence in the concepts of metric units and approximations. In part (a)(i), they failed to add the measurements of mass in decimals. They also failed to arrange the digits in their corresponding place values. Some of the students added the masses, that is, $30.7\,\text{kg} + 13.44\,\text{kg} + 18.26\,\text{kg}$ and got 61.67 kg instead of getting 62.4 kg. Also, the analysis shows that some of them were unable to distinguish between whole numbers and decimals. Thus, they could not approximate the result to the nearest whole number. For instance, they added $30.7\,\text{kg} + 13.44\,\text{kg} + 18.26\,\text{kg} = 62.4\,\text{kg}$ but failed to approximate $62.4\,\text{kg}$ to $62\,\text{kg}$ as the nearest whole number.

Furthermore, analysis shows that the students were unable to round off the masses of a dog, cat, and goat to the nearest ones, one decimal place, and 3 significant figures respectively. For instance, the students wrote the mass of goat as $183\,\text{kg}$ instead of $18.3\,\text{kg}$. There were other students who were not aware of the approximation symbol as they used an equal sign to approximate the numbers. For instance, they wrote $62.4\,\text{kg} = 62\,\text{kg}$ instead of $62.4\,\text{kg} \approx 62\,\text{kg}$ and they were not able to determine the place values of digits for approximations. In part (b), the students failed to convert the

measurements of length into metres. For instance, some of the students added the units to get $13\,\text{km}1099\,\text{m}\,1009\text{mm}$ but failed to make conversions in order to obtain $4\,\text{km}\;100\,\text{m}\;9\,\text{mm}$. Moreover, the students failed to convert one metric unit of length into other similar units. For instance, they used conversion of $1\,\text{m}=1000\,\text{km}$, $1\,\text{mm}=1000\,\text{mm}$ instead of $1\,\text{km}=1000\,\text{m}$ and $1\text{m}=1000\,\text{mm}$.

The analysis also shows that the students who performed poorly on this question converted the metric units of mass into metric units of length such as $1\,\text{km}=1000\,\text{kg}$. This highlights the fact that the students lacked the knowledge and skills of solving problems on measurements. Extract 2.2 is a sample of a response of a student who answered the question incorrectly.





Extract 2.2: A sample of the student's responses to question 2

Extract 2.2 shows that in part (a)(i), the student failed to perform addition on the given masses as he/she wrote $52.30\,\text{kg}$ instead of $62.4\,\text{kg}$. In (a)(ii), the student failed to approximate the given masses into the required system of numbers. In part (b)(i), the student failed to add the measurements of length. In part (b)(ii), the student failed to convert the units of length into metres.

2.3 Question 3: Geometry, Perimeters and Areas

This question had two parts, (a) and (b). In part (a), the students were assessed on whether they were able to draw a circle with centre at a point O and indicate the features; namely, (i) Arc AB, (ii) Chord $\overline{CD}$, (iii) Sector AOB, and (iv) Radius $\overline{AO}$. In part (b), they were provided with the information that "the side of a square carpet is $14\,\text{m}$. If a designer decides to make the largest possible circular carpet" and so, the item required students to determine the area of the circular carpet to be formed as well as the area of the remaining part of the carpet.

A total of 634,775 (100%) students attempted the question, out of whom 538,846 (84.3%) students scored below 3 marks and 422,289 (66.5%) scored zero mark. This shows that the students' performance on this question was weak. Despite the weak performance, 6,617 (1.0%) students managed to score full marks. Figure 3 summarizes the students' performance on question 3.

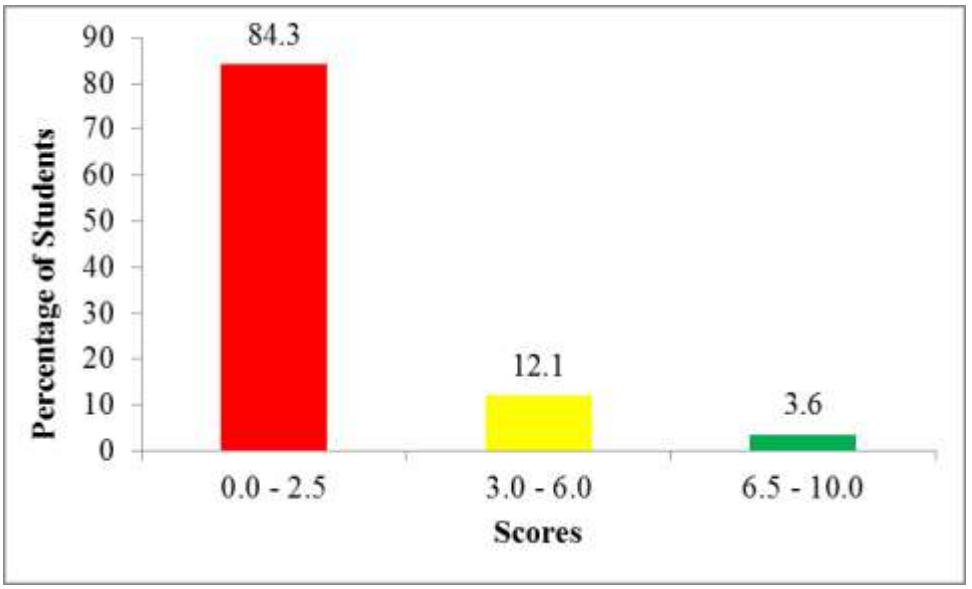
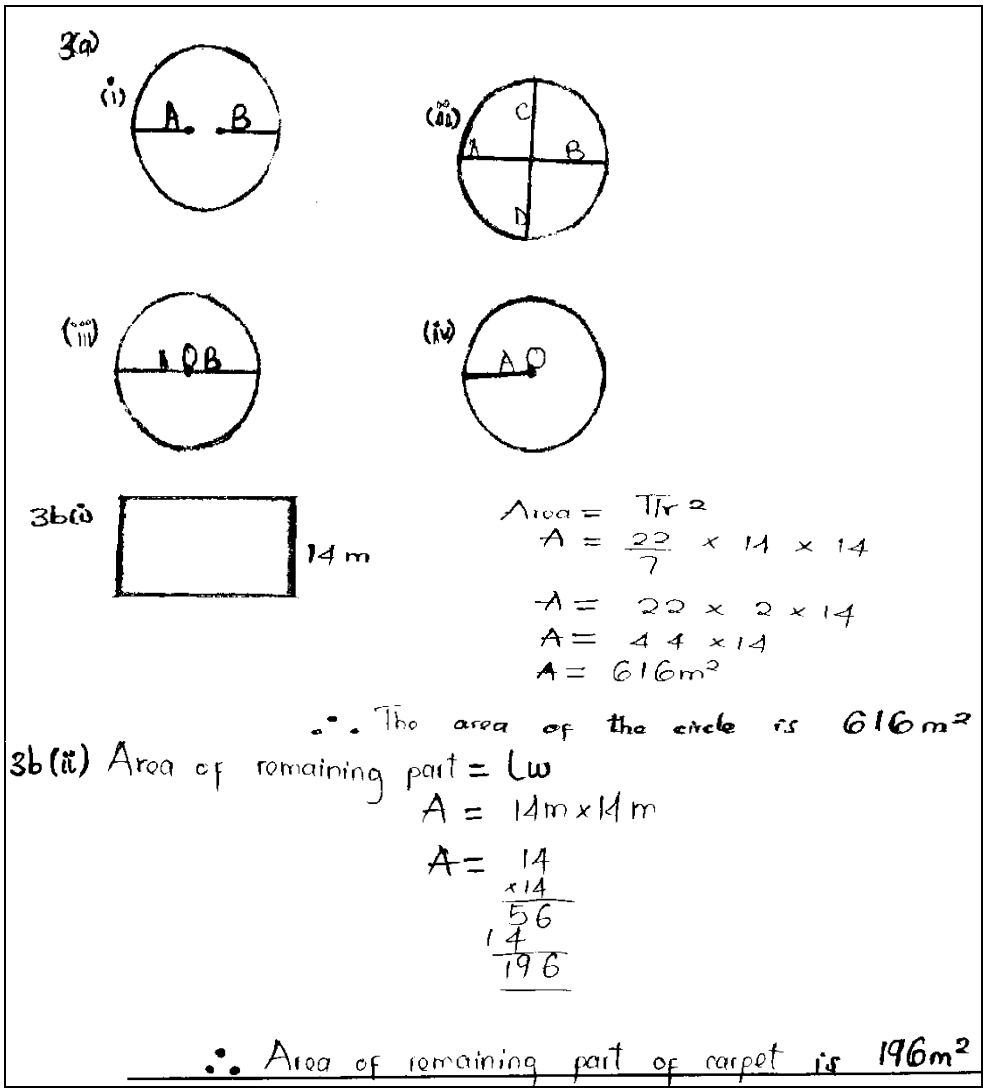


Figure 3: *Students' performance on question 3*

The analysis of students' responses indicates that the students who performed poorly on this question failed to demonstrate the competence of geometry, perimeters and areas. In part (a), the majority of students were not able to accurately locate the centre of the circle. They also lacked knowledge of the concept of an arc, chord, sector and radius of a circle. This shows that, the students could not develop the required competence on the features of the circle during the learning process. In this case, some students failed to distinguish between a chord and a radius. Moreover, students who scored zero marks drew either cylinders, triangles, rectangles or a square instead of a circle. In part (b)(i), students who scored zero were unable to interpret properly the information given in the question. They failed to recognize that the length of the side of the square is the diameter of the circle inscribed in it. They considered the diameter 14m as the

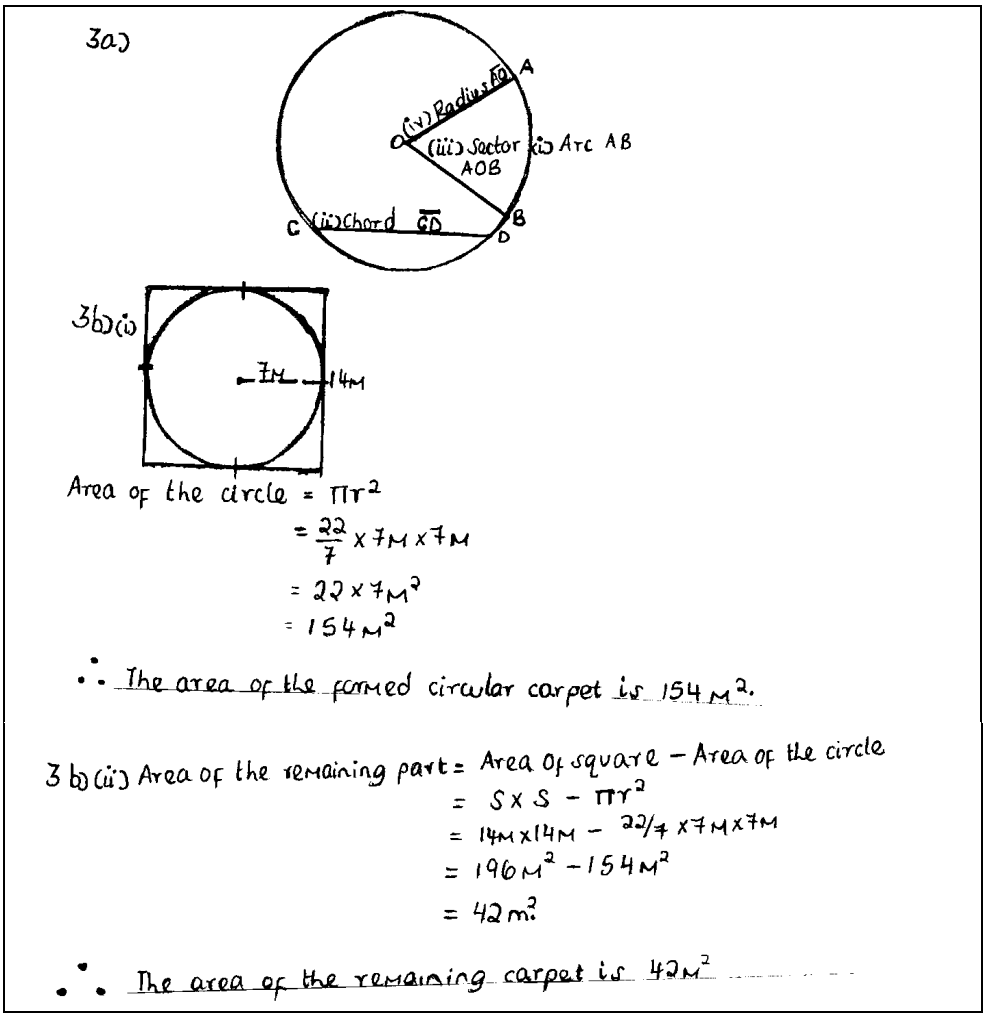
radius of the circle. This led to incorrect computations of the area of the circular carpet, hence the final answers were incorrect. Further analysis shows that, the weak performance on this question was also contributed by the use of inappropriate formula for area of a circle. Some students applied the formula $\text{Area} = \pi d^2$ instead of the formula $\text{Area} = \frac{\pi d^2}{4}$ while others applied $\text{Area} = 2\pi r$ instead of $\text{Area} = \pi r^2$. Extract 3.1 shows a sample of a response of a student who answered this question incorrectly.



Extract 3.1: A sample of the student's responses to question 3

Extract 3.1 shows that in part (a), the student drew circles but he/she lacked the knowledge of the concepts of arc, chord, sector and radius of a circle. Similarly, the student could not distinguish between a chord and a radius as well as a sector and a diameter. In part (b), the student applied the appropriate formula $A_1 = \pi r^2$ but failed to deduce that the radius of the circle was half the diameter. That is, $\text{radius} = \dfrac{\text{diameter}}{2}$, as he/she considered the diameter 14 m as the radius of the circle. Similarly, the student calculated the area of the remaining part of the carpet as $\text{Area} = \text{length} \times \text{width}$ instead of taking the difference between the areas.

On the other hand, the analysis shows that the students who scored all marks on this question applied properly the knowledge and skills gained after learning the concepts of geometry, perimeters and areas. In part (a), the students were able to draw a circle with centre at a point O and they indicated correctly an arc AB, the chord $\overline{\text{CD}}$, the sector AOB, and the radius $\overline{\text{AO}}$. In part (b), the students were able to draw a square with the side of length 14 m which inscribes a circle of radius 7 m. They applied the formula for area A_1 of a circle, that is $A_1 = \pi r^2$, where $\pi = \dfrac{22}{7}$ and $r = 7\text{ m}$ to get the area of circular carpet. Proper computations were performed to obtain the area of a circle as $A_1 = 154\text{ m}^2$. Similarly, the students were able to apply the formula for area A_2 of a square, that is, $A_2 = \text{length} \times \text{length}$ and they obtained $A_2 = 196\text{ m}^2$. Finally, they computed the area of the remaining part of the carpet by taking the difference between A_1 and A_2 to obtain 42 m^2 which was the correct answer. Extract 3.2 shows a sample of a response of a student who answered this question correctly.



Extract 3.2: A sample of the student's responses to question 3

Extract 3.2 shows that, in part (a), the student drew a well described circle with the centre at O and indicated all the required parts of a circle, that is, (i) an arc AB, (ii) chord $\overline{CD}$, (iii) Sector AOB, and (iv) radius $\overline{AO}$. In part (b)(i), the student was able to draw a square with a side of length 14m which inscribes a circle of diameter 14m. He/she calculated the area of a circle and that of a square correctly to get 154m^2 and 196m^2 respectively. Finally, the student computed the area of the remaining part of the carpet, that is, $196\text{m}^2 - 154\text{cm}^2 = 42\text{m}^2$.

2.4 Question 4: Algebra and Quadratic Equations

This question had two parts, (a) and (b). In part (a), the students were assessed if they were able to use elimination method to solve a pair of linear simultaneous equations, $\begin{cases} \frac{a}{2} - \frac{b}{5} = 1 \\ 3b = 24 + a \end{cases}$. In part (b), they were assessed whether they were able to use the given information that read, "The length of a book exceeds its width by 5 cm". They were supposed to calculate the dimensions of the book, given that its area is 50 cm^2.

Analysis shows that a total of 634,775 (100%) attempted the question and among them, 594,806 (93.7%) students scored 0 to 2.5 marks showing that the students' performance on this question was weak. It was further noted that a total of 486,341 (76.6%) students scored zero marks whereas only 6,616 (1.0%) scored full marks. Figure 4 shows the summary of the students' performance on question 4.

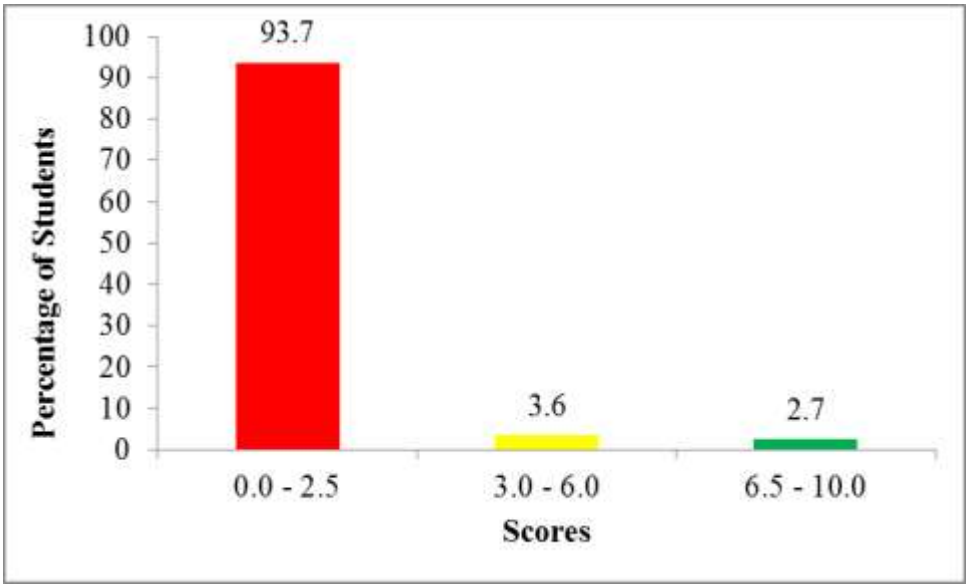
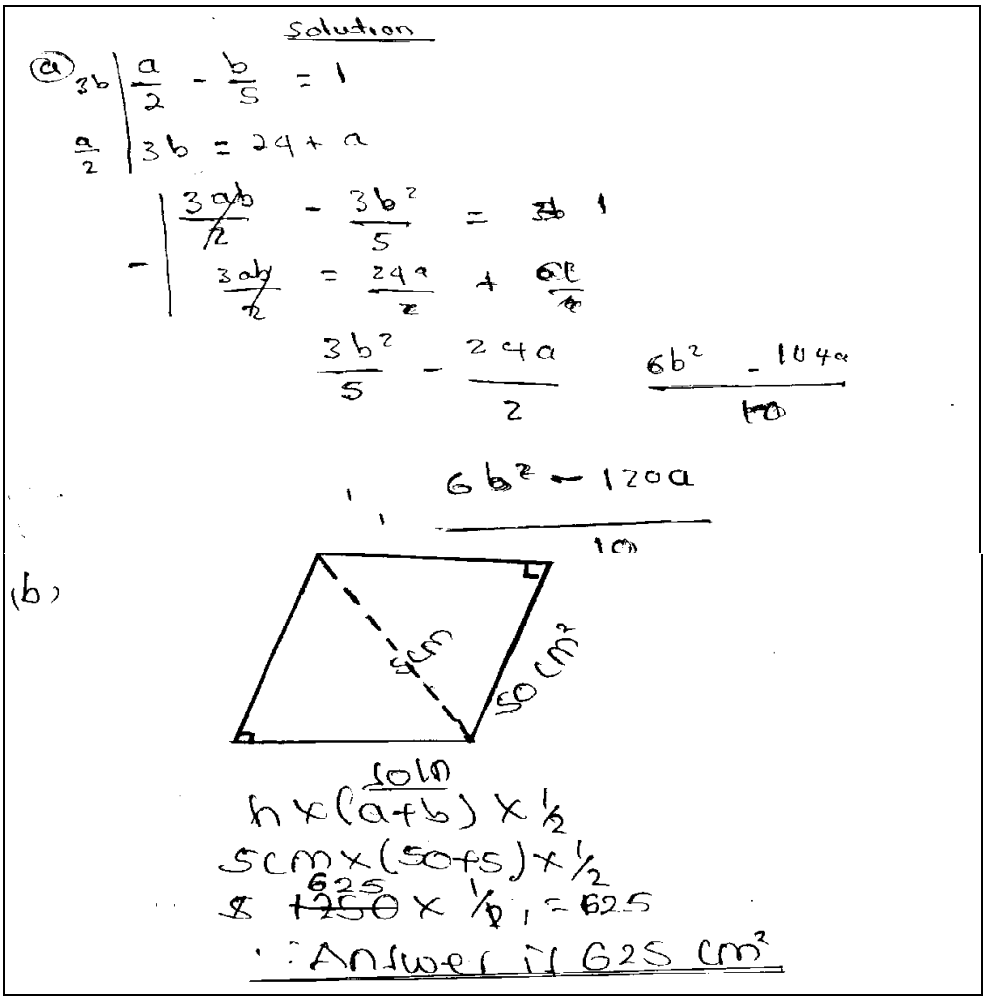


Figure 4: *Students' performance on question 4*

The analysis of the students' responses shows that, the poor performance on this question was due to lack of competence in algebra, perimeters and areas. In part (a), some students failed to notice the need of rewriting the given system of simultaneous equations in standard form. Majority of the

students failed to transform the equations into standard form. Other students managed to write the equations into the standard form. However, they failed to apply elimination method to solve the resulting linear system of simultaneous equations. Also, a few students had the knowledge of applying L.C.M in removing fractional coefficients, but they failed to multiply both sides of the equation by the L.C.M of the denominators. For instance, the students recognized that the L.C.M of the denominators in the equation $\frac{a}{2}-\frac{b}{5}=1$ is 10, but they multiplied wrongly by writing $\frac{2b\text{ - }5a}{10}=1$ instead of $\frac{5a-2b}{10}=1$. A few students tried to eliminate one of the variables without rewriting the two equations in standard form. This led to difficulties in calculations, hence they obtained incorrect answers. Also, the analysis reveals that some students solved the problem by using substitution method which was against the question. This shows that the students were not competent in using the elimination method or they could not understand the question. In part (b), students failed to interpret the word problem and could not formulate the required quadratic equation. Further analysis shows that some students lacked knowledge on geometrical shape. They considered the surface of a book as being triangular in shape, hence they performed incorrect calculations by applying the formula for the area of a triangle. In the analysis, it was noted that, some students calculated the perimeter of a rectangle instead of its area. The formula they applied was $\mathrm{A}=(\mathrm{L}+\mathrm{w})2$ which led to the wrong computations and an incorrect final answer. Extract 4.1 shows a sample of a response of a student who failed to answer the question correctly.

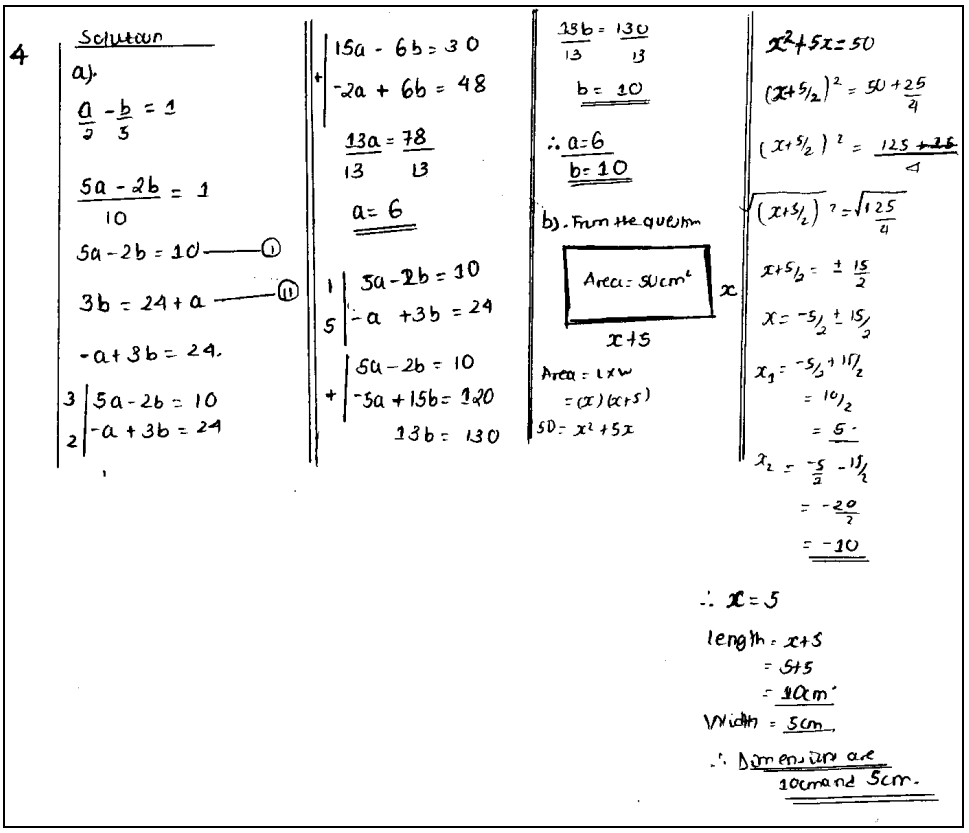


Extract 4.1: A sample of the student's responses to question 4

Extract 4.1 shows that in part (a), the student failed to rearrange the two simultaneous equations in standard form. He/she failed to remove the fractional coefficients in the equation, $\frac{a}{2} - \frac{b}{5} = 1$. In part (b), the student failed to identify the shape of the surface of a book. Thus, he/she applied a wrong formula for calculating its dimensions.

Further analysis shows that the students who scored all the marks on this question applied correctly the knowledge gained after learning algebra, perimeters and areas. In part (a), the students followed all the steps used in solving system of equations by elimination method. First, the two equations

were written in standard form to obtain $\begin{cases} 5a-2b=10 \\ a-3b=24 \end{cases}$. They eliminated one variable from the resulting equations and then solved for the value of the remaining variable. They obtained the correct values of a and b. That is, $a=6$ and $b=10$. In part (b), students correctly interpreted the given word problem, extracted the necessary information and formulated a quadratic equation. They applied the knowledge and skills in the concept of geometrical shapes to identify the fact that the surface of the book is rectangular in shape. Hence, the formula for area of a rectangle was used to calculate the dimensions of the book. That is, $A=L\times W$ to obtain a quadratic equation $x^2+5x-50=0$, where x was the width and $x+5$ was the length of the book. They solved for the value of x to get $x=5\text{ cm}$ or $x=-10\text{cm}$. Since the dimensions of the book cannot be negative, they chose the correct value, $x=5\text{cm}$. The students correctly concluded that, the book had dimensions 10cm by 5cm. Extract 4.2 shows a sample of a response of a student who answered this question correctly.



Extract 4.2: A sample of the student's responses to question 4

Extract 4.2 shows that in part (a), the student used the L.C.M correctly to remove fractional coefficients and arranged the two linear simultaneous equations into standard form. He/she applied the elimination method to obtain the values of a and b as required. In part (b), the student formulated a mathematical equation correctly from the given word problem and calculated the dimensions of the book as required.

2.5 Question 5: Ratios, Profit and Loss

This question consisted of two parts, (a) and (b). In part (a), the students were given the information that Asha and Juma received 630,000 shillings from their father. The father wanted to give Asha twice as much money as the amount that could be given to Juma. The students were assessed whether they were able to calculate the amount of money received by Asha. Part (b) stated that, Mr. and Mrs. Juma deposited some money in a bank that pays a simple interest of 3% per annum. After 4 years they earned an

interest of 900,000 shillings. Item (b)(i) required the students to determine the amount of money that was deposited initially. Finally, item (b)(ii) required the students to determine the amount of money that was accumulated after a period of four years.

The data analysis reveals that the question was attempted by 634,775 (100%) students. Further analysis shows that 558,570 (88.0%) students scored below 3 marks, indicating that the students' performance on this question was weak. It was also noted that, only 12,447 (2.0%) students scored full marks while a total of 424,723 (66.9%) scored zero. Figure 5 summarizes the students' performance on question 5.

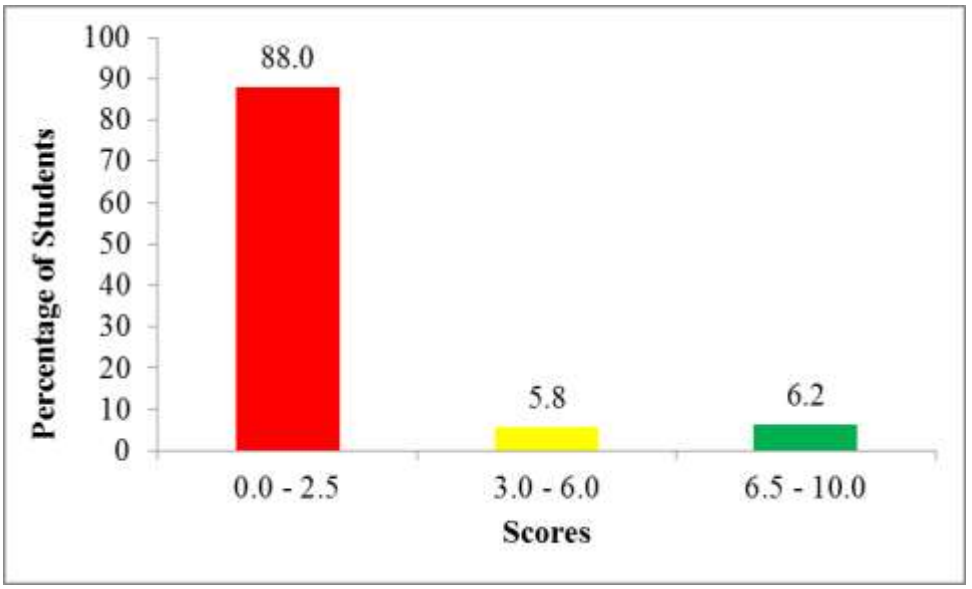
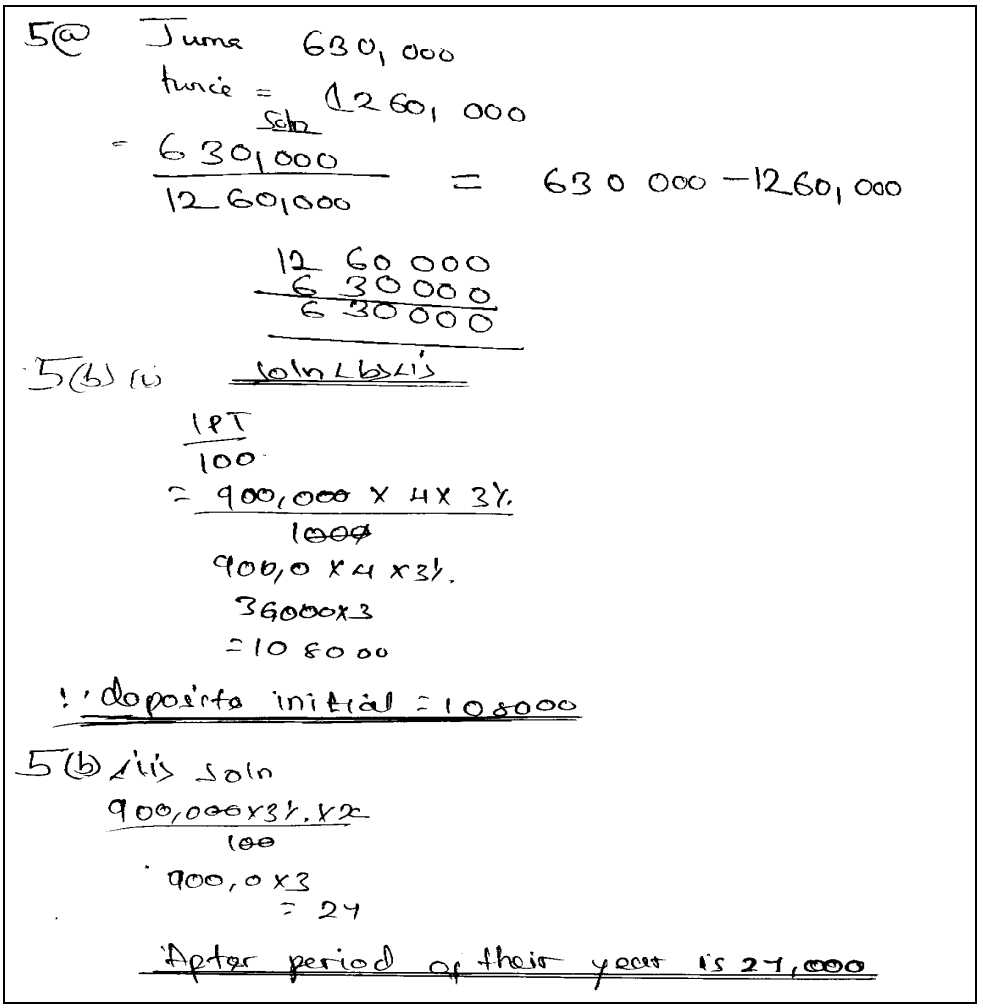


Figure 5: *Students' performance on question 5*

The analysis of students' responses indicates that most of the students who had poor performance on this question were incompetent in dividing quantities in their proportional parts. They lacked knowledge and skills of solving real life problems on ratios, profit and loss. In part (a), the students were not able to interpret the information from word problem, hence they failed to formulate a mathematical equation representing the problem. For instance, some students failed to understand the meaning of the phrase "twice as much" which led them to divide $630,000 \div 2$, multiply $2 \times 630,000$, and add $630,000 + 630,000$. Other students interchanged the

ratios $\text{Asha}:\text{Juma}=1:2$ instead of $2:1$. In part (b), the students used wrong formulae such as, $\%\text{Profit}=\frac{\text{profit made}}{\text{buying price}}\times 100\%$ instead of $I=\frac{PRT}{100}$. Moreover, some students failed to transpose the formula $I=\frac{PRT}{100}$ to express P as the subject. They wrote $P=\frac{IR}{100}$ instead of $P=\frac{100I}{RT}$. Also, other students applied appropriate formula for calculating the simple interest but they wrongly substituted $P=900,000$ shillings instead of $I=900,000$ shillings. Many students failed to find the accumulated amount of money since they were not able to recall the formula, that is, $\text{Accumulated amount}\,(A)=\text{Principal}\,(P)+\text{Interest}\,(I)$. Extract 5.1 shows a sample of a response of a student who answered this question incorrectly.

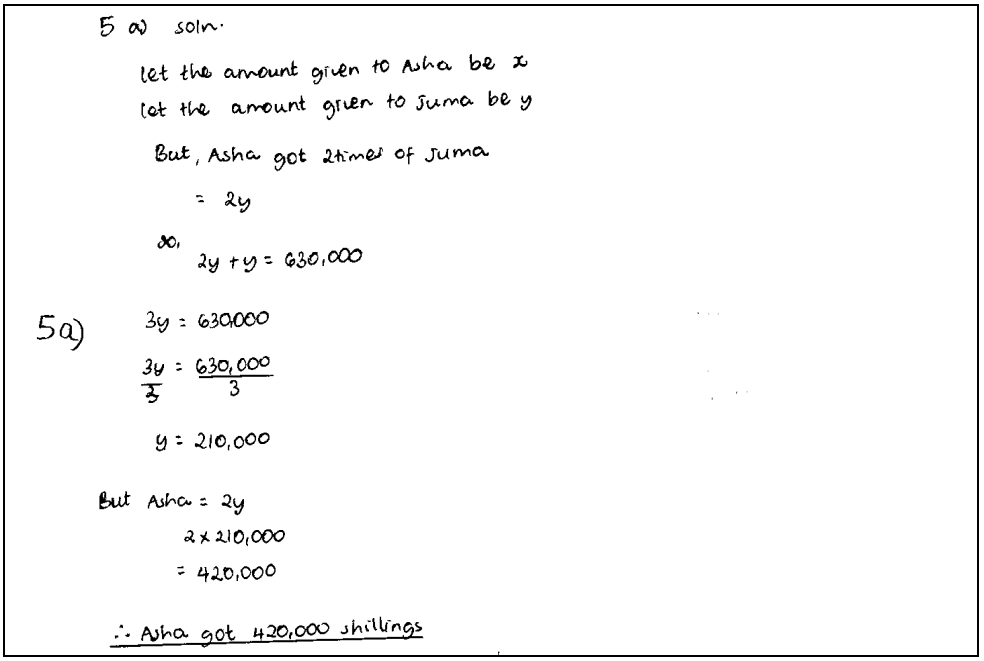


Extract 5.1: A sample of the student's responses to question 5

Extract 5.1 shows that in part (a), the student failed to interpret the information from the word problem; hence he/she could not formulate the required mathematical equation. In part (b), the student failed to apply the correct formula of simple interest.

On the other hand, the analysis shows that the students who scored all the marks in this question applied properly the knowledge and skills gained after learning the concepts of ratios, profit and loss. In part (a), the students were able to understand the question and formulated a mathematical equation from the given word problem. They correctly let x to be the amount received by Juma. Hence, Asha received $2x$. The formulated

equation was $2x + x = 630{,}000$. They were able to solve for the value of x to get $x = 210{,}000$ and obtained the amount of money received by Asha, $2 \times 210{,}000 = 420{,}000$ shillings, which was the correct answer. Alternatively, some students applied the concept of ratios to solve the problem correctly. That is, $\text{Asha} : \text{Juma} = 2:1$, sum of ratios $2+1=3$. They calculated the amount of money Asha received, that is, $\frac{2}{3} \times 630{,}000 = 420{,}000$ shillings. In part (b)(i), the students modified the formula for simple interest. That is, $P = \frac{100I}{RT}$ which was applied correctly, where $I = 900{,}000$ shillings, $R = 3\%$ and $T = 4\,\text{years}$. They substituted the values and made the correct calculations to get $7{,}500{,}000$ shillings. The results implied that the students had knowledge and skills in simple interest. In part (b)(ii), the students were able to apply correctly the formula $A = I + P$, where $I = 900{,}000$ shillings and $P = 7{,}500{,}000$ shillings. They performed proper computations to calculate the accumulated amount of money to get $A = 8{,}400{,}000$ shillings as required. Extract 5.2 shows a sample of a response of a student who answered this question correctly.


5 a) soln.

let the amount given to Asha be x

let the amount given to Juma be y

But, Asha got 2times of Juma

= 2y

∴

2y + y = 630,000

5a)

3y = 630,000

$\frac{3y}{3} = \frac{630{,}000}{3}$

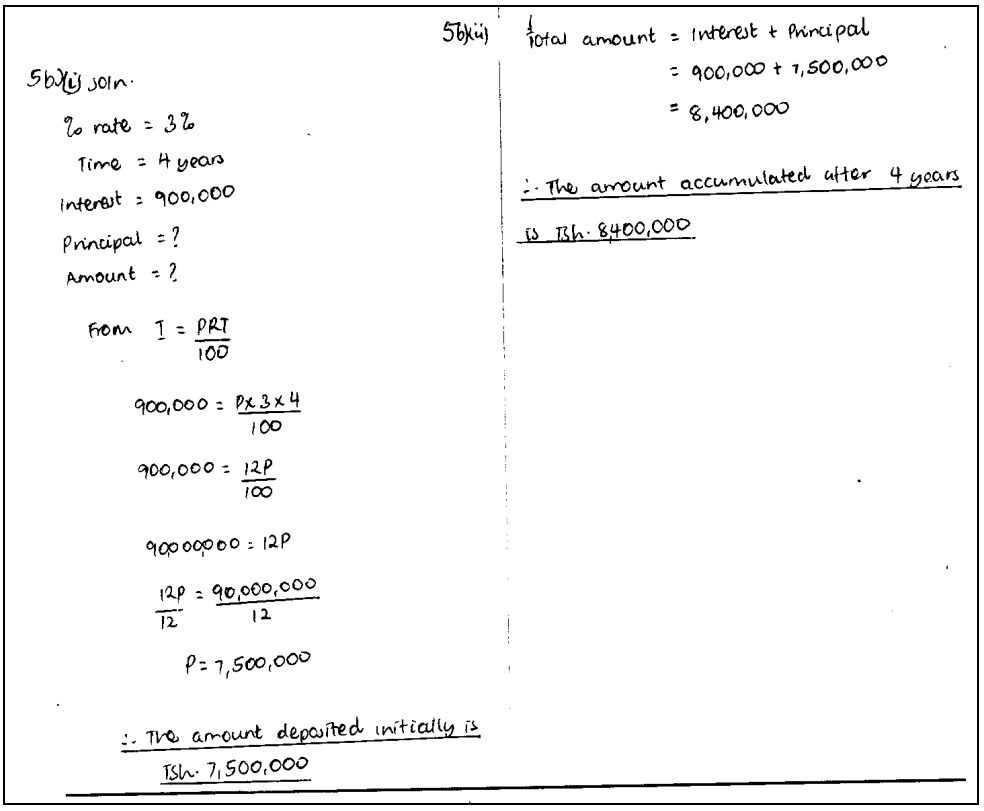
y = 210,000

But Asha = 2y

2 × 210,000

= 420,000

∴ Asha got 420,000 shillings



Extract 5.2: A sample of the student's responses to question 5

Extract 5.2 shows that in part (a), the student was able to interpret the information in the word problem. He/she was able to formulate a mathematical equation from the word problem. In part (b), the student was able to apply the correct formulae and they made proper substitution. This shows that the students had ability to solve real life problems on simple interest.

2.6 Question 6: Coordinate Geometry

This question comprised two parts, (a) and (b). In part (a), the students were given the line whose equation is $y = 3x - p$ and it passes through the points $(6,10)$ and $(q,22)$. They were required to find the values of the integers p and q. In part (b), they were given a word problem which involved a mason who wanted to design a small room of dimensions $500\,\text{cm}$ by $200\,\text{cm}$. In (b)(i), students were required to draw a flat figure of

the room at a scale of $1:100$. In (b)(ii), students were assessed whether or not they could calculate the area of the room by using the result obtained in part (b)(i).

The data show that the question was attempted by 634,775 (100%) students out of whom, 589,278 (92.8%) scored 0 to 2.5 marks. This reveals that the students' performance on this question was weak. However, only 3,030 (0.5%) students scored 10 marks whereas 505,408 (79.6%) scored zero. The summary of the students' performance on question 6 is indicated in Figure 6.

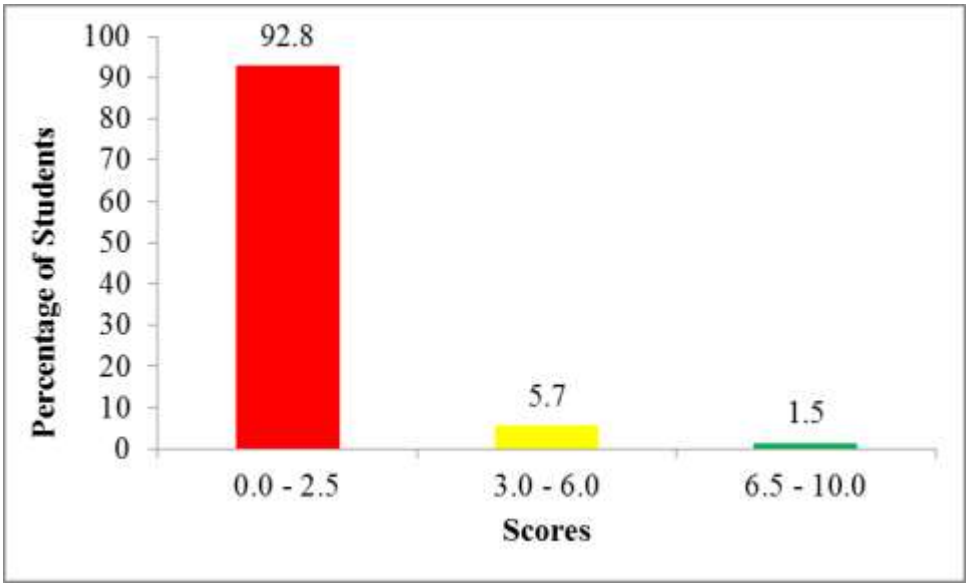
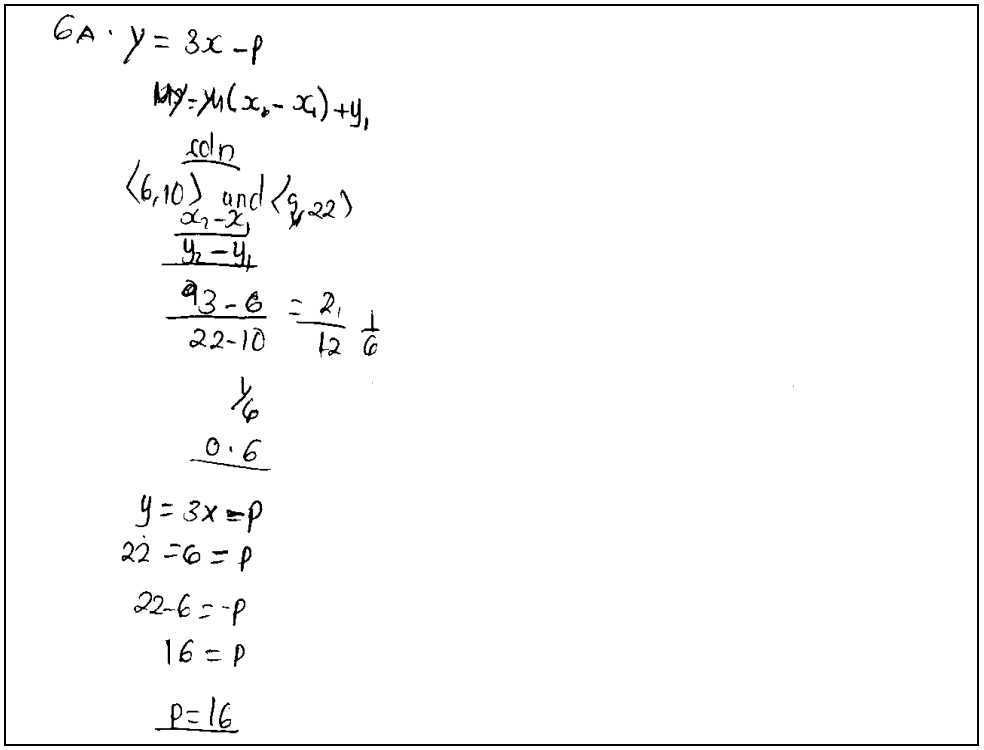
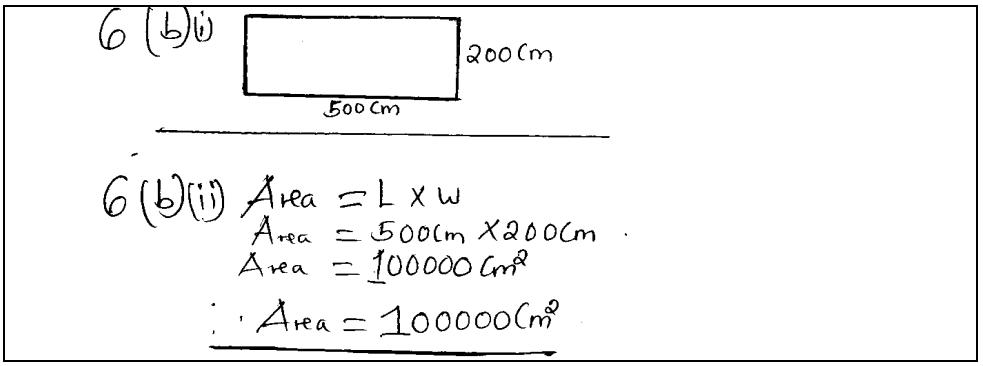


Figure 6: *Students' performance on question 6*

The analysis shows that the students' weak performance on this question was contributed by lack of competence in coordinate geometry. In part (a), the students were unable to recognize the points in the xy-plane. They interchanged the x and y coordinates of the given points. Thus, they substituted the point $(6,10)$ in the line $y=3x-\mathrm{p}$ by taking $x=10$ and $y=6$ to get an incorrect value of $p=24$. Similarly, the point $(q,22)$ was substituted in the equation $y=3x-p$ by taking $x=22$ and $y=q$ to obtain an incorrect value of $q=42$. Also, there were students who failed to recall the correct formula for calculating the slope of a straight line. They

used $m = \frac{\text{change in } x}{\text{change in } y}$ instead of $m = \frac{\text{change in } y}{\text{change in } x}$. Yet other students used the correct formula for finding the slope but they failed to substitute correctly the values of x and y in the formula. In part (b)(i), the students were not able to convert the given dimensions of the room to the scale of $1:100$. They drew the diagram of the room without using the given scale. Further analysis show that in part (b)(ii), the students applied wrong formulae for finding the area of a rectangle such as $\text{Area} = \text{Length} + \text{Width}$ instead of $\text{Area} = \text{Length} \times \text{Width}$. Some of the students calculated the area of the rectangular room without converting the dimensions into the given scale. That is, they calculated the area by using the formula $\text{Area} = 500\,\text{cm} \times 200\,\text{cm} = 100{,}000\,\text{cm}^2$, which was not the correct answer. Extract 6.1 shows a sample of a response of a student who answered this question incorrectly.



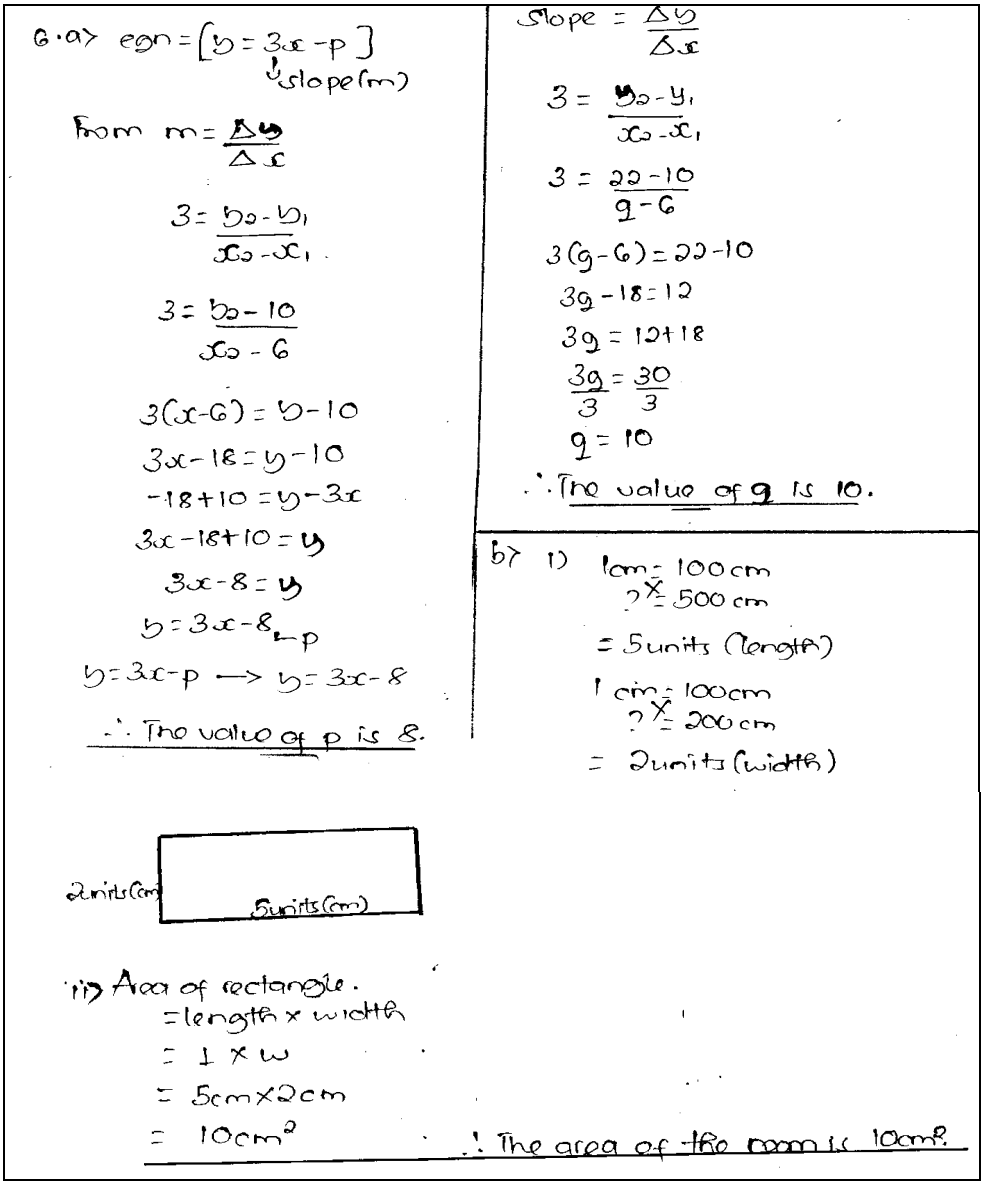


Extract 6.1: A sample of the student's responses to question 6

Extract 6.1 shows that in part (a), the student used a wrong formula when finding the gradient of a straight line, that is, $\frac{x_2 - x_1}{y_2 - y_1}$, which led to getting an incorrect answer. In part (b)(i), the student failed to use the scale given to convert the given dimensions. In part b(ii), the student used the given dimensions to calculate the area of the room without converting the dimensions into the given scale of $1:100$.

On the other hand, the students who scored all the marks in part (a) were able to find the values of p and q. They computed the slope m of the line $y = 3x - p$ and its y-intercept to obtain $m = 3$, y-intercept $= -p$. The formula for finding the slope, $m = \frac{y_2 - y_1}{x_2 - x_1}$ was applied such that $3 = \frac{22 - 10}{q - 6}$ to get $q = 10$. Then the value of p was calculated by relating the equation of the given line with that of a line passing through the points $(6,10)$ and $(10, 22)$ to obtain $p = 8$. Furthermore, some students used the fact that, the two points $(6,10)$ and $(q,22)$ lie on the line, hence the points satisfy the equation $y = 3x - p$. Thus, they substituted the points correctly to obtain $p = 8$ and $q = 10$. In part (b)(i), the students were able to apply the scale of $1:100$ to the given measurements by taking $\frac{500 \text{ cm}}{100} = 5 \text{ cm}$ and $\frac{200 \text{ cm}}{100} = 2 \text{ cm}$. They drew a diagram of the room as a rectangle of

dimensions 5cm by 2cm. In (b)(ii), the students managed to recognize that, the room is rectangular in shape. They applied an appropriate formula when finding the area of a rectangle. That is, Area=Length×Width. The area was calculated to get 10cm^2. Extract 6.2 shows a sample of a response of a student who answered the question correctly.



Extract 6.2: A sample of the student's responses to question 6

Extract 6.2 shows that in part (a), the student was able to identify the slope from the given equation and applied the formula for calculating the slope correctly, then he/she solved for the values of p and q. In part (b)(i), the student managed to draw a diagram of a room by using the given scale of $1:100$ and got the dimensions 5cm by 2cm. In (b)(ii), the student calculated the area of the room by using the formula $\text{Area}=\text{Length}\times\text{Width}$.

2.7 Question 7: Exponents and Radicals

This question had two parts, (a) and (b). In part (a), the students were assessed whether or not they are able to perform basic mathematical operations on the two given quantities, P and Q in radicals. The item (a)(i) required the students to show that $PQ=-1-2\sqrt{2}$, while item (a)(ii) required the students to show that $\frac{P}{Q}=5-4\sqrt{2}$, where $P=\sqrt{2}-3$ and $Q=\sqrt{2}+1$. In part (b), the students were required to transpose the mathematical formula $p=\sqrt{q+x}$ by writing x in terms of the variables p and q, hence to determine the value of x when $p=3$ and $q=-1$.

According to data, out of 634,775 (100%) students who attempted the question, 570,203 (89.8%) students scored below 3 marks. It was further noted that 472007 (74.4%) students scored zero mark, indicating that the students' performance on this question was weak. In spite of weak performance, 9,838 (1.5%) students managed to score all marks for this question. Figure 7 summarizes the students' performance on question 7.

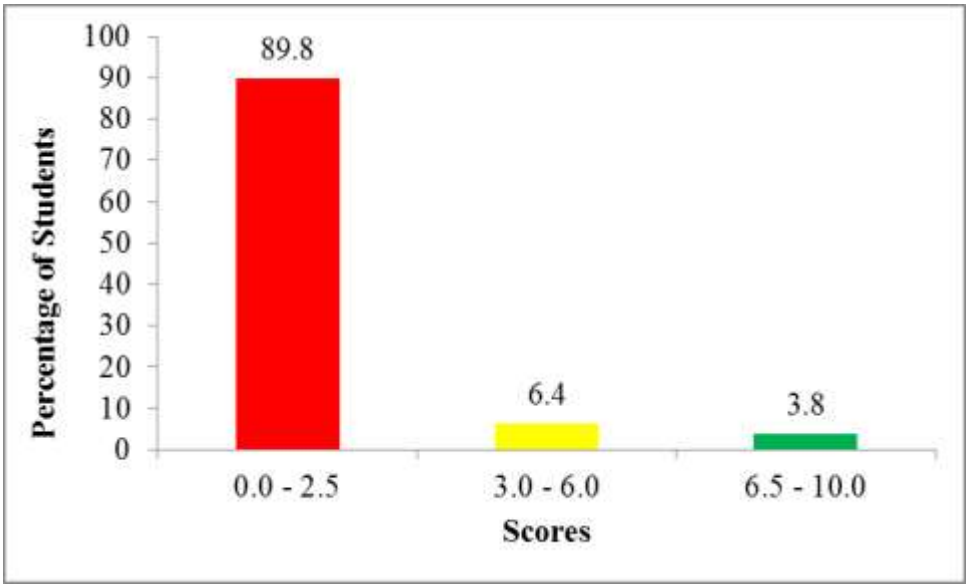


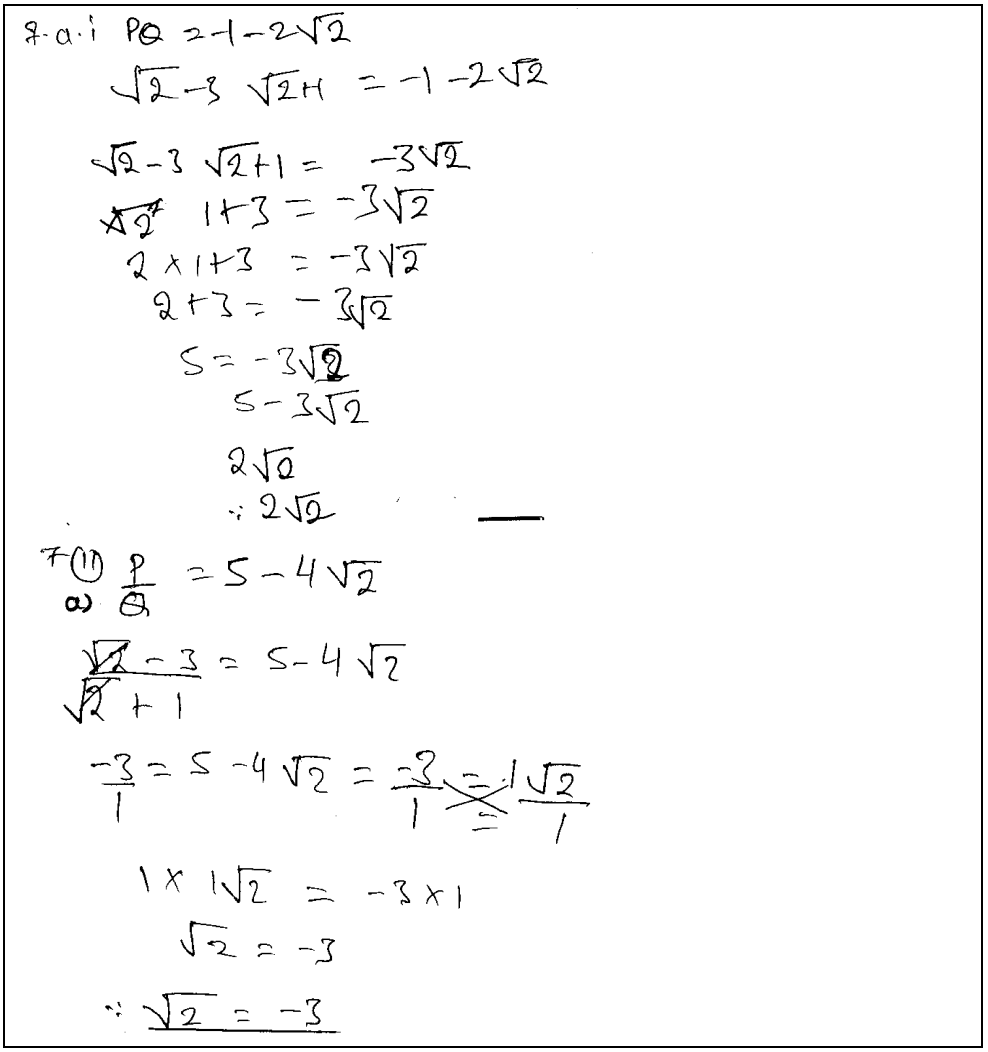
Figure 7: *Students' performance on question 7*

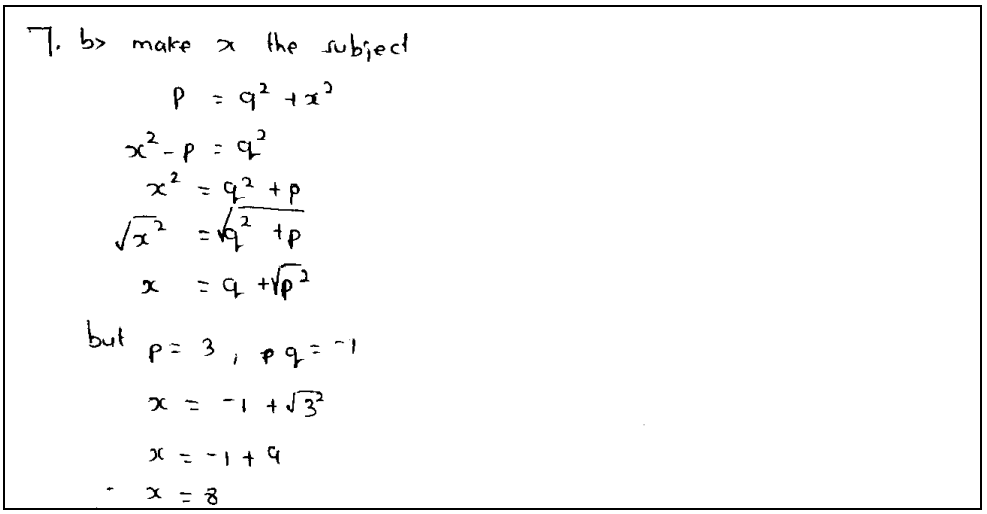
The analysis of students' responses shows that the students who performed poorly on this question lacked competence on operations of exponents and radical expressions. In part (a)(i), majority of the students were unable to perform basic operations on radicals. For instance, some of the students failed to insert brackets between the two expressions. Instead of writing $\left(\sqrt{2}-3\right)\times\left(\sqrt{2}+1\right)$, they wrote $\sqrt{2}-3\times\sqrt{2}+1$ which gives an incorrect answer. Also, the analysis shows that some of the students did not understand the difference between radicals and integers, thus they joined together the two expressions when doing the basic operations. For instance, they wrote $\left(\sqrt{2-3}\right)\times\left(\sqrt{2+1}\right)$ instead of $\left(\sqrt{2}-3\right)\times\left(\sqrt{2}+1\right)$. Moreover, some of the students changed the expression $\left(\sqrt{2}-3\right)\times\left(\sqrt{2}+1\right)$ to $\sqrt{2}-3+\sqrt{2}+1$.

Furthermore, some students failed to multiply radicals and radicals by integers. For instance, some students expanded $\left(\sqrt{2}-3\right)\times\left(\sqrt{2}+1\right)$ as $\sqrt{2}\times\sqrt{2}+\sqrt{2}-\sqrt{6}+3$ which was incorrect. In part (a)(ii), majority of the students failed to rationalize the denominator of the expression $\frac{P}{Q}$. Some

students were able to determine the rationalizing factor $\sqrt{2}-1$ but failed to use it in simplifying the expression.

In part (b), majority of the students lacked knowledge on transposition of the formula. In this case, the students were not able to write x in terms of p and q. They failed to square both sides of the equation to remove the radical sign. For instance, the students wrote $\sqrt{p}=\sqrt{q+x}$ instead of $p^2=\left(\sqrt{q+x}\right)^2$. Extract 7.1 provides a sample of a response of a student who failed to answer this question correctly.



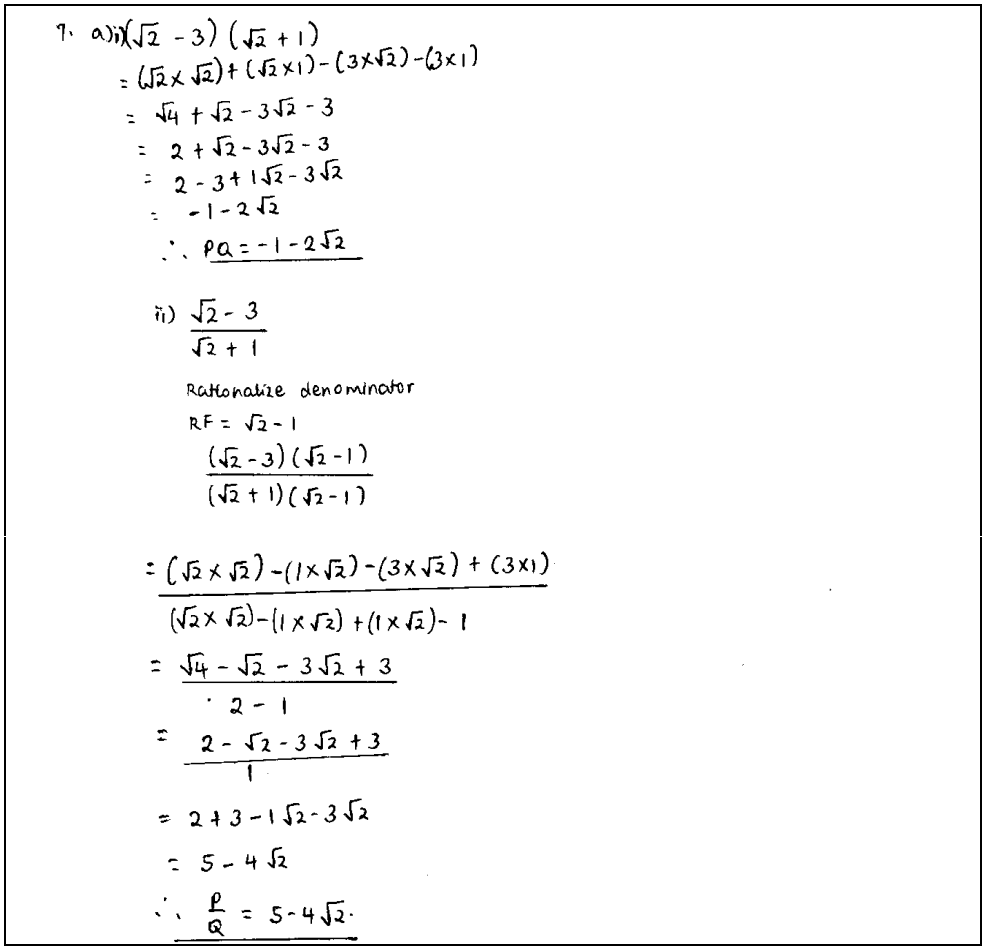


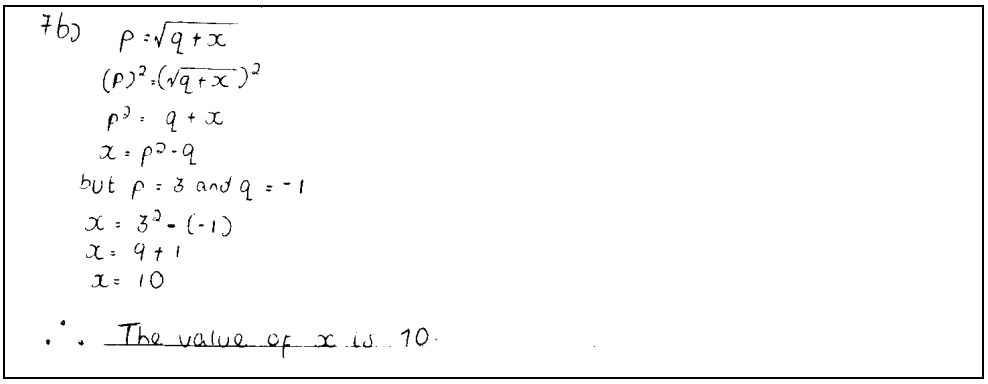
Extract 7.1: A sample of the student's responses to question 7

Extract 7.1 shows that in part (a)(i), the student failed to multiply radicals and radicals by integers. He/she wrote $\left(\sqrt{2}-3\right)\times\left(\sqrt{2}+1\right)$ as $\sqrt{2}\left(1+3\right)$ which was incorrect. In part (a)(ii), the student illogically cancelled the radicals in the numerator and the denominator of the expression $\frac{\sqrt{2}-3}{\sqrt{2}+1}$ to obtain $\frac{-3}{1}$, which is incorrect. In part (b), the student incorrectly removed the radical sign from the right-hand side of the equation.

On the other hand, the analysis shows that the students who correctly answered this question had sufficient knowledge of the concept of exponents and radicals. They demonstrated their competence on exponents and radicals gained during the learning process to answer the question. In part (a)(i), they multiplied the two quantities to obtain $PQ=\sqrt{2\times 2}+\sqrt{2}-3\sqrt{2}-3$. After simplification, they obtained $PQ=-1-2\sqrt{2}$ as required. In part (a)(ii), the students were able to rationalize the denominator by identifying the rationalizing factor which was $\sqrt{2}-1$. That is, $\frac{P}{Q}=\frac{\sqrt{2}-3}{\sqrt{2}+1}\times\frac{\sqrt{2}-1}{\sqrt{2}-1}$. They performed computations using mathematical operations of addition, subtraction and multiplication to

get the correct answer, which was $5-4\sqrt{2}$. In part (b), the students were able to transpose the formula by rewriting the variable x in terms of p and q. The formula $p=\sqrt{q+x}$ was transposed to obtain $x=p^2-q$. They obtained the value of x after substituting $p=3$ and $q=-1$ to get $x=10$. Extract 7.2 presents a sample of a response of a student who answered the question correctly.



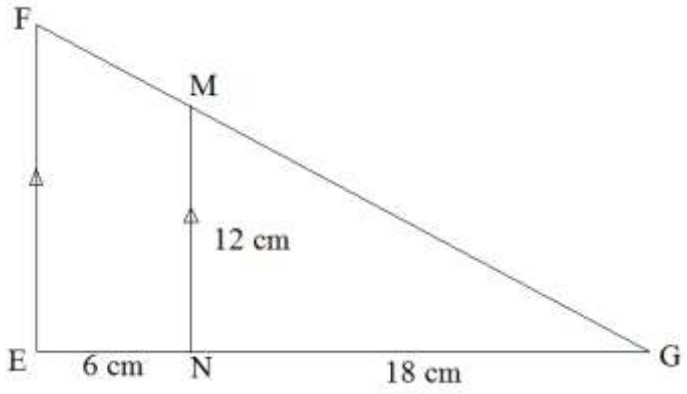


Extract 7.2: A sample of the student's responses to question 7

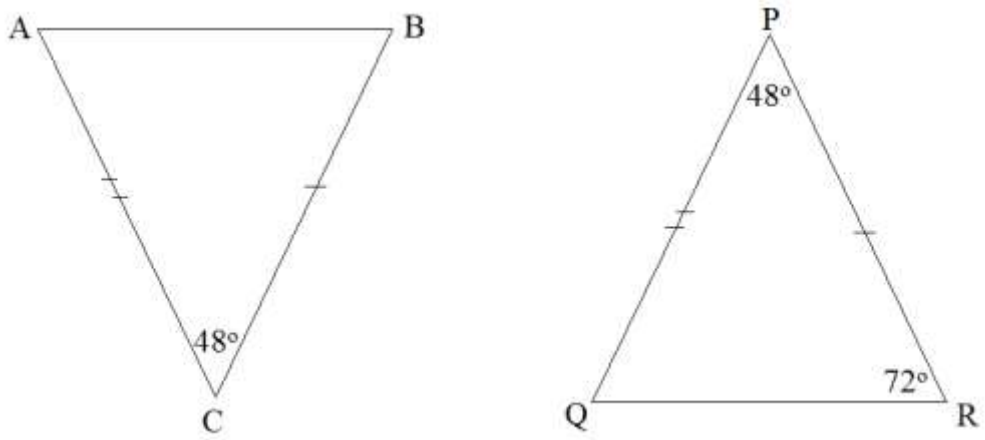
Extract 7.2 shows that in part (a)(i), the student multiplied the two quantities $P=\sqrt{2}-3$ and $Q=\sqrt{2}-1$, then performed basic mathematical operations on radicals to get the correct answer, that is $PQ=-1-2\sqrt{2}$. In part (a)(ii), the student substituted the two quantities P and Q in the expression $\frac{P}{Q}$. Then, he/she rationalized the denominator to get $5-4\sqrt{2}$. In part (b), the student squared both sides of the expression $p=\sqrt{q+x}$ to write x in terms of p and q, that is $x=p^2-q$. He/she substituted the values of p and q to get the correct answer.

2.8 Question 8: Similarity and Congruence

The question had two parts, (a) and (b). In part (a), the students were given the information that read "A young designer has designed a building whose side view structure is represented by the following figure".



Item (a)(i) intended to assess whether the students were able to state with reasons the pair of similar triangles. Item (a)(ii) required students to determine the length of the side $\overline{FE}$ in the given figure. In part (b), students were given the information that "Form Two students were challenged on the use of corresponding angles and sides to prove for the congruence of triangles. One student managed to draw the triangles of the same size and shape as follows".



Item (b)(i) assessed whether the students were able to state why the triangles, ABC and PQR are congruent. Item (b)(ii) required the students to calculate the value of angle RQP.

The analysis shows that a total of 634,775 (100%) students attempted the question and among them, 558,167 (87.9%) students scored 2.5 marks or

less. Furthermore, 414,014 (65.2%) students got zero marks on this question showing that the general performance was weak. However, there were 1,352 (0.2%) students who scored full marks. Figure 8 recaps the students' performance on question 8.

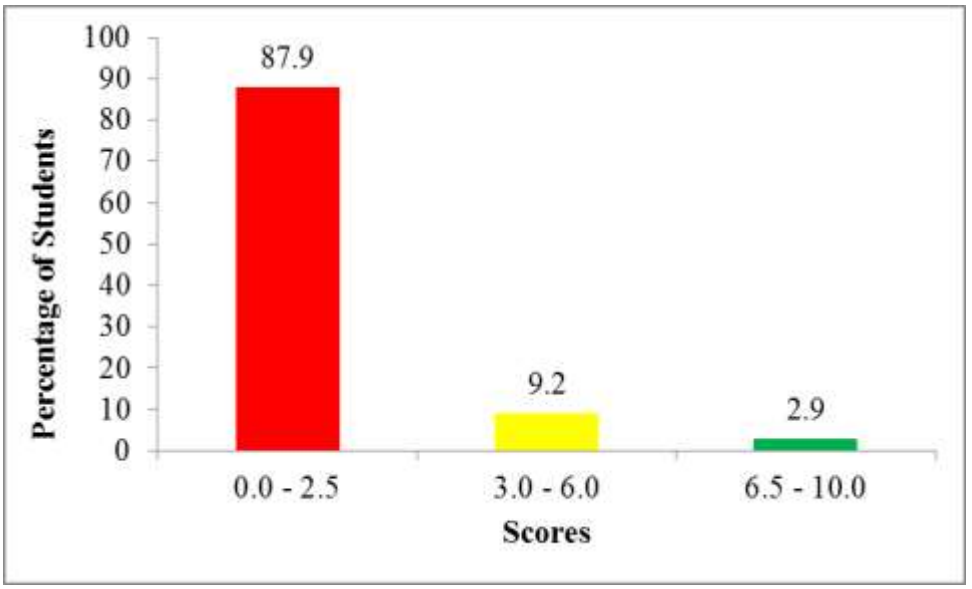
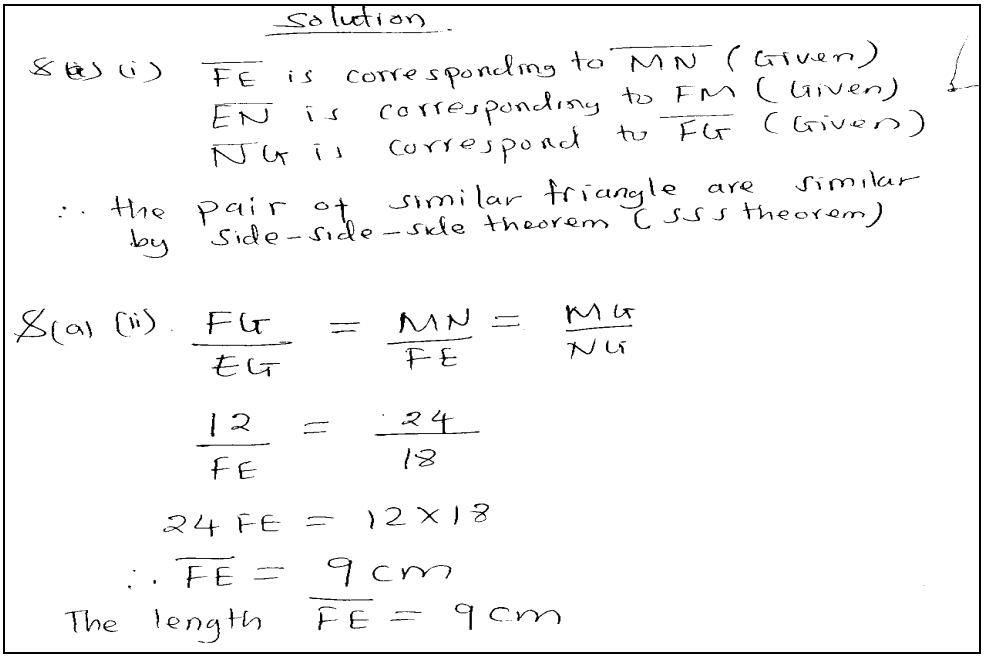


Figure 8: *Students' performance on question 8*

The analysis of students' responses indicates that the students who performed poorly on this question lacked knowledge and skills in the basic concepts of congruence and similarity. In part (a)(i), the students failed to recognize the necessary conditions for two triangles to be similar, thus they were unable to apply an appropriate similarity theorem. For example, some students wrongly used the Side-Side-Side theorem, Angle-Angle-Angle theorem and Angle-Side-Angle theorem. Additionally, other students did not recognize the common angle $\angle EGF$ of the ΔEGF and ΔMGN. In part (a)(ii), majority of the students applied incorrect relationships of sides of the two triangles. For instance, they wrote $\frac{\overline{FE}}{\overline{MN}}=\frac{\overline{EN}}{\overline{NG}}=\frac{\overline{FM}}{\overline{MG}}$. Other students calculated the area of the triangle instead of finding the length of side $\overline{FE}$. Also, there were students who just added the measurements given in the question, that is $6\,\text{cm}+18\,\text{cm}+12\,\text{cm}$ which was incorrect. In part (b)(i), students who scored zero were not able to state why the triangles

ABC and PQR were congruent. Most of those students applied wrong conditions or theorems to prove for the congruence of the two triangles. For example, they either used the Angle-Angle-Side theorem (AAS), or Side-Side-Side theorem (SSS), which were all incorrect. Also, there were students who used the concepts of similarity to prove that the triangles ABC and PQR were congruent. For instance, they wrote $\frac{\overline{AB}}{\overline{QR}}=\frac{\overline{AC}}{\overline{PQ}}=\frac{\overline{BC}}{\overline{PR}}$ which was wrong. Other students used wrong mathematical notations such as $\angle CB=\angle RP$ instead of $\overline{CB}=\overline{RP}$, $\angle AC=\angle PQ$ instead of $\overline{AC}=\overline{PQ}$ for the corresponding sides and $PQ\angle AC$ instead of $\angle ACB=\angle QPR$ for corresponding angles. This implies that the students were not able to apply the conditions of congruence of triangles. In part (b)(ii), the students were not able to recall the sum of interior angles of a triangle. For example, they summed as $48°+72°+\angle RQP=360°$ instead of $48°+72°+\angle RQP=180°$. Other students multiplied the given measured angles and divided the sum of interior angles of the triangle by the product. That is, $48°\times 72°\times x=180°$ that led to an incorrect answer. Extract 8.1 shows a sample of a response of a student who answered this question incorrectly.


Solution

8(b)(i) $\overline{FE}$ is corresponding to $\overline{MN}$ (Given)
$\overline{EN}$ is corresponding to $\overline{FM}$ (Given)
$\overline{NG}$ is correspond to $\overline{FG}$ (Given)

∴ the pair of similar triangle are similar by side-side-side theorem (SSS theorem)

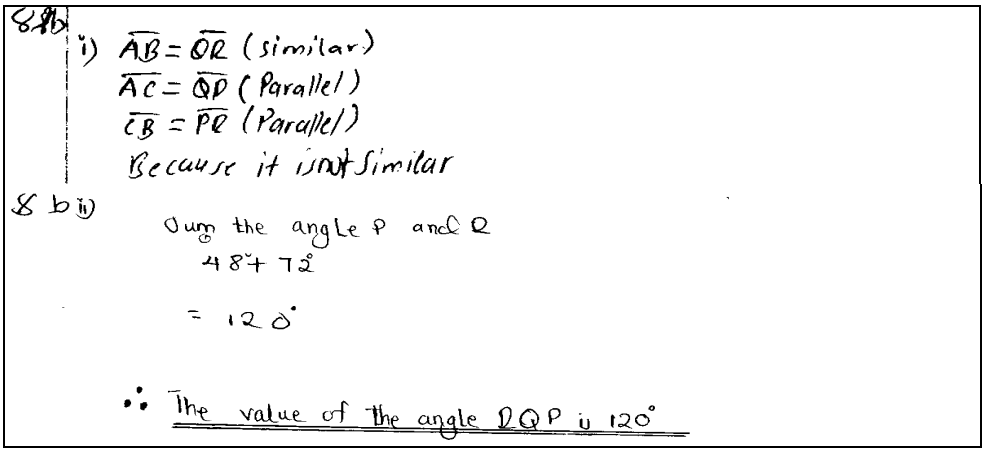
8(a)(ii) $\frac{FG}{EG}=\frac{MN}{FE}=\frac{MG}{NG}$

$\frac{12}{FE}=\frac{24}{18}$

$24\,FE=12\times 18$

$\therefore \overline{FE}=9\,cm$

The length $\overline{FE}=9\,cm$

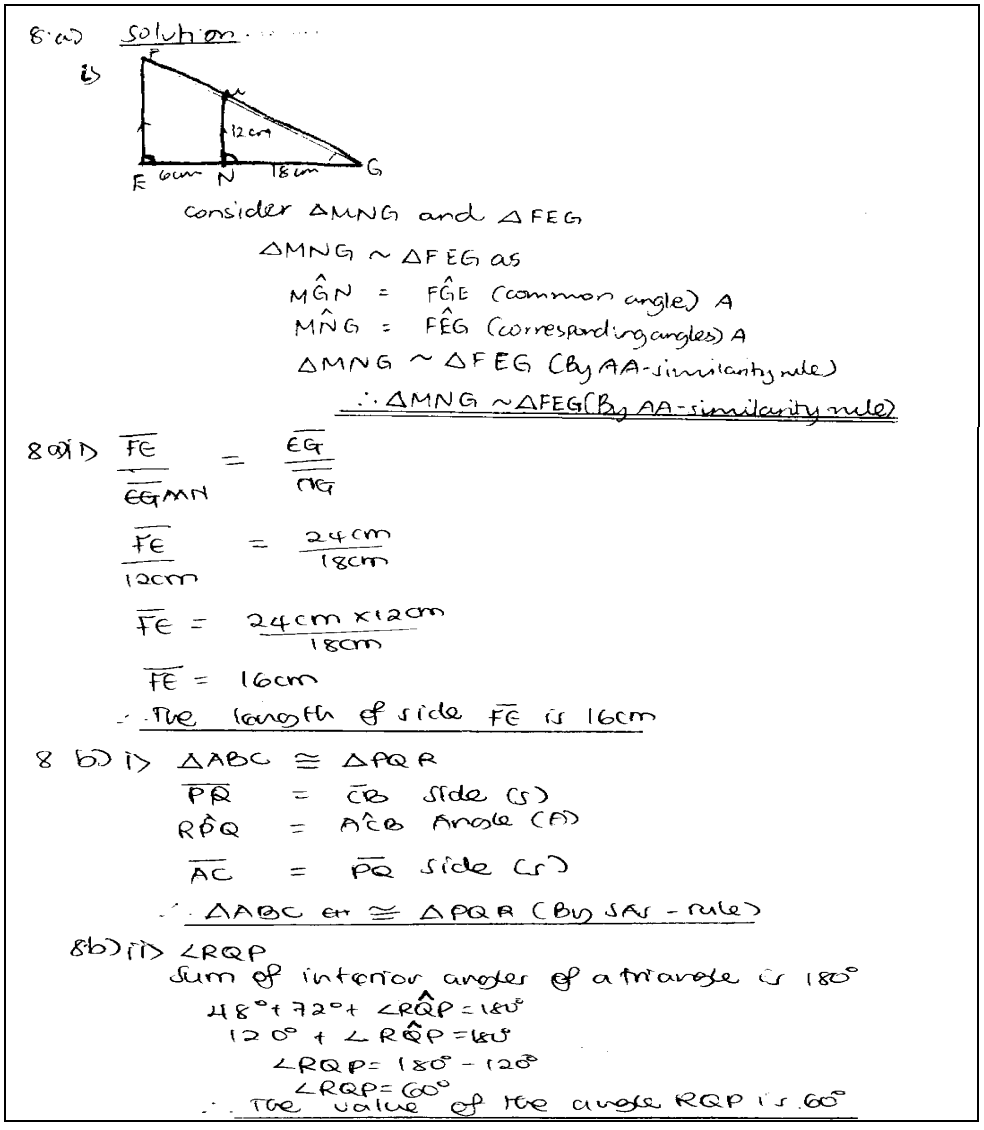


Extract 8.1: A sample of the student's responses to question 8

Extract 8.1 shows that in part (a)(i), the student used wrong conditions to prove that the two triangles were similar. In part (a)(ii), the student lacked competence in finding the length of $\overline{FE}$ by not recognizing the condition for ratios of corresponding sides of similar triangles. In part (b)(i), the student used wrong postulates to prove that the two triangles, ABC and PQR are congruent. In part (b)(ii), the student failed to find the value of angle RQP by using the fact that, the sum of interior angles of a triangle is $180°$.

Despite the weak performance of the students, the response analysis shows that students who scored full marks on this question applied the knowledge and skills gained after learning the concepts of similarity and congruence. In part (a)(i), the students were able to apply Angle-Angle Similarity theorem (AA Similarity theorem) to show the similarity of the triangles, GEF and GNM. They correctly concluded that $\Delta GEF \sim \Delta GNM$. In part (a)(ii), they were able to recognize the relationship between the sides of the two triangles, GEF and GNM. The students correctly used the fact that the ratios of the corresponding sides of similar triangles are equal, that is, $\frac{\overline{GE}}{\overline{GN}} = \frac{\overline{EF}}{\overline{NM}} = \frac{\overline{GF}}{\overline{GM}}$. They performed computations to obtain $\overline{FE} = 16\text{cm}$, which was a correct answer. In part (b)(i), the students responded based on the following arguments: $\overline{AC}$ corresponds to $\overline{QP}$, $\overline{BC}$ corresponds to $\overline{RP}$

and $\angle ACB = \angle QPR = 48°$. Therefore, they concluded that the triangle ABC was congruent to triangle PQR by Side-Angle-Side theorem. Furthermore, in part (b)(ii), the students were able to identify that the sum of interior angles of the triangle PQR is $180°$. That is, $48° + 72° + \angle RQP = 180°$. They computed the value of angle RQP and obtained $\angle RQP = 60°$ as required. Extract 8.2 shows a sample of a response of a student who answered this question correctly.



Extract 8.2: A sample of the student's responses to question 8

Extract 8.2 shows that, in part (a)(i), the student managed to state with reasons why two triangles are similar by using Angle-Angle theorem (AA theorem). In part (a)(ii), the student was able to find the length of $\overline{FE}$ by using the condition that, "the ratios of corresponding sides of similar triangles are equal". In part (b)(i), the student was able to state with reasons why two triangles, ABC and PQR are congruent and in part (b)(ii), the student was able to find the value of angle RQP.

2.9 Question 9: Pythagoras' Theorem and Trigonometry

This question consisted of two parts, (a) and (b). In part (a), the students were assessed if they could determine whether or not a photograph whose diagonal is $7.8\,\text{cm}$ can fit in a frame of length 6cm and width 5cm. In part (b), they were required to evaluate the length of a square tile whose diagonal is 8cm long and makes an angle $45°$ with the side of the tile.

The data indicate that 634,775 (100%) students attempted the question out of whom, 599,972 (94.5%) students scored below 3 marks and 557,517 (87.8%) scored zero. This confirms that the students' performance on this question was weak. The data also indicate that there were 4,064 (0.6%) students who scored full marks. Figure 9 presents a summary of the students' performance on question 9.

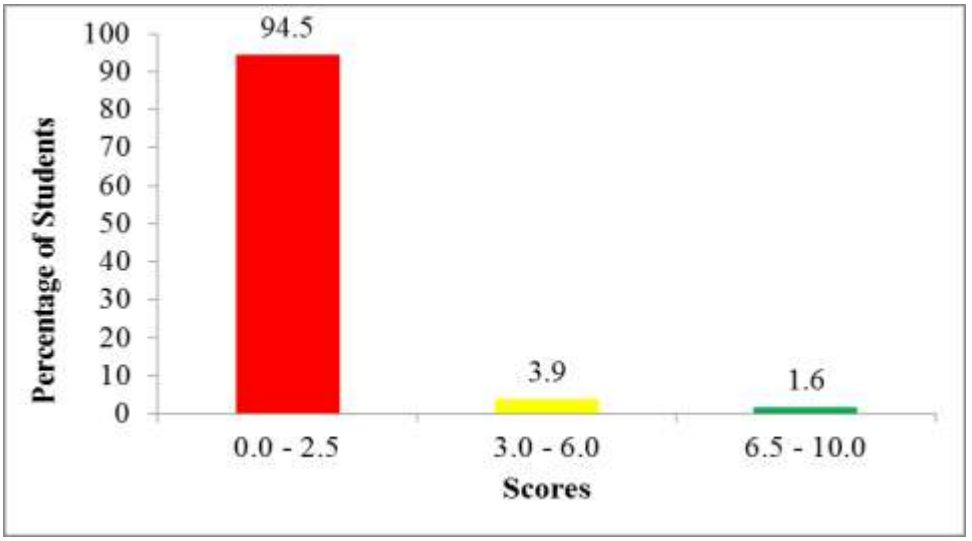
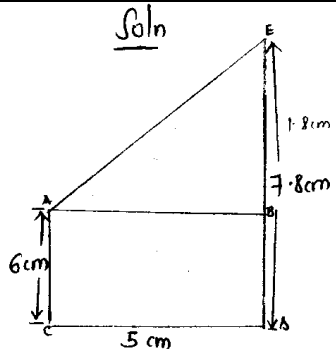


Figure 9: *Students' performance on question 9*

The response analysis shows that, the students failed to answer this question due to lack of competence in the concepts of Pythagoras' theorem and trigonometry. In part (a), most of the students failed to extract a right-angled triangle from a frame-like diagram. They then sketched using the given information. It was further noted that the students failed to demonstrate their knowledge and skills in applying the Pythagoras' theorem $a^2 + \mathrm{b}^2 = \mathrm{c}^2$ to help them get the correct length of the diagonal of the frame. For instance, some of the students applied the incorrect formula $a^2 - \mathrm{b}^2 = \mathrm{c}^2$. As a result, they got incorrect answers. Also, the students used various formulae such as $A = \frac{1}{2}bh$, $A = L \times W$, and $A = \frac{1}{2}(a+b) \times h$ which also produced incorrect answers. Further analysis shows that, some of the students were completely incompetent in interpreting and solving the real life problem related to the Pythagoras' theorem. They drew incorrect diagrams which were not relevant to the problem. For example, they drew rectangles, cuboids, non-right-angled triangles, parallelograms, and trapeziums. The analysis also shows that some of the students added the values given in the problem, that is, $7.8\,\mathrm{cm} + 6\,\mathrm{cm} + 5\,\mathrm{cm} = 18.8\,\mathrm{cm}$ while others calculated the L.C.M of 5, 6, and 7.8. Other students calculated the perimeter and area of the rectangle or the area of a trapezium. Although some students were competent in using the Pythagoras' theorem, they lacked the knowledge required to find square roots of numbers. In part (b), they failed to extract a right-angled triangle from the square-like diagram which represented a tile. The students lacked competence in drawing the right-angled triangle using the given information. They drew different triangles that could not give the correct answer. Moreover, some students used wrong trigonometric ratios such as $\tan 45°$ instead of $\sin 45°$ or $\cos 45°$. Those students lacked appropriate knowledge of applications of trigonometric ratios. Extract 9.1 provides a sample of a response of a student who answered this question incorrectly.

9.(a) Soln



EB = 7·8cm - 6cm

= 1·8cm

Area = Width x length

A = 6cm x 5cm

A = 30cm²

Area = $\frac{1}{2}bh$

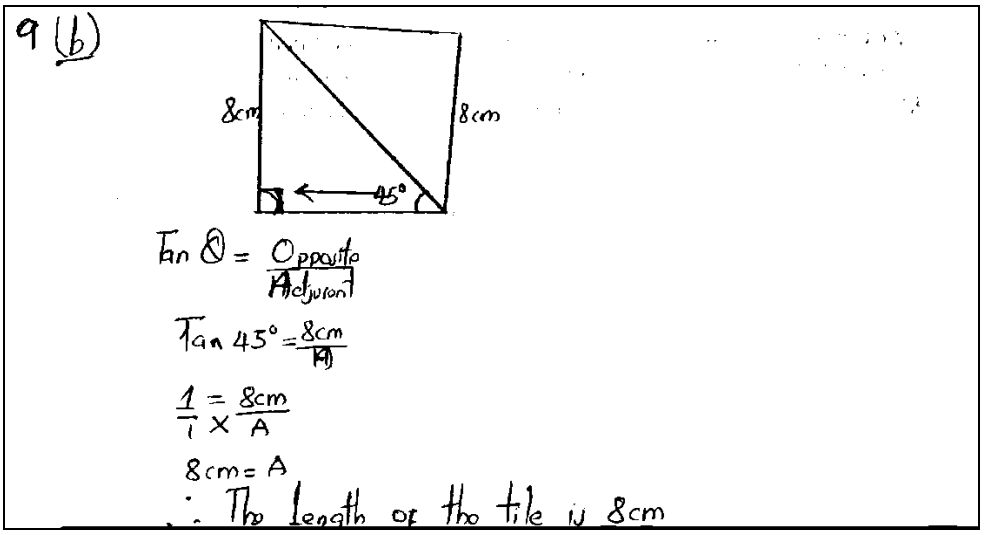
= $\frac{1}{2}$ x 5cm x 0·9cm

Area = 4·5cm²

Area of photograph is 4·5cm²

Area of the frame is 30cm²

∴ The photograph will fit of area 4·5cm² will fit in a frame of 30cm²



Extract 9.1: A sample of the student's responses to question 9

Extract 9.1 shows that, in part (a), the student drew a trapezium-like figure containing a rectangle and calculated its area. That is, $A = \text{Length} \times \text{Width}$ to get the area of the frame as $30\,\text{cm}^2$ and then calculated the area of a triangle $A = \frac{1}{2}bh$ to obtain the area of the photograph as $4.5\,\text{cm}^2$. Finally, the student compared the two areas to conclude that the photograph will fit in the frame.

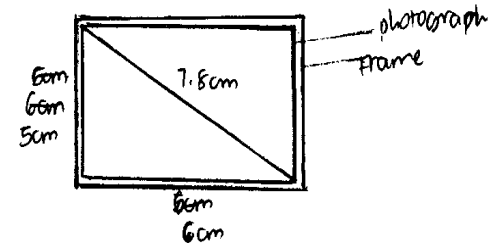
In part (b), the student labeled the diagram incorrectly and used the relation $\tan\theta = \frac{\text{Opposite}}{\text{Adjacent}}$. Lastly, he/she performed the calculations that produced incorrect length.

On the other hand, the analysis shows that the students who answered the question correctly were able to apply well the knowledge and skills gained after learning the concept of Pythagoras' theorem and trigonometry. In part (a), the students were able to demonstrate their competence by sketching a frame-like diagram with sides 6 cm by 5 cm and extracted a right-angled triangle from it. Thereafter, they applied the Pythagoras' theorem $a^2 + b^2 = c^2$. That is, $5^2 + 6^2 = c^2$ to obtain $c = 7.81\,\text{cm}$, which is the diagonal of the frame. Such students demonstrated their competences in

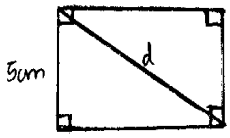
using mathematical tables to find the square root of 61. However, other students used the concept of antilogarithm to find the square root of 61. The condition for the photograph to fit in the frame is that, "the diagonal of the frame must be greater or equal to that of the photograph". The students who answered correctly were able to understand this condition. They concluded that the photograph will fit in the frame since the length of the diagonal $c = 7.81\,\text{cm}$ is approximately equal to that of the photograph. In part (b), the students who managed to answer the question correctly sketched the diagram representing a right-angled triangle extracted from a square tile with the diagonal which is 8 cm long as the hypotenuse of the triangle. They located the acute angle $45°$ formed by the diagonal and the side of the tile. Moreover, the students demonstrated their competence in applying trigonometric ratios. They used the relation $\sin\theta = \dfrac{\text{Opposite}}{\text{Hypotenuse}}$ or $\cos\theta = \dfrac{\text{Adjacent}}{\text{Hypotenuse}}$, where $\theta = 45°$. They realized that the length of the side of the tile is given by the opposite side or adjacent side to the angle $45°$. The students were competent enough to read $\sin 45° = 0.7071$ or $\cos 45° = 0.7071$ from trigonometric tables or extract it from the special angles correctly. That is, $\sin 45° = \dfrac{\sqrt{2}}{2}$ or $\cos 45° = \dfrac{\sqrt{2}}{2}$. They correctly calculated the length of the opposite or adjacent side to obtain $5.6568\,\text{cm}$. However, some of the students let x represent the side of the square tile and applied the Pythagoras' theorem, $x^2 + x^2 = 8^2$. They solved for the value of x to get the correct length of the side of the tile, that is, $x = 5.6568\ \text{cm}$. Extract 9.2 shows a sample of responses of a student who answered this question correctly.

Soln

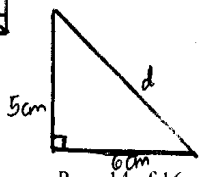
9 @



from the frame,



Let the diagonal be d



Let a = 5
b = 6
c = d

9@ from Pythagoras theorem,

$a^2 + b^2 = c^2$

$6^2 + 5^2 = c^2$

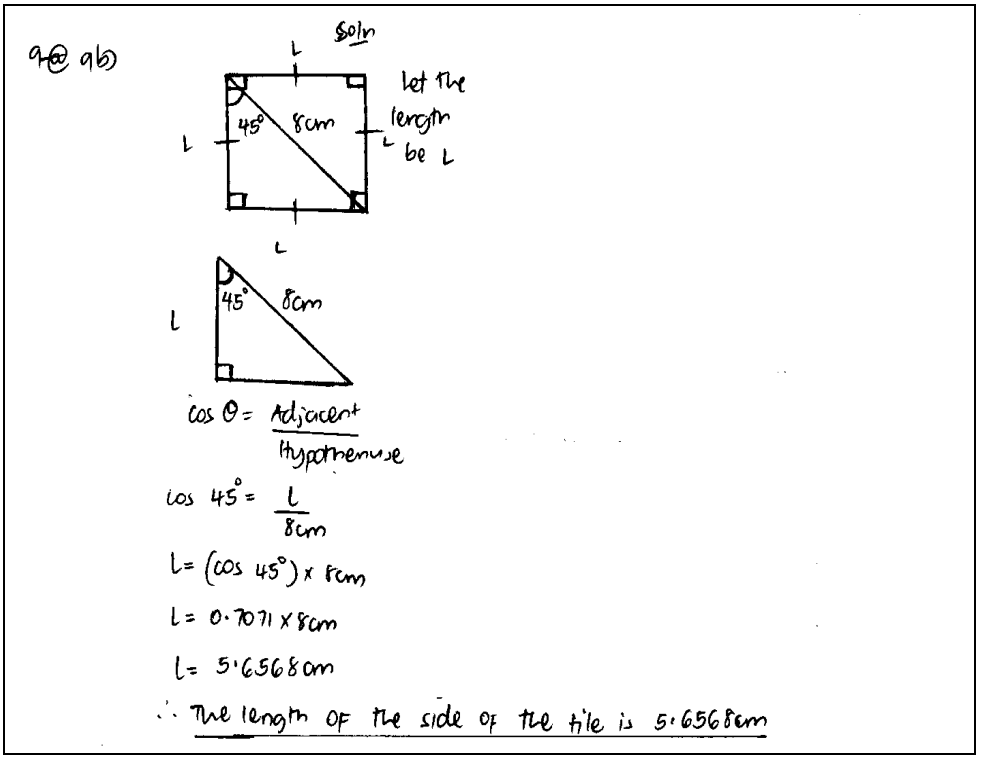
$c^2 = 36 + 25$

$c^2 = 61$

$c = \sqrt{61}$

$c = 7.810\text{cm} \approx 7.8\text{ cm}$

∴ The photograph will fit in the frame of 6cm by 5cm since the diagonals are equal.



Extract 9.2: A sample of the student's responses to question 9

Extract 9.2 shows that, in part (a), the student extracted well a right-angled triangle and applied the Pythagoras' theorem correctly to get the length of the diagonal of the frame. He/she made the comparison of the lengths of the diagonals of the frame and photograph. Finally, he/she concluded that the photograph will fit in the frame since the length of the diagonal of the frame is approximately equal to that of the photograph. In part (b), the student used trigonometric ratio of cosine correctly and obtained the required length of the side of the tile.

2.10 Question 10: Sets and Statistics

This question consisted of two parts, (a) and (b). Part (a) stated that "In a village of 1500 villagers, 600 keep goats, 700 keep cows, and 300 do not keep any of these animals". The students were assessed whether they could use a Venn diagram to find the number of villagers who keep: (i) both goats and cows, (ii) goats only, (iii) cows only, and (iv) goats or cows. Part (b)

stated that "The grades on a Mathematics test taken by 100 students are as shown in the following distribution table".

Marks	50 – 59	60 – 69	70 – 79	80 – 89	90 – 99
Number of students	3	21	32	27	17

The students were required to find: (i) the size of each class interval of the distribution, (ii) the class interval with the highest number of students, (iii) the class mark of the highest class interval, (iv) the number of students who passed if the pass mark was 70 , and (v) the number of students who failed the test by using the condition given in (b)(iv).

Analysis shows that out of 634,775 (100%) students who attempted the question 538,635 (84.9%) students scored 2.5 marks or less and 381,626 (60.1%) scored zero marks. Based on these scores, the students' performance on this question was weak. On the contrary, 3,706 (0.6%) students scored all the marks allotted to the question. Figure 10 recapitulates the students' performance on question 10.

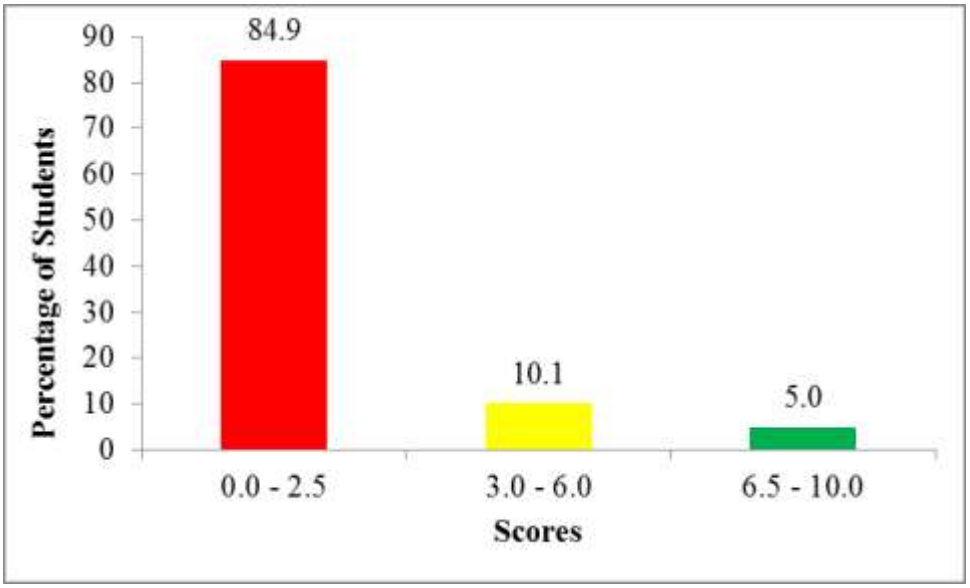
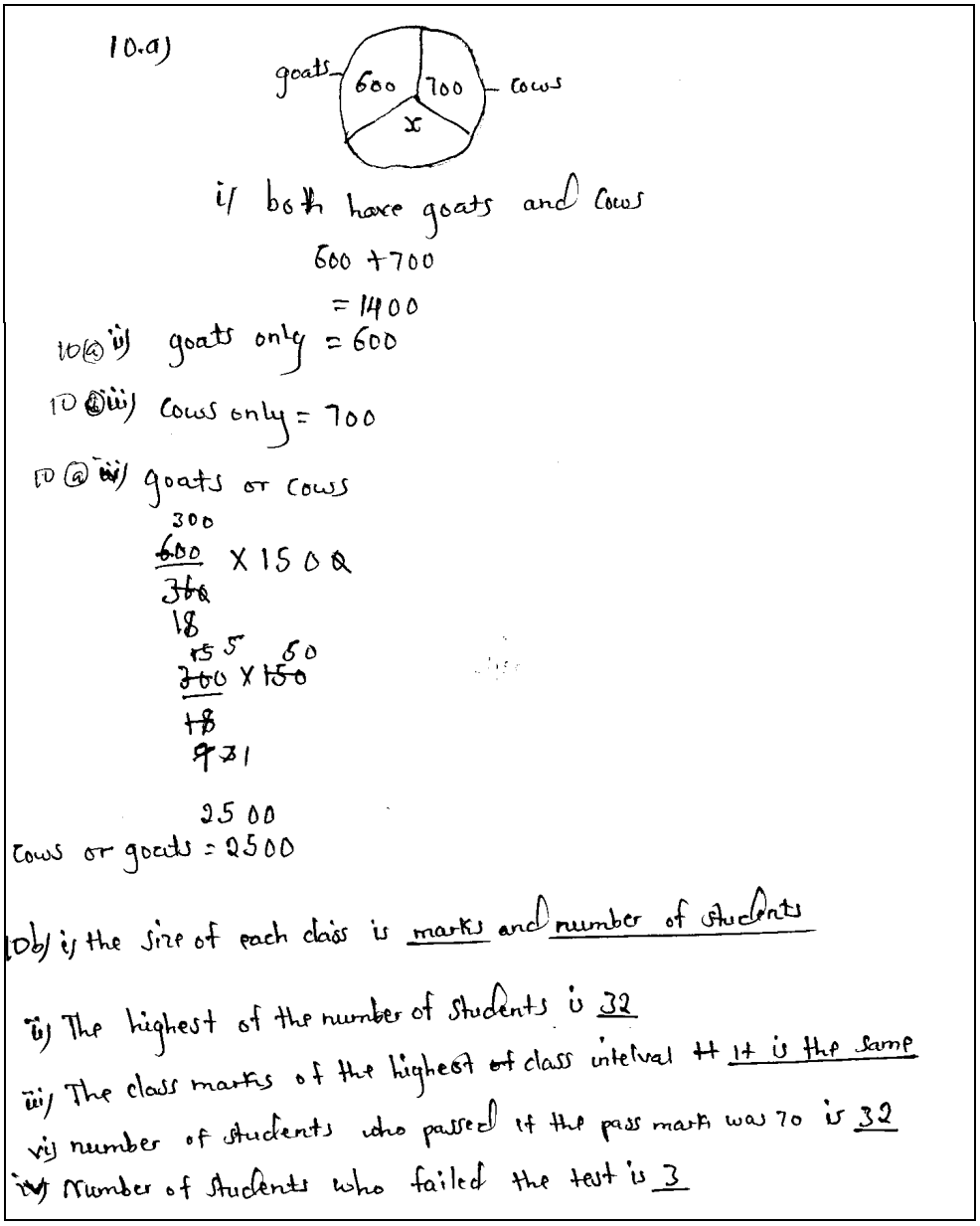


Figure 10: *Students' performance on question 10*

The analysis of the students' responses shows that, the students who performed poorly on this question lacked competence in the basic

principles of sets and statistics. In part (a), the students were not able to extract and present the given data in a Venn diagram. They either drew pie charts or circles of disjoint sets. Moreover, some of the students wrongly presented the data in a Venn diagram. They failed to subtract the number of elements in the intersection of the two sets to obtain the number of villagers who keep "goats only" and "cows only". Those students showed inadequate knowledge of the concept of sets as they were not able to recognize that "goats or cows" represents the union of the two sets. Most of the students just added the number of villagers who keep goats and cows that was given in the question. That is, $(600+700)$ villagers to obtain 1300 villagers, which is incorrect. In part (b)(i), the majority of students confused the concept of class interval and class size. They were not competent enough to make interpretation of the given data and use statistical information correctly. They wrote class interval instead of finding the class size. Also, the students lacked skills in determining the class size of grouped data. They calculated the class size wrongly and obtained 9 instead of 10. That is, $59-50=9,\ 69-60=9,\ 79-70=9,\ 89-80=9$ or $99-90=9$ instead of $59.5-49.5=10,\ 69.5-59.5=10,\ 79.5-69.5=10,\ 89.5-79.5=10,$ or $99.5-89.5=10$. In part (b)(ii), the students failed to identify the class interval with the highest frequency. They wrote 32 instead of $70\text{-}79$. In part (b)(iii), the students lacked enough skills on how to compute the class marks of grouped data. They considered the upper and lower class limits as class marks. For instance, $89.5\text{-}99.5$ instead of $\frac{90+99}{2}=94.5$. In part (b)(iv), they failed to use the given condition of pass mark to determine the number of students who passed the test. This weakness was also observed in part (b)(v) where they were not able to find the number of students who failed the test. Majority of the students presented the highest frequency of 32 and the lowest frequency of 3 as the number of students passed and those who failed the test, respectively, all of which were incorrect answers. The correct answer for the number of students who passed the test was the sum of the frequencies of the classes starting from $70-79$ and above. That is, $32+27+17=76$ students. The number of students who failed the test was the sum of the frequencies of the classes below $70-79$. That is,

$3+21=24$ students. Extract 10.1 provides a sample of a response of a student who answered the question incorrectly.

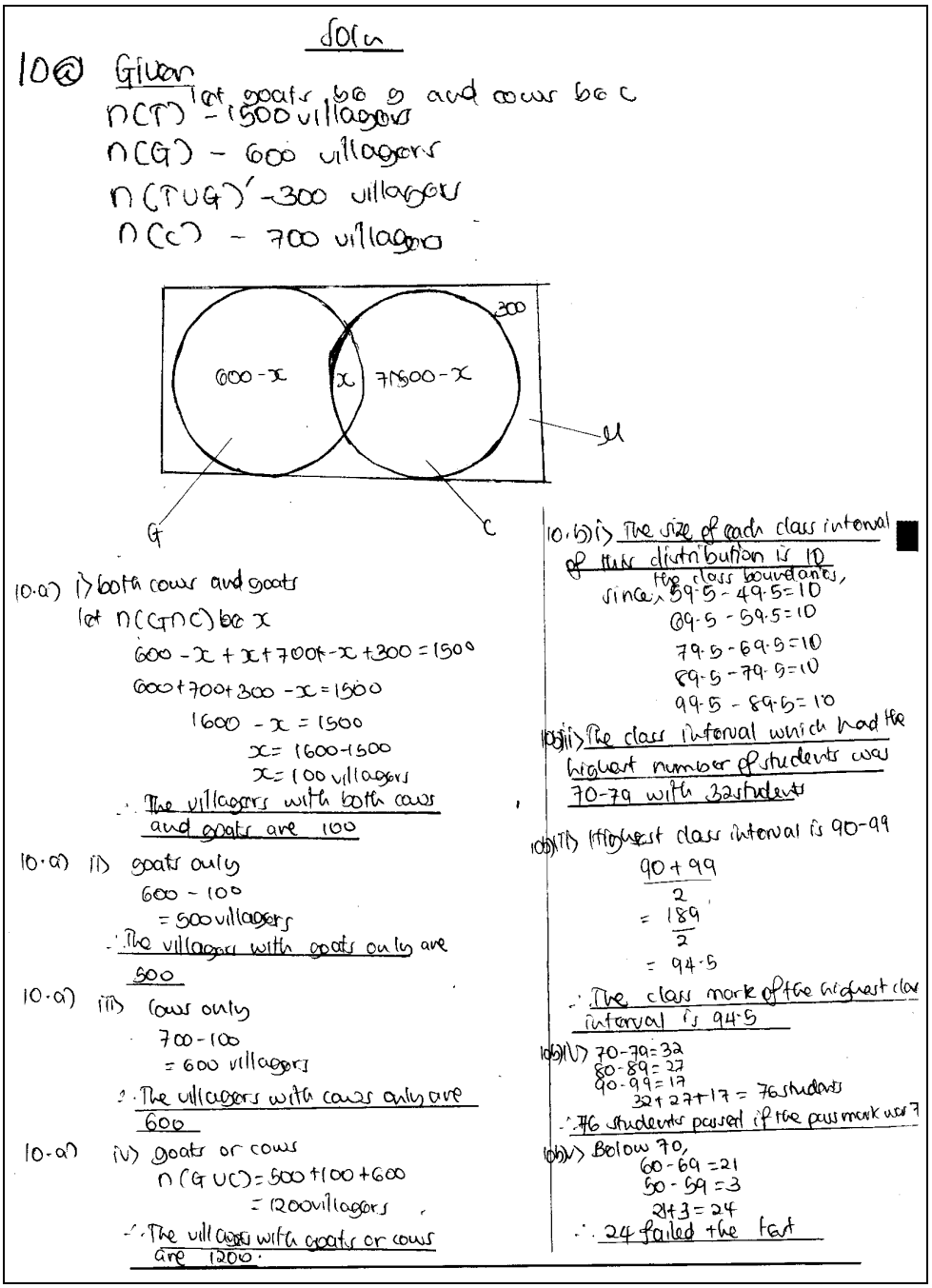


Extract 10.1: A sample of the student's responses to question 10

Extract 10.1 shows that in part (a), the student drew a pie chart-like figure instead of a Venn diagram and failed to present meaningful information. The student performed incorrect calculations because he/she lacked basic

principles of sets. In part (b), the student was not able to make interpretation of statistical concepts. This indicated that the student lacked knowledge and skills for all parts of the question.

Despite the poor performance, the analysis shows that there were students who performed well on this question. The students were competent enough on the basic principles of sets and statistics. In part (a), the students demonstrated adequate skills in representing the given data in a Venn diagram. They used the Venn diagram to obtain the intersection of the two sets. The intersection of the sets of goats and cows in a Venn diagram represented the correct answer to item (a)(i). Thus, they obtained the correct answer. That is, the number of villagers who keep both goats and cows is 100. In parts (a)(ii) and (a)(iii), the students demonstrated a good understanding of the use of the word "only" to get the number of villagers who keep goats only and cows only as $600-100=500$ and $700-100=600$, respectively. In part (a)(iv), the students were able to understand the meaning of the word "or" as used in the set theory. They correctly used for the union of two sets to find the number of villagers who keep goats or cows. In part (b), they interpreted correctly the frequency distribution table and determined the class size, class interval, class mark, highest frequency, number of students who passed, and the number of students who failed the test. Extract 10.2 presents a sample of a response of a student who answered the question correctly.



Extract 10.2: A sample of the student's responses to question 10

Extract 10.2 shows that, in part (a), the student constructed correctly a Venn diagram and used it effectively to identify the number of students who keep both goats and cows, goats only, cows only, and goats or cows by

using the correct formulae of sets. In part (b), the student managed to apply statistical skills to make clear interpretations of the given concepts from the frequency distribution table.

3.0 ANALYSIS OF STUDENTS' PERFORMANCE ON EACH TOPIC

The Basic Mathematics assessment paper consisted of 10 questions that were set from 18 topics. The analysis shows that only two (2) topics of *Units* and *Approximations* were rated good as 65.15 per cent of the students passed. The other remaining topics were poorly performed. The topics include: *Numbers* and *Decimals and Percentages* (17.44%), *Geometry* and *Perimeters and Areas* (15.74%), *Sets* and *Statistics* (15.15%), *Congruence* and *Similarity* (12.07%), *Ratios, Profit and Loss* (12.00%), *Exponents and Radicals* (10.17%), *Coordinate Geometry* and *Geometrical transformations* (7.17%), *Algebra* and *Quadratic Equations* (6.30%), and *Pythagoras theorem* and *Trigonometry* (5.48%). The Appendix attached with this report summarizes the students' performance on each topic.

4.0 CONCLUSION

The analysis shows that 16.56 per cent of the students passed the assessment in FTNA 2022 in the Basic Mathematics subject compared to 19.52 per cent of the students who passed the assessment in FTNA 2021. From the analytical point of view, the students performed well only in the topics of *Units* and *Approximations* by 65.15 per cent in 2022 compared to that of 2021 in which the performance was weak (27.65%). The remaining topics had constantly been poorly performed in both 2021 and 2022.

5.0 RECOMMENDATIONS

The analysis of the performance of 2022 Form Two National Assessment showed that the performance was not good as 16 out of 18 topics had weak performance. Based on the findings, the following recommendations are provided for future improvements:

(a) Teachers should use examples of real objects related to right angled triangles to effectively teach students the mathematical relationship between the hypotenuse and the perpendicular lengths of a right

angled triangle and solve word problems related to right angled triangles using the Pythagorean Theorem.

(b) Teachers should instruct students step by step on how to formulate trigonometric ratios and calculate sine, cosine and tangent of right angled triangles without using mathematical tables and calculators in order to enhance the intended competence in the topic on Trigonometry.

(c) Teachers should demonstrate to students the methods and steps to be followed in solving simultaneous equations in the topic on Algebra.

(d) Teachers should guide students through discussion on how to solve word problems related to the topic on Quadratic Equations by applying various methods such as factorization, completing the square, general formula and graph.

(e) In the topic on Coordinate Geometry, teachers should lead students' discussion in groups on how to determine the unknown coordinates, given the other points and the equation that satisfies the points by using the teaching and learning aids such as pencil, graph paper and ruler.

(f) Teachers should use various examples related to shapes of real objects to teach students the concepts of arc, chord, sector and radius and how to present those concepts in a circle. Also, they should fully involve students in drawing a circle that is inscribed in a square and determining the difference in their areas using the appropriate formulae in the topics on Geometry and Perimeters and Areas.

(g) In the topic on Exponents and Radicals, teachers should guide students through discussion on how to simplify radicals by using the basic operation on exponents, number charts and laws of exponents.

(h) In the topic on Ratios, Profit and Loss, teachers should instruct students on how to calculate the simple interest and the total amount of money accumulated after a certain period of time by using formulae and real objects available in their environment such as money and worksheets.

(i) In the topics on Congruence and Similarity, teachers should fully involve students in discussing the properties of congruent triangles and similar figures using various teaching and learning aids such as shapes of real objects, pictures and geometric drawings. They should also demonstrate to students how to use theorems to prove arguments and calculate angles and lengths using various questions from those topics.

(j) In the topic on Sets, teachers should guide students through discussing the main operations on sets such as set complement, union and intersection of two sets. This includes demonstrating to students step by step how to use the formulae and Venn diagrams to calculate the number of elements basing in the concepts of complement, union and intersection of sets.

(k) In the topic on Statistics, teachers should instruct students on how to present statistical information by using the frequency distribution tables, assess their ability to interpret the information presented in the frequency distribution and assist them accordingly.

(l) In the topic on Decimals and Percentages, teachers should instruct and then assess students in solving word problems involving decimals and percentages that represent real-life events, including all the necessary steps to be taken to convert repeating decimals into simple fractions.

APPENDIX: Analysis of Students' Performance per Topic – FTNA 2022

S/N	Topics	Question number	Percentage of students who scored 30 per cent or more	Remarks
1	Units and Approximations	2	65.15	Good
2	Numbers, and Decimals and Percentages	1	17.44	Weak
3	Geometry, and Perimeters and Areas	3	15.74	Weak
4	Sets and Statistics	10	15.15	Weak
5	Congruence and Similarity	8	12.07	Weak
6	Ratios, Profit and Loss	5	12.00	Weak
7	Exponents and Radicals	7	10.17	Weak
8	Coordinate Geometry and Geometrical transformations	6	7.17	Weak
9	Algebra and Quadratic Equations	4	6.30	Weak
10	Pythagoras theorem and Trigonometry	9	5.48	Weak