



THE UNITED REPUBLIC OF TANZANIA  
MINISTRY OF EDUCATION, SCIENCE AND TECHNOLOGY  
NATIONAL EXAMINATIONS COUNCIL OF TANZANIA



**STUDENTS' ITEM RESPONSE ANALYSIS REPORT  
ON THE FORM TWO NATIONAL ASSESSMENT  
(FTNA) 2022**

**ADDITIONAL MATHEMATICS**



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**042 ADDITIONAL MATHEMATICS**

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## FOREWORD

This report presents Students' Items Response Analysis (SIRA) on Form Two Additional Mathematics National Assessment which was conducted in November 2022. The report aims to provide feedback to all educational stakeholders on the factors that contributed to the students' performance in Additional Mathematics.

The Form Two National Assessment (FTNA) is a formative evaluation which intends to monitor students' learning to provide feedback that teachers, students and other educational stakeholders can use to improve teaching and learning. This analysis shows justification for the students' performance in the Additional Mathematics subject. The students who scored high marks had adequate knowledge and skills of the tested concepts, correct interpretation of word problems, computation skills, understanding of the questions asked, and the ability to recall the appropriate formulae. However, students who scored low marks faced difficulties in responding to the questions due to insufficient knowledge of the tested concepts.

This report will help students to identify strengths and weaknesses for them to improve learning before sitting for Certificate of Secondary Education Examination (CSEE). It will help teachers to identify the challenging areas and take appropriate measures during teaching and learning.

The National Examinations Council of Tanzania (NECTA) expects that the feedback provided in this report will highlight the challenges which education stakeholders should take proper measures to improve teaching and learning the Additional Mathematics subject. Consequently, students will acquire knowledge, skills and competences indicated in the syllabus for better performance in future assessments and examinations.

The Council appreciates the contribution of all those who prepared this report.



Dr. Said Ally Mohamed  
**EXECUTIVE SECRETARY**

## 1.0 INTRODUCTION

The Students' Item Response Analysis (SIRA) report in Additional Mathematics subject has been written to provide feedback to stakeholders on the students' performance in the Form Two National Assessment (FTNA) 2022. The assessment paper was set based on Form Two National Assessment Format of 2019 which is based on the Additional Mathematics syllabus of 2010 for secondary schools. The paper comprised 10 compulsory questions; each carrying 10 marks.

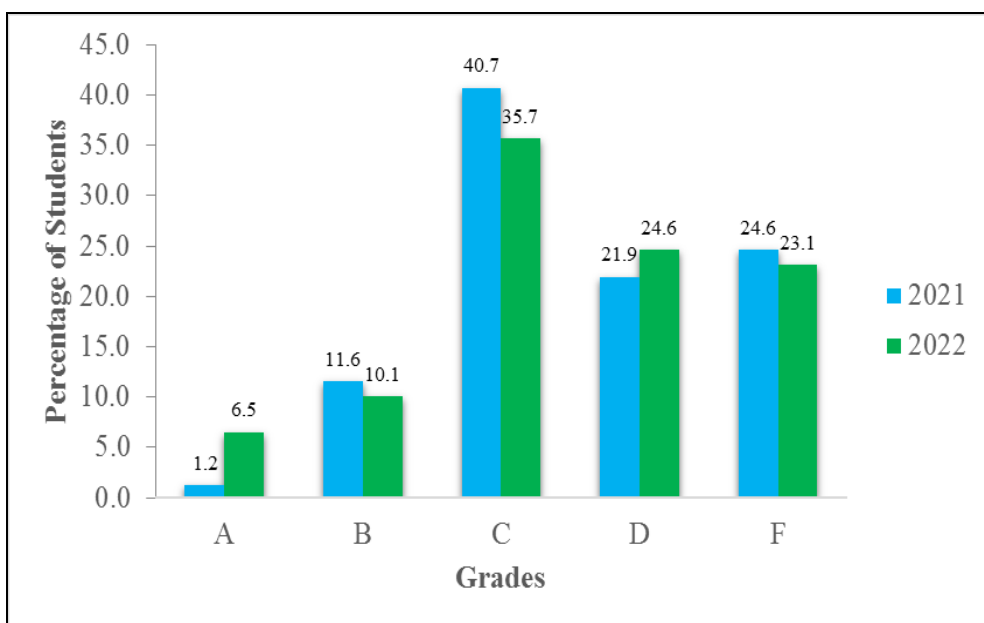
In 2022, a total of 398 students sat for the Additional Mathematics Assessment. When compared to the FTNA 2021 results, the statistics shows a decrease in the number of students who sat for this assessment by 17.8 per cent and an increase in performance by 1.47 per cent compared to the performance of 2021. Table 1 presents a summary of the students' performance in 2021 and 2022.

**Table 1: The Students' Performance in Additional Mathematics (FTNA) 2021 and 2022**

Year	Code	Subject Name	Students Sat	Passed		Grades				
				No.	%	A	B	C	D	F
2021	042	Additional Mathematics	484	365	75.41	6	56	197	106	119
2022			398	306	76.88	26	40	142	98	92

Table 1 reveals that the students' performance in Additional Mathematics in 2022 was good as 76.88 per cent who sat for assessment passed. Comparatively in 2021, a total of 484 students sat for Additional Mathematics. Out of whom, 365(75.41%) passed the assessment.

The percentages of students who passed the assessment in Additional Mathematics in different grades are shown in Figure 1.



**Figure 1:** *Performance of Students in Different Grades*

Observation of data in Figure 1 shows that the performance in Additional Mathematics in 2021 was low compared to that of the year 2022, since the percentages of students who passed the assessment were high. Meanwhile in 2022, the percentage of the students who got grade A is 6.5 while 23.1 per cent got grade F compared to 2021 where 1.2 per cent got grade A and 24.6 per cent got grade F.

The analysis of students' performance on each question is presented in section 2.0. The section includes a brief explanation of the questions' demand as well as an analysis of the students' responses. Extract from the scripts of the students for both well and poorly performed questions are inserted to illustrate cases presented. Factors that contributed to performance on each question are also illustrated. Teachers and students can use this analysis to improve their teaching and learning and eventually improve students' performance in future assessments.

The analysis of the students' performance on each topic assessed is presented in section 3.0. In the analysis green, yellow and red colours are used to show good, average and weak performance respectively. At the end of this report, recommendations are made to assist students, teachers, and the government in improving students' future performance.

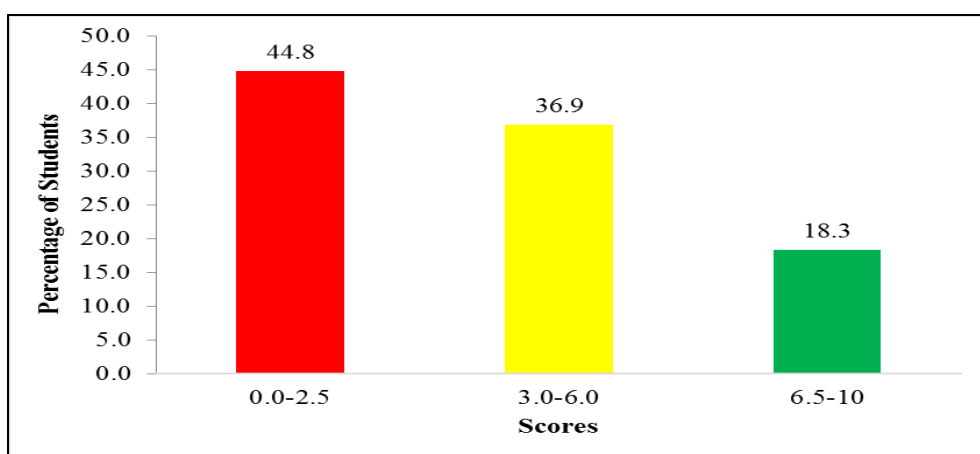
## 2.0 ANALYSIS OF THE STUDENTS' PERFORMANCE ON EACH QUESTION

This section provides the analysis of the students' performance on each question. The national assessment results are based on the score interval of 75 – 100, 65 – 74, 45 – 64, 30 – 44 and 0 – 29 to mean excellent, very good, good, satisfactory and fail respectively. The percentage of performance on each question is divided into three categories, which are weak performance (0 – 29 per cent), average performance (30 – 64 per cent) and good performance (65 – 100 per cent).

### 2.1 Question 1: Numbers

The question consisted of parts (a) and (b). In part (a), the students were asked to determine the first 5 prime numbers of the Fibonacci sequence. While in part (b), they were asked in (i) to find the next four numbers in the sequence 1, 4, 10, 22, ..., ..., ..., ... and in (ii) to use the divisibility rule to determine whether 9142 is divisible or not by 5, 6, 7 and 9.

The analysis of data shows that out of 398 (100%) students who attempted this question, 178 (44.8%) scored 0 to 2.5 marks, 147 (36.9%) students scored 3 to 6 marks and 73 (18.3%) students scored 6.5 to 10 marks. Generally, the data analysis shows that students had average performance on this question. The performance summary is presented in Figure 2.



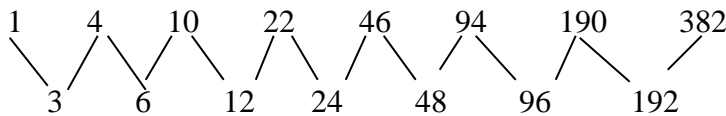
**Figure 2:** *Students' Performance on Question 1*

The data analysis shows that some students were able to respond to the question in accordance with the instructions. In part (a), the students were



able to apply their knowledge of Fibonacci sequence by listing numbers such as, 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, ... As a result, 2, 3, 5, 13 and 89 were correctly identified.

In part (b) (i), the students demonstrated good application of their knowledge of numbers. They were able to recognize that each term is obtained by multiplying the proceeding term by 2 then adding 2 to the answer. That is,  $4 = 1 \times 2 + 2$ ,  $10 = 4 \times 2 + 2$ ,  $22 = 10 \times 2 + 2$ . So, they worked on the fifth, sixth, seventh and eighth terms to get 46, 94, 190 and 382 respectively. Alternatively, other students used the tree diagram approach such as the following;



Then, from the tree diagram they obtained the next four numbers required which were 46, 94, 190 and 382.

In part (b) (ii), the students were able to state and apply correctly the divisibility rule of numbers. In order for a number to be divisible by 5, the last digit should be either 0 or 5. The students applied correctly the rule to conclude that the number 9142 is not divisible by 5. Furthermore, for a number to be divisible by 6 or 9, the sum of the digits must be divisible by 3 or 9 respectively. So they worked on the sum of the digits of 9142 as follows;  $9 + 1 + 4 + 2 = 16$  which is not divisible by 3 or 9. Therefore, the number 9142 is neither divisible by 6 nor by 9.

Likewise, for a number to be divisible by 7, the difference between the remaining digits and 2 times the last digit should be divisible by 7, that is

$$9142 \rightarrow 914 - (2 \times 2) = 910$$

$$910 \rightarrow 91 - (2 \times 0) = 91$$

$$91 \rightarrow 9 - (2 \times 1) = 7$$

Therefore, the number 9142 is divisible by 7.

Extract 1.1 is a sample of a response from one of the students who responded correctly to this question.

1. (a) Determine the first 5 prime numbers of the Fibonacci sequence.

*Solution.*  
 5 prime numbers  
 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144.  
 $= 2, 3, 5, 13$  and 89.  
 $\therefore$  the first 5 prime numbers of the fibonacci sequence  $= 2, 3, 5, 13$  and 89

- (b) (i) Find the next four numbers in the following sequence:

1, 4, 10, 22, ..., ..., ..., ...

*Solution*

1, 4, 10, 22, 46, 94, 190, 382  
 3 6 12 24 48 96 192

$\therefore$  the next four numbers in the Sequence  
 1, 4, 10, 22 are 46, 94, 190 and 382.

- (ii) Use divisibility rule to determine whether 9142 is divisible by 5, 6, 7 and 9 or not.

*Solution*  
 9142  
 Since the last digit is not zero or 5 then 9142 is not divisible by 5

9142  
 $9+1+4+2=16$   
 $=16/3$

Since the sum total of 9142 is not divisible by 3 then number 9142 is not divisible by 6.

9142  $2 \times 2 = 4$   
 9142  
 $\frac{9142}{910} \quad 2 \times 0 = 0$   
 910  
 $\frac{910}{91} \quad 2 \times 1 = 2$   
 91  
 $\frac{91}{9} \quad 2 \times 1 = 2$   
 9  
 $\frac{9}{2} \quad 2 \times 1 = 2$   
 2  
 $\frac{2}{1} \quad 2 \times 1 = 2$   
 1

$\therefore$  The number 9142 is divisible by 7 but is not divisible by 5, 6 and 9

**Extract 1.1:** A sample of the student's responses to question 1

In Extract 1.1, part (a), the student managed to identify correctly the property of Fibonacci sequence and then determined the first 5 prime numbers required. In part (b), the student had sufficient knowledge on divisibility rule of numbers and applied it to identify whether the given number is divisible by 5, 6, 7 and 9.

On the other hand, the analysis of data revealed that 32 (8.0%) students scored zero as they failed to answer this question correctly due to various challenges including the following:

In part (a), some students perceived the work of the Fibonacci sequence of numbers by responding to the question using the technique of squaring prime numbers such as  $1^2 = 1$ ,  $3^2 = 9$ ,  $5^2 = 25$ ,  $7^2 = 49$ ,  $9^2 = 81$ . Later they incorrectly recommended that the required numbers of the Fibonacci sequence were 1, 9, 25, 49 and 81 instead of 2, 3, 5, 13 and 89.

Furthermore, the analysis shows that other students responded to the question by listing incorrect prime numbers 1, 3, 5, 7, 11, 13, 15 ... Thereafter, the first 5 prime numbers identified were 1, 3, 5, 7 and 9. Not only that, there were other students who perceived the Fibonacci sequence as Pascal's triangle, and so they attempted the question by applying Pascal's triangle, which is contrary to the question asked. After applying Pascal's triangle, they concluded wrongly that the first 5 prime numbers were 1, 2, 3, 5, and 7 instead of 2, 3, 5, 13 and 89. These responses were due to lack of knowledge of Fibonacci sequence.

In part (b) (i), a considerable number of students responded wrongly to the question, due to their insufficient knowledge of numbers. The majority applied inappropriate formula  $n^2 + 2$  and substituted the values of  $n$  with 5, 6, 7 and 8, which resulted to 27, 38, 51, 66 and 83, which were wrong. If they could apply  $2n + 2$ , where  $n$  is the number before the preceding term, they could end up with the correct terms as 46, 94, 190 and 382. Likewise, there were students who applied tree method as their approach, but due to insufficient knowledge of the method, they ended up with incorrect responses which were 43, 76, 124 and 290.

In part (b) (ii), some students were not familiar with divisibility rule of numbers. So, they applied the normal method to the division of numbers to check whether the number 9142 is divisible by 5, 6, 7 and 9. For instance,  $9142 \div 5 = 1828.4$ ,  $9142 \div 6 = 1523.67$ ,  $9142 \div 7 = 1306$  and

$9142 \div 9 = 1015.78$ . The approach was not related to the requirements of the question that was about the divisibility rule.

Some students responded to the question with wrong concepts. They applied techniques of finding LCM of the given numbers. For example,

2	5	6	7	9
3	5	3	7	9
3	5	1	7	3
5	5	1	7	1
7	1	1	7	1
	1	1	1	1

Then, they found  $\text{LCM} = 2 \times 9 \times 5 \times 7 = 630$ . Thereafter, they divided 9142 by 630 and got 14.5. Lastly they concluded that 9142 is not divisible by 5, 6, 7 and 9 because 9142 is not divisible by LCM. All this was due to their inability to understand the question or that was due to their failure to recall the divisibility rule of the given numbers. Extract 1.2 is sample response from one of the students who failed to attempt the question correctly.

1. (a) Determine the first 5 prime numbers of the Fibonacci sequence.

$$\begin{aligned} 1+2 &= 4 \\ 1+3+5 &= 9 \end{aligned}$$

$$\begin{aligned} \text{Ans. } \frac{n(n+1)}{2} - & \begin{aligned} 1^2 &= 1 \\ 3^2 &= 9 \\ 5^2 &= 25 \\ 7^2 &= 49 \\ 9^2 &= 81 \end{aligned} \end{aligned}$$

Fibonacci sequence first 5 is 1, 1, 2, 3, 5

- (b) (i) Find the next four numbers in the following sequence:

1, 4, 10, 22, ..., ..., ..., ...

Solution

1, 4, 10, 22

1, 4, 10, 22, 36, 84, 180, 372

- (ii) Use divisibility rule to determine whether 9142 is divisible by 5, 6, 7 and 9 or not.

Solution

9142 by rules find L.C.M

2 | 5, 6, 7, 9  
3 | 5, 3, 7, 9  
3 | 5, 1, 7, 3  
5 | 5, 1, 7, 1  
7 | 1, 1, 7, 1  
1 | 1, 1, 1, 1

$$\text{L.C.M} = 2 \times 9 \times 35$$

$$18 \times 35 = 630$$

The number 9142 is not divisible by 5, 6, 7 and 9, because their L.C.M is not multiple of 9142

**Extract 1.2:** A sample of the student's responses to question 1

In Extract 1.2, part (a), the student faced difficulties in identifying the correct property governing the Fibonacci sequence. Instead the student determined the square of the 5 numbers starting with 1 and ending with 9 which also are not prime numbers. In part (b) (i), the student faced

difficulties in completing the pattern of numbers for the given sequence. Hence, he/she failed to get the next four numbers. While in part (b) (ii), the student used an incorrect approach to test the divisibility of 9142 by the given numbers.

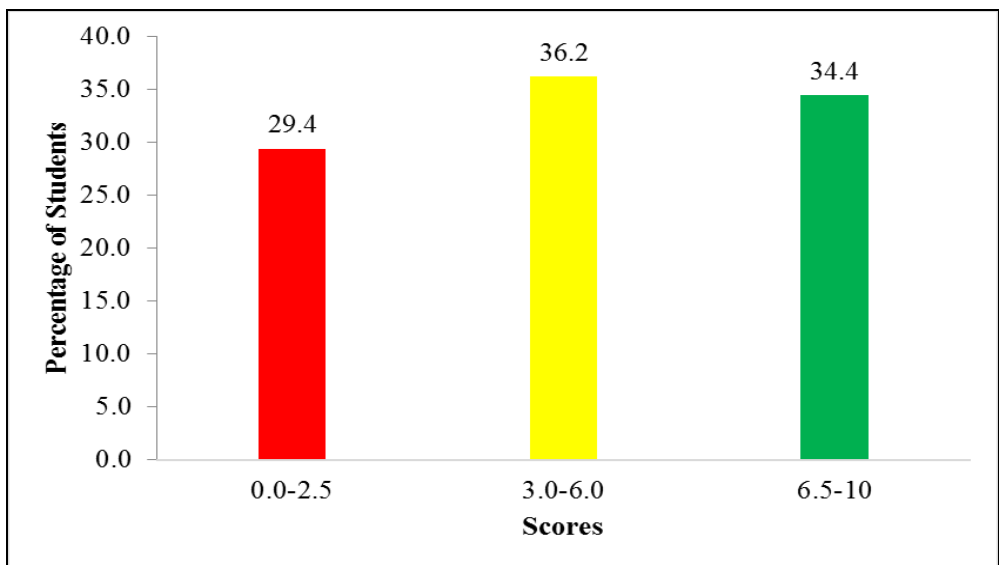
## 2.2 Question 2: Algebra

The question comprised parts (a) and (b). In part (a), the students were required to express  $f$  in terms of  $A$ ,  $b$ , and  $p$  given that  $\frac{A}{b} = \sqrt{\left(\frac{f+p}{f-p}\right)}$ .

In part (b), they were asked to use the elimination method to solve the

simultaneous equations 
$$\begin{cases} \frac{p}{5} + \frac{2q}{3} = \frac{49}{15} \\ \frac{3p}{7} - \frac{q}{2} + \frac{5}{7} = 0 \end{cases}.$$

The analysis of data shows that out of 398 (100%) students who attempted this question, 117 (29.4%) scored 0 to 2.5 marks, 144 (36.2%) students scored 3 to 6 marks and 137 (34.4%) students managed to score from 6.5 to 10 marks. Generally, students had good performance on this question as 70.6 per cent scored 3 to 10 marks. The students' performance summary is presented in Figure 3.



**Figure 3:** Students' Performance on Question 2

Most of the students were able to respond appropriately to every part of the question. In part (a), they were able to rearrange the given formula to express  $f$  in terms of other letters. The students managed to recall and correctly apply the technique of simplification by squaring both sides of the

given equation  $\left(\frac{A}{b}\right)^2 = \left(\sqrt{\frac{f+p}{f-p}}\right)^2$  to obtain  $\frac{A^2}{b^2} = \frac{f+p}{f-p}$ . Thereafter, they

applied the cross multiplication technique and got  $A^2 f - b^2 f = A^2 p + b^2 p$ .

Finally, they used proper manipulation and simplification techniques to obtain the required equation  $f = \frac{p(A^2 + b^2)}{A^2 - b^2}$ .

In part (b), the students were able to transform the given equations

$$\begin{cases} \frac{p}{5} + \frac{2q}{3} = \frac{49}{15} & \text{.....(i)} \\ \frac{3p}{7} - \frac{q}{2} + \frac{5}{7} = 0 & \text{.....(ii)} \end{cases}$$

by multiplying equations (i) and (ii) by 15 and 14 respectively to obtain the

$$\text{new equations } \begin{cases} 3p + 10q = 49 & \text{.....(i)} \\ 6p - 7q = -10 & \text{.....(ii)} \end{cases}.$$

Following that, they demonstrated a good understanding of algebra by eliminating variable  $q$  by multiplying the new equations (i) and (ii) by 7

$$\text{and 10 respectively to obtain } \begin{cases} 21p + 70q = 343 \\ 60p - 70q = -100 \end{cases}.$$

Then, they performed both addition and division to obtain  $p = 3$ .

Furthermore, the students applied the same techniques to obtain  $q = 4$ .

Extract 2.1 is a sample response from one of the students who responded correctly to this question.

2. (a) Given that  $\frac{A}{b} = \sqrt{\frac{f+p}{f-p}}$ , express  $f$  in terms of  $A$ ,  $b$  and  $p$ .

Solution

$$\frac{A}{b} = \sqrt{\frac{f+p}{f-p}}$$

$$\frac{A^2}{b^2} = \left( \sqrt{\frac{f+p}{f-p}} \right)^2$$

$$\frac{A^2}{b^2} = \frac{f+p}{f-p}$$

$$A^2(f-p) = b^2(f+p)$$

$$A^2f - A^2p = b^2f + b^2p$$

$$A^2f - b^2f = b^2p + A^2p$$

$$f(A^2 - b^2) = b^2p + A^2p$$

$$\frac{f(A^2 - b^2)}{A^2 - b^2} = \frac{b^2p + A^2p}{A^2 - b^2}$$

$$f = \frac{b^2p + A^2p}{A^2 - b^2}$$

$$\therefore f = \frac{b^2p + A^2p}{A^2 - b^2}$$

- (b) Use elimination method to solve the following simultaneous equations:

$$\begin{cases} \frac{p}{5} + \frac{2q}{3} = \frac{49}{15} \\ \frac{3p}{7} - \frac{q}{2} + \frac{5}{7} = 0 \end{cases}$$

Solution.

$$\frac{p}{5} + \frac{2q}{3} = \frac{49}{15}$$

$$15 \times \frac{p}{5} + 15 \times \frac{2q}{3} = 15 \times \frac{49}{15}$$

$$3p + 10q = 49 \quad \dots \dots i$$

and

$$\frac{3p}{7} - \frac{q}{2} + \frac{5}{7} = 0$$

$$\frac{3p}{7} - \frac{q}{2} = -\frac{5}{7}$$

$$14 \times \frac{3p}{7} - 14 \times \frac{q}{2} = 14 \times -\frac{5}{7}$$

$$6p - 7q = -10 \quad \dots \dots ii$$

So to combine 2 equations

$$\begin{cases} 3p + 10q = 49 \\ 6p - 7q = -10 \end{cases}$$

$$\begin{cases} 18p + 60q = 294 \\ -18p - 21q = -30 \end{cases}$$

$$\frac{81q}{81} = \frac{324}{81}$$

$$q = 4$$

and to eliminate  $q$  in order to get  $p$ .

$$\begin{cases} 3p + 10q = 49 \\ 6p - 7q = -10 \end{cases}$$

$$\begin{cases} 21p + 70q = 343 \\ -60p - 70q = -700 \end{cases}$$

$$\frac{81p}{81} = \frac{273}{81}$$

$$p = 3$$

**Extract 2.1:** A sample of the student's responses to question 2

In Extract 2.1, in part (a), the student applied the appropriate technique to express  $f$  in terms of  $A$ ,  $b$ , and  $p$ . In part (b), the student was knowledgeable about the concepts of solving simultaneous equations using elimination method and got correct answers for  $p$  and  $q$ .



Despite the strength demonstrated by most of the students, 27.9 per cent were unable to respond to the question correctly as they failed to score at least 2.5 marks. These students faced the following difficulties:

In part (a), some of the students ignored the radical sign and responded to the question without considering it. For example, from  $\frac{A}{b} = \sqrt{\frac{f+p}{f-p}}$ , they

responded as  $\frac{A}{b} = \frac{f+p}{f-p}$ . Then the cross multiplication yielded

$A(f-p) = b(f+p)$ , and it was concluded as  $f = \frac{bf-bp}{a-ap}$  instead of

$f = \frac{p(A^2+b^2)}{A^2-b^2}$ . Some students attempted the question by squaring

incorrectly on both sides of the given formula  $\frac{A}{b} = \sqrt{\left(\frac{f+p}{f-p}\right)}$  such as

$\left(\frac{A}{b}\right)^2 = \sqrt{\left(\frac{f+p}{f-p}\right)^2}$ . Then, they wrote  $\frac{A^2}{b} = \frac{f+p}{f-p}$ . Thereafter, they cross

multiplied to obtain  $A^2(f-p) = b(f+p)$ . Finally, they concluded that

$f = \frac{b+pb}{A^2-Ap}$  instead of  $f = \frac{p(A^2+b^2)}{A^2-b^2}$ . The analysis also shows that,

there were students who failed to remove the radical sign from the given

formula and responded to the equation as  $\frac{A}{b} = \sqrt{\left(\frac{f+p}{f-p}\right)} \Rightarrow \sqrt{\frac{A}{b}} = \frac{f+p}{f-p}$

and ended there. All these responses were due to lack of computational skills.

In part (b), some students faced challenges when writing the given simultaneous equations without fractions. Instead of finding the LCM of the denominator, they concentrated on multiplying each term by its denominator and as a result, they ended up with incorrect equations as well as solutions. For example, one of the students solved as follows;

$$\begin{cases} \frac{p}{5} + \frac{2p}{3} = \frac{49}{15} \Rightarrow 5 \times \frac{p}{5} + \frac{2p}{3} \times 3 = \frac{49}{15} \times 15 \\ \frac{3p}{7} - \frac{q}{2} + \frac{5}{7} = 0 \Rightarrow 7 \times \frac{3p}{7} - \frac{q}{2} \times 2 + \frac{5}{7} \times 7 = 0 \end{cases}$$

Later, the student got the incorrect equations

$$\begin{cases} p + 2p = 49 \dots\dots\dots(i) \\ 3p - q = -5 \dots\dots\dots(ii) \end{cases}.$$

When solved the equations, he/she got  $q = 440$  and  $p = 142$  instead of  $p = 3$  and  $q = 4$ .

Moreover, some other students faced the same difficulties on removing fractions from the given equations by multiplying the first equation by  $\frac{3p}{7}$

and the second equation by  $\frac{p}{5}$  as follows:

$$\begin{array}{l|l} \frac{3p}{7} & \frac{p}{5} + \frac{2q}{3} = \frac{49}{15} \\ \frac{p}{5} & \frac{3p}{7} - \frac{q}{2} + \frac{5}{7} = 0 \end{array}$$

When computed the equations, one student ended up with more complex equations such as;

$$\begin{cases} \frac{3p^2}{35} + \frac{6qp}{21} = \frac{147}{105} \\ \frac{3p^2}{35} - \frac{pq}{35} = 0 \end{cases}$$

Then, a student solved the equation and arrived at the incorrect responses  $\frac{6qp}{21} = \frac{147}{105}$ , instead of  $p = 3$  and  $q = 4$ . Extract 2.2 is a sample of a response from one of the students who failed to attempt the question correctly.

2. (a) Given that  $\frac{A}{b} = \sqrt{\left(\frac{f+p}{f-p}\right)}$ , express  $f$  in terms of  $A$ ,  $b$  and  $p$ .

solution:

$$\frac{a}{b} = \sqrt{\frac{f+p}{f-p}}$$

$$\frac{a}{b} = \frac{f+p}{f-p}$$

$$a(f-p) = b(f+p)$$

$$af - ap = bf - bp$$

$$f = \frac{bf - bp}{a - ap}$$

$$\therefore f = \frac{bf - bp}{a - ap}$$

- (b) Use elimination method to solve the following simultaneous equations:

$$\begin{cases} \frac{p}{5} + \frac{2q}{3} = \frac{49}{15} \\ \frac{3p}{7} - \frac{q}{2} + \frac{5}{7} = 0 \end{cases}$$

solution.

$$\frac{p}{5} + \frac{2q}{3} = \frac{49}{15}$$

$$3\left(\frac{p}{5}\right) + 2\left(\frac{2q}{3}\right) = 49 \times \frac{15}{15}$$

$$p + 2q = 735 \rightarrow i)$$

$$\left(\frac{3p}{7} - \frac{2q}{2} + \frac{5}{1}\right) = 0$$

$$3p - 2q + 5 = 0 \rightarrow ii)$$

$$1 \mid p + 2q = 735$$

$$2 \mid 3p + q = 5$$

$$- \mid p + 2q = 735$$

$$6p - 2q = 10$$

$$p - 6p = 735 - 10$$

$$-5p = 725$$

$$-5 \quad -5 \quad p = 145 \quad \therefore \text{Hence } q = 440 \text{ and } p = 145$$

$$1 \mid p + 2q = 735$$

$$1 \mid 3p + q = 5$$

$$- \mid 3p + 6q = 2205$$

$$3p + q = 5$$

$$6q - q = 2205 - 5$$

$$5q = 2200$$

$$\frac{5q}{5} = \frac{2200}{5}$$

$$q = \frac{2200}{5}$$

$$q = 440$$

Again.

$$1 \mid p + 2q = 735$$

$$2 \mid 3p + q = 5$$

**Extract 2.2:** A sample of the student's responses to question 2

In Extract 2.2, part (a), the student ignored the radical sign when transposing the given formula. While in part (b), the student encountered challenges of writing simultaneous equations without fractions.

### 2.3 Question 3: Geometrical Constructions

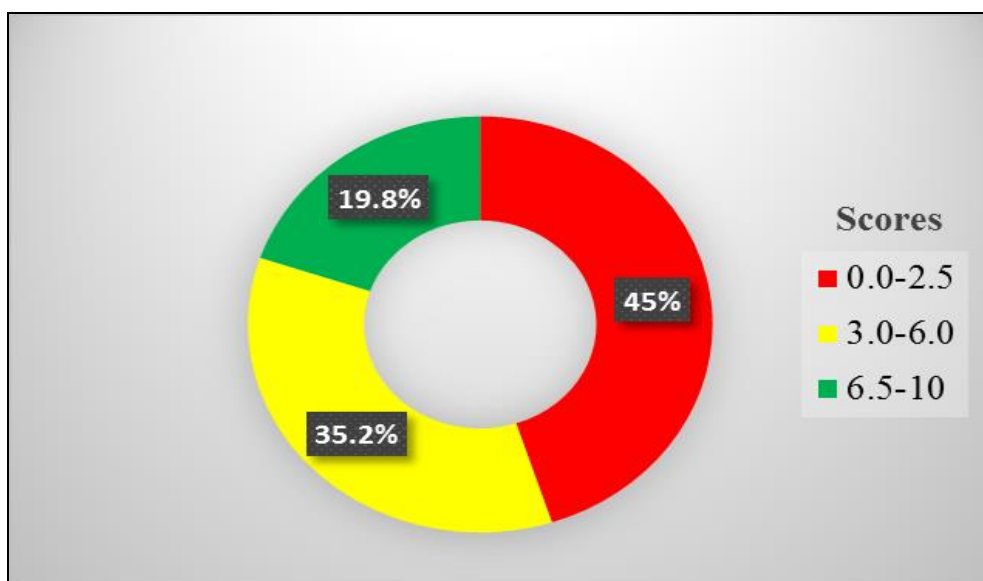
The question comprised two parts, (a) and (b). In part (a), the students were given that the sum of interior angles of a regular polygon is  $1440^\circ$ . They were asked to determine:

- (i) the number of sides of the polygon.
- (ii) the name of the polygon.
- (iii) the maximum number of triangles that can be formed from the polygon.

In part (b), it was given that, if the exterior angle of a regular polygon is  $30^\circ$  less than a half of the interior angle, then they were asked to calculate:

- (i) the size of the interior angle.
- (ii) the size of the exterior angle.
- (iii) the number of angles of regular polygon.

The analysis of data shows that out of 398 (100%) students who attempted this question, 45 per cent scored 0 to 2.5 marks, while 35.2 per cent scored 3 to 6 marks and 19.8 per cent of the students managed to score 6.5 to 10 marks.



**Figure 4:** *Students' Performance on Question 3*

Generally, students had average performance on this question as seen in the summary of the students' performance presented in Figure 4.

The students who answered this question correctly had adequate knowledge of the concept being tested. In part (a) (i), a considerable number of students were able to recall correctly the formula for the sum of the interior angles of a polygon, that is  $(n-2)180^\circ$  where  $n$  is the number of sides of a regular polygon. The students then computed  $(n-2)\times 180^\circ = 1440^\circ$  and obtained the number of sides of a regular polygon,  $n=10$ . In part (a) (ii), the students recognized that the polygon with 10 sides is called decagon. In part (a) (iii), the students further determined the maximum number of triangles by using the formula  $(n-2)$ , where  $n$  is the number of sides. Finally, by using the formula, they obtained the maximum number of triangle as 8. That is,  $n-2=10-2=8$ .

In part (b) (i), competent students correctly interpreted the word problem and formulated the mathematical equation as  $\text{exterior angle} = \frac{1}{2}x - 30^\circ$ , where  $x$  is the interior angle. Then, they used the appropriate formula,  $\text{exterior angle} + \text{interior angle} = 180^\circ$  to calculate the degree measure of an interior angle. Thereafter, the students managed to substitute  $\frac{1}{2}x - 30^\circ + x = 180^\circ$  and finally concluded that the size of an interior angle is  $140^\circ$ . In part (b) (ii), the students calculated the size of the exterior angle as  $\frac{1}{2}x - 30^\circ$  by substituting  $x=140^\circ$  from (i) and got the value of the exterior angle  $= 40^\circ$ . In part (b) (iii), the students correctly recalled the formula that; the size of the exterior angle  $= \frac{360^\circ}{n}$  where  $n$  is the number of sides. Later, they computed for  $n$ , that is,  $40^\circ = \frac{360^\circ}{n}$  and concluded that the number of angles of a regular polygon was 9. Extract 3.1 is a sample response selected from one of the students who attempted this question correctly.

3. (a) The sum of interior angles of a regular polygon is  $1440^\circ$ , determine:

- (i) the number of sides of the polygon.
- (ii) the name of the polygon.
- (iii) the maximum number of triangles that can be formed from the polygon.

Soln

$$(i) \text{ Sum of Interior angle} = 1440^\circ$$

But

$$(n-2)180^\circ = 1440^\circ$$

$$180n - 360^\circ = 1440^\circ$$

$$180n = 1440^\circ + 360^\circ$$

$$\frac{180n}{180} = \frac{1800}{180}$$

$$n = \underline{10}$$

$$\therefore \text{Number of Side} = \underline{10}$$

(ii) The name of the polygon is "DECAEEN"

$$(iii) \text{ Sum of Interior angle of triangle} = \underline{180^\circ}$$

$$\text{Therefore} = \underline{1440^\circ}$$

$$180^\circ$$

$$= \underline{144}$$

$$18)$$

$$= \underline{8}$$

$\therefore$  Maximum number of triangles that can be formed  
8 triangles.

(b) If the exterior angle of a regular polygon is  $30^\circ$  less than a half of the interior angle. Calculate:

- (i) the size of the interior angle.
- (ii) the size of the exterior angle.
- (iii) the number of angles of the polygon.

Soln:

$$(i) \quad \text{Ext} + \text{Inter} = 180$$

$$\frac{1}{2}x - 30^\circ + x = 180 \times 2$$

$$x - 60^\circ + 2x = 360^\circ$$

$$(x + 2x) - 60^\circ = 360^\circ$$

$$3x = 360^\circ + 60^\circ$$

$$3x = 420^\circ$$

$$\frac{3x}{3} = \frac{420^\circ}{3}$$

$$x = 140^\circ$$

$\therefore$  The size of interior angle =  $140^\circ$

(ii) Size of Exterior angle:

$$\left(\frac{1}{2}x - 30^\circ\right)$$

$$x = 140^\circ$$

$$\left(\frac{1}{2} \times 140^\circ\right) - 30^\circ$$

$$= 70^\circ - 30^\circ$$

$$= 40^\circ$$

$\therefore$  The size of exterior angle is  $40^\circ$

(iii) Number of Angles:

We know

$$360^\circ = \text{Ext. angle.}$$

$$\frac{360^\circ}{n} = \text{Ext.}$$

$$\frac{360^\circ}{n} = 40^\circ$$

$$\frac{360^\circ}{40} = \frac{360^\circ}{40}$$

$$n = 9$$

$\therefore$  The number of Angles of polygon is 9

**Extract 3.1:** A sample of the student's responses to question 3

In Extract 3.1, the student had adequate knowledge of the geometrical figures and its properties. Hence, in part (a), he/she managed to determine the number of sides, the name of the polygon and the maximum number of triangles that can be formed. While in part (b), the student managed to

interpret the given word problem and correctly calculated the required angles.

In spite of good responses, there were students who encountered challenges in responding to this question. According to the analysis, this was due to students' inability to recall the appropriate formula and misinterpretation of word problems in mathematical models. In part (a) (i), some students perceived the word sum of interior angles as one interior angle, thus they responded by considering the given sum as an interior angle. For instance,  $\text{exterior angle} + \text{interior angle} = 180^\circ$ . But the interior angle was  $1440^\circ$ . The student substituted it as  $\text{exterior} + 1440^\circ = 180^\circ$ , which implies  $\text{exterior} = 180^\circ - 1440^\circ$  and ended up with  $-1260^\circ$ . However, the question required the students to recall the formula for the sum of interior angles, that is,  $(n-2)180^\circ$ . Which could result in the number of sides of a polygon being 10.

The analysis also revealed that, there were students who applied an incorrect formula for finding the sum of interior angles. For example, instead of using  $(n-2) \times 180^\circ$ , a certain student used the formula  $(2n+4) \times 90^\circ$  and responded as follows;  $(2n+4) \times 90^\circ = 1440^\circ \Rightarrow 180^\circ + 360^\circ = 1440^\circ$ , but simplified it wrongly and got  $n = 10$ .

In part (a) (ii), some students failed to identify the name of the polygon, which led them to give different names of the polygon, such as hexagon, nonagon while others named the polygon as octagon instead of decagon. In part (a) (iii), some of the students failed to recall correctly the formula used to determine the maximum number of triangles  $(n-2)$  where  $n$  is the number of sides. For example, one student used  $n+2$  and substituted  $n = 7$  which resulted in  $7+2=9$ . Likewise, some other students also applied incorrect formulas such as  $\frac{n(n+1)}{2} \Rightarrow \frac{10(10+1)}{2} = 55$  hence they ended up with incorrect responses of the number of triangles instead of 8.

In part (b) (i), some students were not able to correctly interpret the question, especially on the statement that; the exterior angle is  $30^\circ$  less than half of the interior angle, such as one of the students wrote  $30x + x = 180^\circ$ .



Thereafter, he/she simplified and got  $x = 45^\circ$  as an interior angle instead of  $140^\circ$ . Also, another student formulated the equation  $30^\circ + \text{interior angle} = 90^\circ$  and then simplified and got an interior angle of  $60^\circ$ . Furthermore, some students failed to apply the correct formula. For example, one among the students who responded to the question used the formula  $\text{exterior angle} - \text{interior angle} = 180^\circ$  that is  $30^\circ - \text{interior angle} = 180^\circ$  to obtain the interior angle.

In part (b) (ii), a considerable number of students failed to interpret the question correctly. This led them to give incorrect responses to this part. For instance, some interpreted it as  $\text{exterior angle} - 30^\circ = \frac{\text{interior angle}}{2}$ . Then,

$$e - 30^\circ = \frac{i}{2} \Rightarrow 2e - i = 30^\circ \text{----- (i)}$$

$$\text{From, } \text{exterior angle} + \text{interior angle} = 180^\circ \rightarrow e + i = 180^\circ \text{----- (ii)}$$

Thereafter, they solved simultaneously the two equations  $\begin{cases} 2e - i = 30^\circ \\ e + i = 180^\circ \end{cases}$  and got  $3e = 210^\circ$  which implied an exterior angle  $= 70^\circ$ .

Some other students formulated the equation,  $\text{exterior angle} = 30^\circ - \frac{1}{2}i$ , instead of  $\frac{1}{2}i - 30^\circ$ . That incorrect equation led them to give the response  $30^\circ - \frac{1}{2}i + i = 180^\circ$ , and simplified to  $\frac{1}{2}i = 150^\circ$ . Finally, they got the interior angle  $= 300^\circ$  and hence the exterior angle  $= -120^\circ$  instead of  $40^\circ$ .

In part (b) (iii), some students used an incorrect formula. For example, one student used the formula that, the number of angles  $= \frac{360^\circ}{30^\circ} - 1$  which resulted in 11 instead of 9. Likewise, a few students used an inappropriate formula. For instance, one among the students used the formula  $n - 2$ , which gave the number of triangles instead of the number of angles of the polygon. Extract 3.2 provides a sample of a response from one of the students who encountered challenges when attempting this question.

3. (a) The sum of interior angles of a regular polygon is  $1440^\circ$ , determine:

- (i) the number of sides of the polygon.
- (ii) the name of the polygon.
- (iii) the maximum number of triangles that can be formed from the polygon.

Soln

$$\begin{array}{r} 180 \times \text{side} = 1440 \\ 180 \quad 180 \end{array}$$

$$\text{side} = 8$$

The sides is 8

iii pentagon

$$\begin{array}{r} \text{iii} \quad n (2 \times 180) \\ 9 \times 2 (180) \\ 18 (180) \end{array}$$

$$= 3240$$

The maximum number of triangle is  
3240

(b) If the exterior angle of a regular polygon is  $30^\circ$  less than a half of the interior angle. Calculate:

- (i) the size of the interior angle.
- (ii) the size of the exterior angle.
- (iii) the number of angles of the polygon.

$$\begin{aligned} & \text{Soln} \\ & \text{Exterior} - \text{Interior} = 180^\circ \\ & 30^\circ - \text{In} = 180^\circ \\ & \therefore \text{In} = 180^\circ - 30^\circ \\ & \text{In} = 150^\circ \\ & \underline{\text{The size of Interior is } 150^\circ} \end{aligned}$$

$$\begin{aligned} & \text{ii} \quad \text{Exterior} - \text{Interior} = 180^\circ \\ & \text{Exterior} - 150 = 180^\circ \\ & \text{Exterior} = 180 + 150^\circ \\ & \text{Exterior} = 330 \\ & \underline{\text{The size of exterior is } 330} \end{aligned}$$

$$\begin{aligned} & \text{iii} \quad \text{In} = 360 - 1 \\ & \frac{\text{In}}{\text{In}} = \frac{360 - 1}{\text{In}} \\ & \text{In} = \frac{360 - 1}{\text{In}} \\ & \text{In} = \frac{360 - 1}{30} \\ & \text{In} = 12 - 1 \\ & \text{In} = 11 \\ & \underline{\therefore \text{The number of angles is } 11} \end{aligned}$$

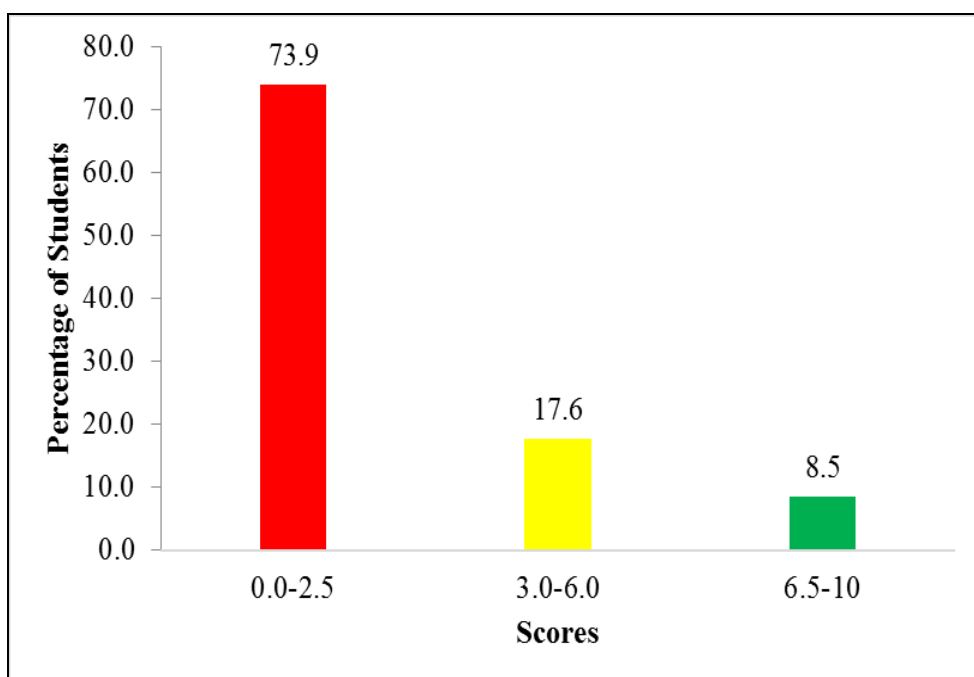
**Extract 3.2:** A sample of the student's responses to question 3

In Extract 3.2, the student failed to interpret the given word problems. Likewise in part (a), the student failed to recall the correct formula. Hence, he/she got an incorrect answer. While in part (b), the student applied incorrect formulas resulting in obtaining wrong answers.

## 2.4 Question 4: Locus

The question comprised two parts, (a) and (b). In part (a), the students were required to find the locus of point  $Q(x, y)$ , given that a point  $Q(x, y)$  moves in such a way that its distance from point  $B(3, 4)$  is twice its distance from the line  $x = 3$ . In part (b), the students were asked to determine the locus of point  $P(x, y)$ . Given that point  $P(x, y)$  moves in such a way that its distance from point  $S(2, -3)$  is equal to its distance from the point of intersection between the lines whose equations are  $x - 2y = 4$  and  $2x + 3y = 15$ .

The analysis of data shows that 398 students attempted this question. Among those, 294 (73.9%) scored marks from 0 to 2.5, 70 (17.6%) students scored marks from 3 to 6 and 34 (8.5%) students scored marks from 6.5 to 10. Generally, most students had weak performance on this question. The summary of students' performance is presented in Figure 5.



**Figure 5:** *Students' Performance on Question 4*

It is observed that a larger number of the students failed to answer the question correctly due to the challenges that faced them while answering this question.

In part (a), some students failed to recall the correct distance formula, that is  $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ . Hence, they applied an improper formula. For example, a certain student used the formula for finding the equation of a straight line to get the locus of the given point. Such as student understood that the given equation  $x = 3$  represents the slope of a straight line, with  $Q(x, y)$  and  $B(3, 4)$  representing the points on the line. Therefore, he/she substituted it into the equation  $m = \frac{y_2 - y_1}{x_2 - x_1}$  to get  $3 = \frac{y - 4}{x - 3}$ . Thereafter, he/she simplified it by cross multiplication and obtained  $y - 4 = 3x - 9$ . Finally, the student obtained an incorrect locus of point;  $y = 3x - 5$ .

Not only that, other students applied the mid-point formula  $m = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$  to find the locus of point  $Q(x, y)$  instead of the distance formula  $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ . Thereafter, they substituted  $x$  and  $y$  with incorrect values and obtained  $m = \left( \frac{6 + 3}{2}, \frac{8}{2} \right)$ , which was simplified to  $(4.5, 5)$  as the required locus.

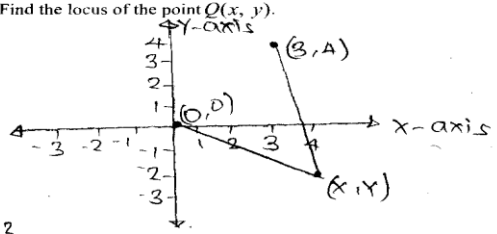
Furthermore, some other students managed to recall the correct distance formula but failed to correctly interpret the given condition of the locus. For example, one student used  $B(3, 4) \times 2$  to get  $B(6, 8)$  and line  $x = 3$  to determine the locus, then he/she substituted the values to the distance formula, such that  $3 = \sqrt{(x - 6)^2 + (y - 8)^2}$  to obtain  $9 = (x - 6)^2 + (y - 8)^2$ . Further simplification resulted in  $x^2 - 12x + y^2 + 36 - 16y + 91 = 0$  as the locus of the given point.

In part (b), some students failed to recall and apply the correct distance formula. For example, a certain student identified the gradients of the equations  $x - 2y = 4$  and  $2x + 3y = 15$  as  $\frac{1}{2}$  and  $\frac{2}{3}$  respectively. Thereafter, he/she added them to obtain  $\frac{7}{6}$  and concluded that, this was the required locus of a point. Further analysis revealed that some of the students responded to the question by solving the given pairs of simultaneous

equations incorrectly. For instance, they solved  $\begin{cases} x - 2y = 4 \\ 2x - 3y = 15 \end{cases}$  and obtained  $x = 18$  and  $y = 7$ . This response was due to inadequate knowledge about basic operations as well as failure to adhere to what the question required.

Moreover, a few students interchanged the value of  $(x, y)$ . For example, one student substituted the point as  $S(-3, 2)$  instead of  $S(2, -3)$  into the distance formula  $d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$  and obtained  $d^2 = (-3 - x)^2 + (2 - y)^2$  expanded to  $13 + 6x - 4y + x^2 + y^2$ . Finally, he/she concluded that the locus of point  $P(x, y) = x^2 + y^2 + 6x + 13$ . Extract 4.1 provides a sample response from one of the students who incorrectly responded to this question.

4. (a) A point  $Q(x, y)$  moves in such a way that its distance from the point  $B(3, 4)$  is twice its distance from the line  $x = 3$ . Find the locus of the point  $Q(x, y)$ .



$d_1 = d_2$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(3 - 0)^2 + (4 - 0)^2} = \sqrt{(3 - x)^2 + (4 - y)^2}$$

$$= 3(3 - 0)(3 - 0) + 4(4 - 0)(4 - 0) = (3 - x)(3 - x) + (4 - y)(4 - y)$$

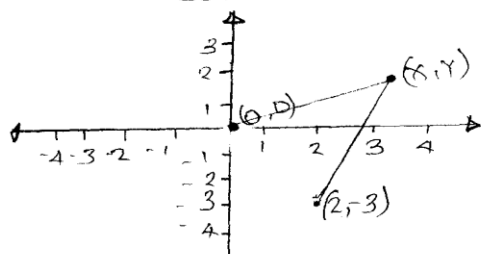
$$= 9 - 0 + 0 + 16 - 0 - 0 + 0 = 9 - 3x - 3x + x^2 + 16 - 4y - 4y + y^2$$

$$= 9 = 9 + x + y$$

$\therefore$  The locus is 18

(b) A point  $P(x, y)$  moves in such a way that its distance from the point  $S(2, -3)$  is equal to its distance from the point of intersection between the lines whose equations are  $x - 2y = 4$  and  $2x + 3y = 15$ . Determine the locus of the point  $P(x, y)$ .

Soln



$$\begin{aligned}
 d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
 &= \sqrt{(2 - x)^2 + (-3 - y)^2} = \sqrt{(2 - x)^2 + (-3 - y)^2} \\
 &= (2 - 0)^2 + (-3 - 0)^2 = (2 - x)^2 + (-3 - y)^2 \\
 &= (2 - 0)(2 - 0) + (-3 - 0)(-3 - 0) = (2 - x)(2 - x) + (-3 - y)(-3 - y) \\
 &= 2(2 - 0) - 0(2 - 0) + -3(-3 - 0) - 0(-3 - 0) = -2(2 - x) - x(2 - x) - 3(-3 - y) - y(-3 - y) \\
 &= 4 - 0 - 0 + 0 + 9 - 0 - 0 + 0 = -4 - 2x - 2x + x + 9 - 3y - 3y + y^2 + y^2 \\
 &1 = x + y \\
 \therefore \text{The locus of point } P(x, y) &= 1
 \end{aligned}$$

**Extract 4.1:** A sample of the student's responses to question 4

In Extract 4.1, part (a), the student failed to apply properly the distance formula. Hence, he/she failed to get the required locus. In part (b), the student had a wrong interpretation of the question, resulting in wrong responses.

Apart from a larger number of students who failed to answer this question correctly, there were some students who scored high marks as they were able to answer correctly in all parts of this question.

In part (a), the students correctly remembered the distance formula from one point to another as  $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$  and correctly substituted the given points  $Q(x, y)$  and  $B(3, 4)$  to obtain  $d_1 = \sqrt{(x - 3)^2 + (y - 4)^2}$ . Then, squaring both sides of the equation as  $d_1^2 = (x - 3)^2 + (y - 4)^2$  and thereafter simplified to get  $d_1^2 = x^2 - 6x + y^2 - 8y + 25$ . On the other hand, the students substituted the coordinates of point  $Q(x, y)$  and line  $x = 3$  into the distance formula as  $d_2 = \sqrt{(x - 3)^2}$ . Then, they simplified to obtain  $d_2^2 = x^2 - 6x + 9$ .

Furthermore, the students correctly interpreted the locus as  $d_1 = 2d_2$  and thus  $d_1^2 = 4d_2^2$  after which they equated the two equations and obtained

$x^2 - 6x + y^2 - 8y + 25 = 4(x^2 - 6x + 9)$ . Later, they performed further simplification to obtain  $3x^2 - 18x - y^2 + 8y + 11 = 0$  which is the required locus of point.

In part (b), the analysis depicts that students had sufficient knowledge of algebra as they managed to get the point of intersection of the given loci by solving the equations simultaneously and obtained the value of  $x$  and  $y$ , that is, 6 and 1 respectively. The coordinates of points  $P(x, y)$  and  $S(2, -3)$  were then substituted into the distance formula,  $d_1 = \sqrt{(x-2)^2 + (y+3)^2}$  which was then simplified by squaring both sides of the equation to obtain  $d_1^2 = (x-2)^2 + (y+3)^2$  and simplified more to obtain  $x^2 - 4x + 4 + y^2 + 6y + 9$ . Following that, the students realized that  $d_1 = d_2$  as the locus condition, thus  $d_1^2 = d_2^2$  which implies  $(x-2)^2 + (y+3)^2 = (x-6)^2 + (y-1)^2$  and simplified to obtain  $x^2 - 4x + 4 + y^2 + 6y + 9 = x^2 - 12x + 36 + y^2 - 2y + 1$ . Further simplification resulted in  $8x + 8y - 24 = 0$  which is equal to  $x + y = 3$ , the locus of point  $P$ . Extract 4.2 is a sample response of one of the students who responded correctly to this question.



4. (a) A point  $Q(x, y)$  moves in such a way that its distance from the point  $B(3, 4)$  is twice its distance from the line  $x = 3$ . Find the locus of the point  $Q(x, y)$ .

Solution:

Let  $Q(x_1, y_1)$  and  $B(3, 4)$   
 $x_1, y_1$   $x_2, y_2$

$$\text{From } Q_1 = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$(Q_1)^2 = \left( \sqrt{(3-x)^2 + (4-y)^2} \right)^2$$

$$\begin{aligned} (Q_1)^2 &= (3-x)^2 + (4-y)^2 \\ &= (3-x)(3-x) + (4-y)(4-y) \\ &= 9 - 3x - 3x + x^2 + 16 - 4y - 4y + y^2 \\ &= x^2 - 6x + y^2 - 8y + 25 \\ (Q_1)^2 &= x^2 - 6x + y^2 - 8y + 25 \end{aligned}$$

But distance from line segment is  $x=3$ .

$$\begin{aligned} (Q_2)^2 &= \left( \sqrt{(x-3)^2} \right)^2 \\ (Q_2)^2 &= (x-3)(x-3) \\ &= x^2 - 3x - 3x + 9 \\ &= x^2 - 6x + 9 \\ (Q_2)^2 &= x^2 - 6x + 9 \end{aligned}$$

From:  $Q_1 = 2Q_2$  Condition of locus.

$$\begin{aligned} \text{Thus } (Q_1)^2 &= (2Q_2)^2 \\ (Q_1)^2 &= 4Q_2^2 \end{aligned}$$

$$\begin{aligned} x^2 - 6x + y^2 - 8y + 25 &= 4(x^2 - 6x + 9) \\ x^2 - 6x + y^2 - 8y + 25 &= 4x^2 - 24x + 36 \\ 3x^2 - 18x + y^2 - 8y + 11 &= 0 \end{aligned}$$

$\therefore$  Equation of the Locus is  
 $3x^2 - 18x - y^2 + 8y + 11 = 0$

- (b) A point  $P(x, y)$  moves in such a way that its distance from the point  $S(2, -3)$  is equal to its distance from the point of intersection between the lines whose equations are  $x - 2y = 4$  and  $2x + 3y = 15$ . Determine the locus of the point  $P(x, y)$ .

Solution:

Let find the point of intersection of equations  $x - 2y = 4$  and  $2x + 3y = 15$

$$\begin{array}{r} 2 \mid x - 2y = 4 \text{ --- (1)} \\ 1 \mid 2x + 3y = 15 \text{ --- (2)} \end{array}$$

$$\begin{array}{r} - \mid 2x - 4y = 8 \\ \quad 2x + 3y = 15 \\ \hline \quad -7y = -7 \\ \quad \quad -7 \quad -7 \\ \quad \quad y = 1 \end{array}$$

$$\begin{array}{r} 2 \mid x - 2y = 4 \text{ --- (1)} \\ 2 \mid 2x + 3y = 15 \text{ --- (2)} \end{array}$$

$$\begin{array}{r} + \mid 3x - 6y = 12 \\ \quad 4x + 6y = 30 \\ \hline \quad 7x = 42 \\ \quad \quad 7 \quad 7 \\ \quad \quad x = 6 \end{array}$$

$\therefore$  Point of intersection is  $(6, 1)$ .

<p>Let <math>P(x, y)</math> and <math>Q(2, -3)</math>  <math>x_1, y_1</math>                      <math>x_2, y_2</math></p> <p>From: <math>d_1 = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}</math></p> <p><math>(d_1)^2 = \sqrt{(2-x)^2 + (-3-y)^2}</math></p> <p><math>(d_1)^2 = (2-x)^2 + (-3-y)^2</math>  <math>= (2-x)(2-x) + (-3-y)(-3-y)</math>  <math>= 4 - 2x - 2x + x^2 + 9 + 3y + 3y + y^2</math>  <math>= x^2 - 4x + y^2 + 6y + 13</math></p> <p><math>\therefore (d_1)^2 = x^2 - 4x + y^2 + 6y + 13</math></p> <p>Let <math>P(x, y)</math> and <math>B(6, 1)</math>  <math>x_1, y_1</math>                      <math>x_2, y_2</math></p> <p>From: <math>d_2 = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}</math></p>	<p><math>(d_2)^2 = \sqrt{(6-x)^2 + (1-y)^2}</math></p> <p><math>(d_2)^2 = (6-x)^2 + (1-y)^2</math>  <math>= (6-x)(6-x) + (1-y)(1-y)</math>  <math>= 36 - 6x - 6x + x^2 + 1 - y - y + y^2</math>  <math>\therefore (d_2)^2 = x^2 - 12x + y^2 - 2y + 37</math></p> <p>But <math>d_1 = d_2</math>  <math>(d_1)^2 = (d_2)^2</math></p> <p><math>x^2 - 4x + y^2 + 6y + 13 = x^2 - 12x + y^2 - 2y + 37</math>  <math>- 4x + 12x + 6y + 2y + 13 - 37 = 0</math>  <math>\frac{8x}{8} + \frac{8y}{8} - \frac{24}{8} = \frac{0}{8}</math>  <math>x + y - 3 = 0</math>  <math>x + y = 3</math></p> <p><math>\therefore</math> Equation of the locus if point <math>P(x, y)</math> is <math>x + y = 3</math>.</p>
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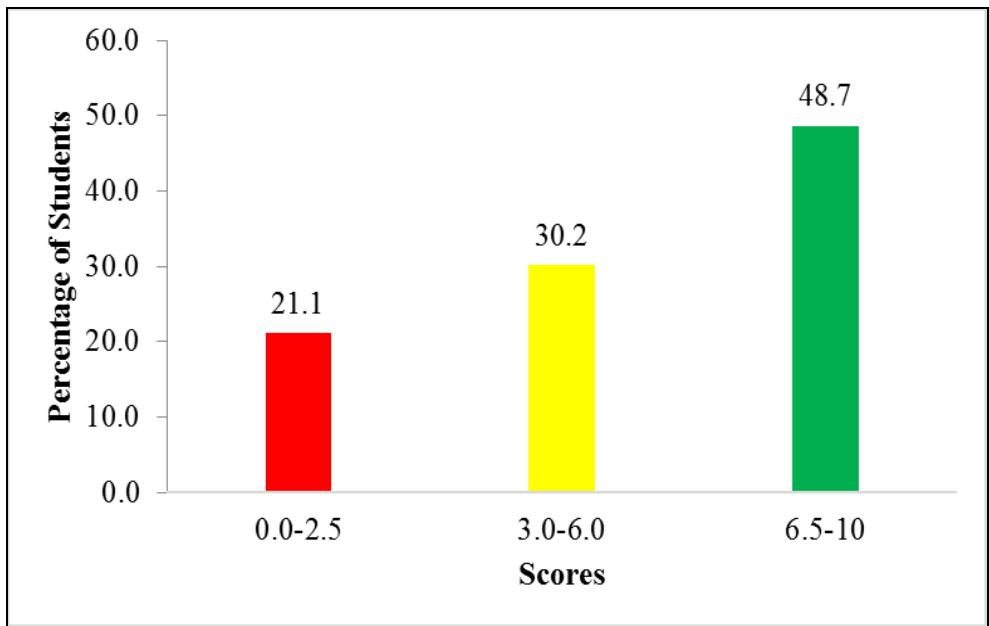
**Extract 4.2:** A sample of the student's responses to question 4

In Extract 4.2, part (a), the student applied correctly the distance formula from one point to another to find the locus of the point correctly. In part (b), the student correctly determined the point of intersection and then used it to get the required locus of a point  $P(x, y)$ .

## 2.5 Question 5: Coordinate Geometry

The question had parts (a) and (b). In part (a), the students were given three points  $A(8, 10)$ ,  $B(5, n)$  and  $C(0, 2)$  that are collinear. Then, they were asked to find the value of  $n$ . While in part (b), it was given that a line passing through point  $A(4, 6)$  crosses  $y$ -axis at point  $B$ . If the line is parallel to another line whose equation is  $2y = x - 2$ , then they were required to determine the coordinates of point  $B$ .

The analysis of data shows that out of 398 (100%) students who attempted this question, 84 (21.1%) scored 0 to 2.5 marks, 120 (30.2%) students scored 3 to 6 marks and 194 (48.7%) students managed to score 6.5 to 10 marks. The general performance on this question was good. Figure 6 provides a summary of students' performance on this question.



**Figure 6:** *Students' Performance on Question 5*

The majority of students who performed well on this question were able to respond to this question accurately. In part (a), the students correctly recalled the condition for collinear points. That is, the points lie on the same line. Therefore, the slopes of the line joining the points are equal ( $M_1 = M_2$ ). Thereafter, they recalled the formula for determining a slope as,  $\text{slope}(m) = \frac{y_2 - y_1}{x_2 - x_1}$  and worked on the slope of  $AB$  as  $M_1 = \frac{n-10}{5-8}$  and got  $M_1 = \frac{n-10}{-3}$ . Also, they worked on the slope of line  $BC$  as  $M_2 = \frac{2-n}{-5}$ . Thereafter, the students applied the concept of parallel lines, that is,  $M_1 = M_2$  to find the value of  $n$  as  $\frac{n-10}{-3} = \frac{2-n}{-5}$ . Then, they simplified and obtained  $n = 7$  as the required value.

In part (b), competent students worked on the first slope  $m_1$  by rearranging  $2y = x - 2$ , obtaining  $y = \frac{1}{2}x - 1$ . Thereafter, they determined that  $m_1 = \frac{1}{2}$ . Similarly, they worked on the second slope using the condition of parallel lines to obtain  $m_2 = \frac{1}{2}$ . Later, they correctly applied the formula for the

equation of the line  $m_2 = \frac{y_2 - y_1}{x_2 - x_1}$  and substituted the coordinates of points

$A(4, 6)$  and  $B(0, y)$  that resulted in  $\frac{1}{2} = \frac{y-6}{0-4}$ . After simplifying, they

came up with at  $y = 4$ . Therefore, the coordinates of point  $B$  are  $(0, 4)$ .

Extract 5.1 provides a sample of a response from one of the students who correctly attempted the question.

(a) Three points  $A(8, 10)$ ,  $B(5, n)$  and  $C(0, 2)$  are collinear. Find the value of  $n$ .

Solution:

CONDITION:  $M_1 = M_2 = M_3$

Then:

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{y_3 - y_2}{x_3 - x_2}$$

$$\frac{n - 10}{5 - 8} = \frac{2 - n}{0 - 5}$$

$$\frac{n - 10}{-3} = \frac{2 - n}{-5}$$

$$-3(2 - n) = -5(n - 10)$$

$$-6 + 3n = -5n + 50$$

$$3n + 5n = 50 + 6$$

$$\frac{8n}{8} = \frac{56}{8}$$

$$n = 7$$

∴ The value of  $n$  is 7.

- (b) A line passing through the point  $A(4, 6)$  crosses the  $y$ -axis at point  $B$ . If the line is parallel to another line whose equation is  $2y = x - 2$ , determine the coordinates of the point  $B$ .

Solution:

From the equation:

$$2y = x - 2 \text{ (Rearranging well)}$$

$$\frac{2y}{2} = \frac{x-2}{2}$$

$$y = \frac{x}{2} - 1$$

Then;

$$m = \frac{1}{2}$$

For parallel:  $m_1 = m_2$

From

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

But:  $B(0, y)$

Then:

$$\frac{1}{2} = \frac{y-6}{0-4}$$

$$\frac{1}{2} = \frac{y-6}{-4}$$

$$2(y-6) = -4$$

$$2y - 12 = -4$$

$$2y = 12 - 4$$

$$\frac{2y}{2} = \frac{8}{2}$$

$$y = 4$$

$\therefore$  The coordinates of the point  $B$  is  $(0, 4)$ .

**Extract 5.1:** A sample of the student's responses to question 5

In Extract 5.1, part (a), the student interpreted correctly the term collinear and applied the correct formula while in part (b), the student recalled the condition for parallel lines and applied the correct formula. Hence, he/she got a correct answer.

However, the analysis of data shows that 21.1 per cent of the students scored less than 3 marks as they failed to respond to this question correctly due to various reasons as explained below:

In part (a), basically the students lacked knowledge about the word “collinear”. Some students confused the term “collinear” as the points which lie on the same line with “translation”. For example, a certain student applied translation concepts, that is  $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix} \Rightarrow \begin{pmatrix} 8 \\ 10 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix} + \begin{pmatrix} 5 \\ n \end{pmatrix}$

and incorrectly wrote  $\frac{8}{10} = \frac{5}{2+n}$  and then simplified it to get  $n = 4.2$ .

Furthermore, other students seemed to have no idea about collinear points. They responded by adding all the points and equated to  $360^\circ$ . For instance, they found the sum of the coordinates of the points  $A(8,10)$ ,  $B(5,n)$  and  $C(0,2)$ . Then, they responded as  $8+10+5+n+0+2=360^\circ$ . Thereafter, they simplified and got  $25+n=360^\circ$  which implied  $n=22$ .

Moreover, other students failed to recall the formula for slope, that is,  $m = \frac{y_2 - y_1}{x_2 - x_1}$ . Instead, they applied the incorrect formula  $m = \frac{x_2 - x_1}{y_2 - y_1}$  and

substituted the values from points  $A(8,10)$ ,  $B(5,n)$  and  $C(0,2)$  to obtain

$m_1 = \frac{0-5}{2-n} = \frac{-5}{2-n}$  and,  $m_2 = \frac{0-8}{2-10} = \frac{-8}{-8} = 1$ . Then, they applied the

condition for perpendicular lines. Thus,  $m_1 \times m_2 = -1 \Rightarrow \frac{-5}{2-n} \times 1 = -1$  was

computed and they ended up with  $n = -3$  instead of  $n = 7$ . Furthermore,

some students used the incorrect concept that  $\frac{8 \times 5}{10 \times n} = \frac{5 \times 10}{n \times 2} \Rightarrow \frac{4}{n} = \frac{25}{n}$  and

then they obtained  $4n = 25n$  and finally got the incorrect value  $n = \frac{25}{4}$ .

In part (b), some students applied the concept of  $x$ -intercept and  $y$ -intercept which are contrary to the demand of the question. For example, some students wrote  $2y = x - 2$  and set  $x = 0$  worked for  $y$ . Thus,  $2y = 0 - 2 \Rightarrow y = -1$ . Also, they wrote  $y = 0$  and then they wrote

$2(0) = x - 2$ . After performing basic operations, they finally obtained  $B(2, -1)$  instead of  $B(0, 4)$ .

Not only that, other students used interchangeably the condition for parallel lines as  $m_1 = m_2$  and the condition for perpendicular lines as  $m_1 \times m_2 = -1$ .

For instance, one wrote  $2y = x - 2 \Rightarrow y = \frac{x}{2} - 1$ . Thus,  $m_1 = \frac{1}{2}$  and then

he/she considered that  $m_1 \times m_2 = -1$ . Therefore,  $\frac{1}{2} \times m_2 = -1 \Rightarrow m_2 = -2$ .

Thereafter, they substituted the values of  $m_2$  in the formula  $m = \frac{y_2 - y_1}{x_2 - x_1}$  to

get  $\frac{-2}{1} = \frac{y - 6}{x - 4}$ . After further simplification, they obtained  $-2x + 8 = y - 6$

which implies  $y = -2x + 14$ . As a result, they concluded that  $(x, y) = (-2, 14)$  instead of  $(0, 4)$ . Extract 5.2 provides a sample of a response from one of the students who faced difficulties when responding to the question.

5. (a) Three points  $A(8, 10)$ ,  $B(5, n)$  and  $C(0, 2)$  are collinear. Find the value of  $n$ .

Solution.

$$\text{Slope } m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\text{Slope } m = \frac{5 - 8}{n - 10}$$

$$\frac{1}{1} = \frac{-3}{n - 10}$$

$$-3 = n - 10$$

$$n = 10 + 3$$

$$\underline{n = 13}$$

- (b) A line passing through the point  $A(4, 6)$  crosses the  $y$ -axis at point  $B$ . If the line is parallel to another line whose equation is  $2y = x - 2$ , determine the coordinates of the point  $B$ .

Solution.

$$\begin{aligned}
 \text{slope } m &= \frac{y_2 - y_1}{x_2 - x_1} \\
 2y &= x - 2 \quad \times \frac{y_2 - 4}{x_2 - 6} \\
 &= 2y = x - 2 \times y_2 - 6 = x_2 - 4 \\
 &= 2y \times y_2 = x - 2 - 6 = x_2 - 4 \\
 &= 2y_2^2 = x + 2 - 6 = x_2 - 4 \\
 &\quad \cancel{2y_2^2} = \cancel{-6} + 4 \neq \cancel{x} \\
 &= 2y_2^2 = -2 - 6 + 4 = x_2 - x \\
 &= 2y_2^2 = -8 + 4 = x_2 - x \\
 &= 2y_2^2 = -4 = x_2 - x \\
 &= \sqrt{2y_2^2} = \sqrt{-4} = \sqrt{x_2 - x} \\
 &= 2y = 2 = \sqrt{x_2 - x} \\
 &= 2y = 2 = \sqrt{x_2 - x} \\
 &\quad = 2y - 2 = \sqrt{x_2 - x} \\
 &\quad = 2y - 2 = x_2^{\frac{1}{2}} - x^{\frac{1}{2}} \\
 &\quad = 2y - 2.
 \end{aligned}$$

**Extract 5.2:** A sample of the student's responses to question 5

In Extract 5.2, part (a), the student applied an incorrect gradient formula while in part (b), the student applied an incorrect gradient formula as well as a wrong approach leading to a wrong response according to the question.

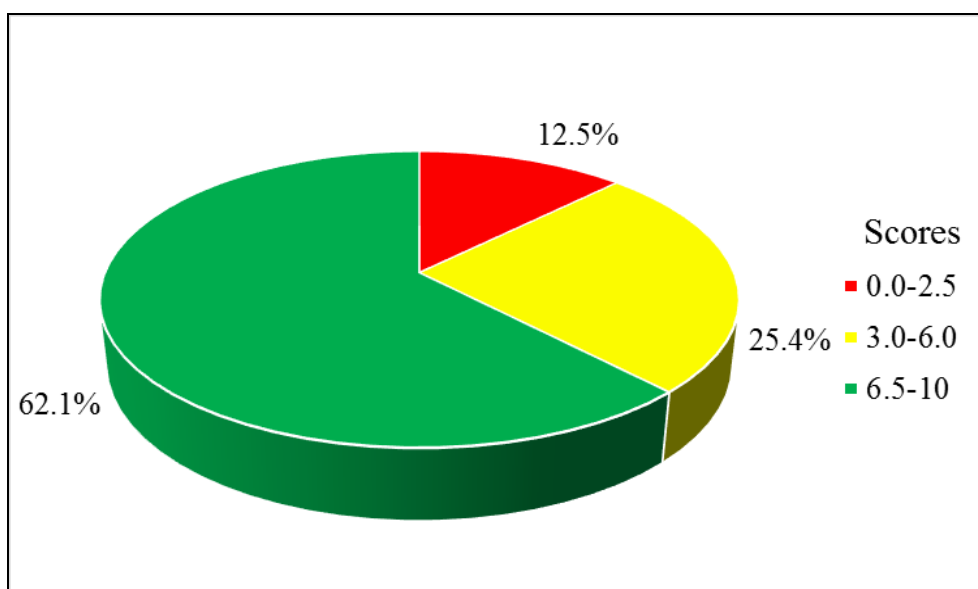


## 2.6 Question 6: Symmetry

This question consisted of parts (a) and (b). In part (a), the students were asked to draw and state the number of line(s) of symmetry for each of the four given figures. While in part (b), they were required to state the order of rotational symmetry for each of the following polygons:

- (i) Circle
- (ii) Equilateral triangle
- (iii) Rectangle
- (iv) Square

The analysis of data shows that 398 (100%) students attempted this question, out of whom 50 (12.5%) scored 0 to 2.5 marks. 101 (25.4%) students scored 3 to 6 marks, while 247 (62.1%) students scored 6.5 to 10 marks. Generally, most students had good performance on this question. The students' performance summary is presented in Figure 7.



**Figure 7:** *Students' Performance on Question 6*

According to the data, most of the students were able to respond to this question according to the instructions provided, showing that they had adequate knowledge and skills on symmetry. In part (a), they managed to draw lines of symmetry of the given figures as illustrated in extract 6.1 and obtained the following results:

- (i) Figure number one had two lines of symmetry.
- (ii) Figure number two had one line of symmetry.

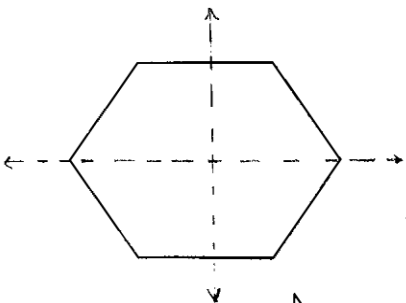
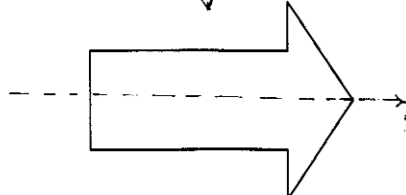
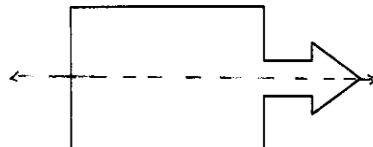
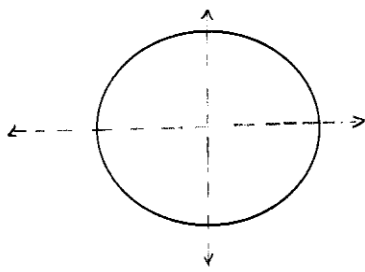
- (iii) Figure number three had one line of symmetry.
- (iv) Figure number four had two lines of symmetry.

In part (b), the students demonstrated their competence as they managed to state the order of rotational symmetry of the given figures as follows:

- (i) A circle has many (infinity) orders of rotational symmetry.
- (ii) An equilateral triangle has 3 orders of rotational symmetry.
- (iii) A rectangle has 2 orders of rotational symmetry.
- (iv) A square has 4 orders of rotational symmetry.

Extract 6.1 provides a sample of a response from one of the students who performed well this question and managed to score all the marks.

6. (a) Draw and state the number of the line(s) of symmetry for each of the following figures:

- (i)  ∴ Has two line of symmetry
- (ii)  ∴ Has one line of symmetry
- (iii)  ∴ Has one line of symmetry
- (iv)  ∴ Has two line of symmetry

(b) State the order of rotational symmetry for each of the following polygons:

- (i) Circle
- (ii) Equilateral triangle
- (iii) Rectangle
- (iv) Square

Solution:

i) Circle

=> It has got infinity order of rotational symmetry.

ii) Equilateral triangle

=> It has got three order of rotational symmetry.

iii) Rectangle

=> It has got only two order of rotational symmetry.

iv) Square

=> It has got four order of rotational symmetry.

**Extract 6.1:** A sample of the student's responses to question 6

In Extract 6.1, part (a), the student demonstrated sufficient knowledge and skills on the tested concept and in part (b), the student stated the correct orders of rotational symmetry for each given polygon.

On the other hand, the analysis of data depicts that 29 (7.3%) students failed to attempt the question correctly and scored zero. Those students were encountered with different challenges. In part (a), the students lacked knowledge and skills on symmetry, especially in drawing lines and stating the number of line(s) of symmetry. Also, some of them concentrated on stating the number of sides of the figures.

Likewise, other students, due to their failure of drawing lines of symmetry as required, they drew lines from one diagonal to another. As a result, they came up with different responses such as;

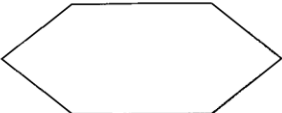
- (i) Has four lines of symmetry
- (ii) Has two lines of symmetry
- (iii) Has three lines of symmetry
- (iv) Has infinite lines of symmetry

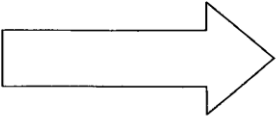
Not only that, other students considered opposite sides as the number of lines of symmetry.


In part (b), a few students were not familiar with the meaning of the word “order” of rotational symmetry. As a result, they considered the angle as the order of rotational symmetry. Also, some of the students, instead of stating the order of rotational symmetry, they responded to the question by drawing polygon figures.

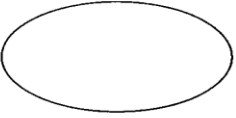
Moreover, some students considered the sum of the degree measures of the angles of the polygon as the order of rotational symmetry, and finally they came up with different degrees of the figures. Extract 6.2 is a sample of a response from one of the students who faced challenges when attempting this question.

6. (a) Draw and state the number of the line(s) of symmetry for each of the following figures:

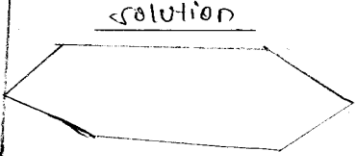
(i) 

(ii) 

(iii) 

(iv) 

Solution

i)  Number of Lines  
~~540°~~  
~~720°~~ The line is 6

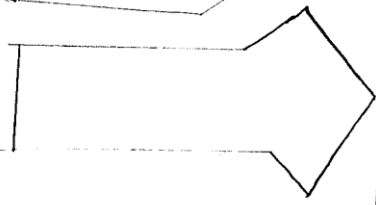
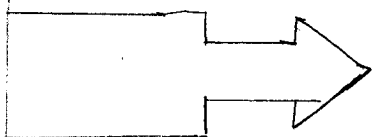

ii)  ~~540°~~ The line is 7

	Diagram	Number of Lines
(iii)		The number of Line is 9
(iv)		The number of lines is 1

(b) State the order of rotational symmetry for each of the following polygons:

- (i) Circle
- (ii) Equilateral triangle
- (iii) Rectangle
- (iv) Square

Solution

i) Circle =  $360^\circ$

ii) Equilateral triangle =  $180^\circ$

iii) Rectangle =  $90^\circ$

iv) Square =  $360^\circ$

**Extract 6.2:** A sample of the student's responses to question 6

In Extract 6.2, part (a), the student failed to draw and state the correct number of line(s) of symmetry. Instead, the student gave the number of sides on each figure. While in part (b), the student provides the sum of angles instead of stating the order of rotational symmetry for each polygon.

## 2.7 Question 7: Logic

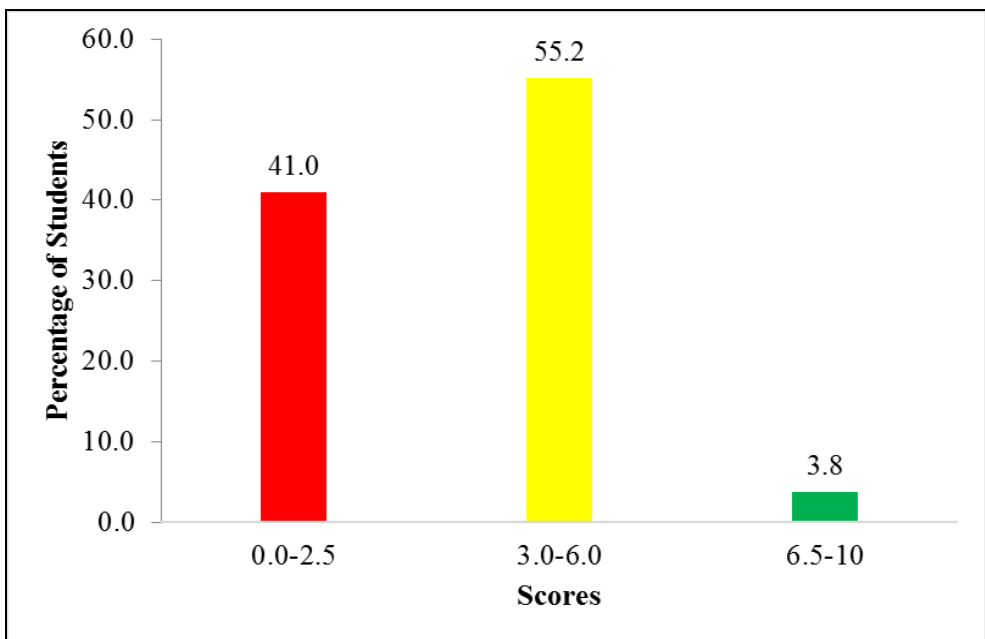
The question consisted of parts (a) and (b). In part (a), the students were required to construct a truth table for each of the following propositions:

(i)  $(p \wedge q) \rightarrow (p \vee q)$

(ii)  $(\sim p \wedge (p \rightarrow q)) \rightarrow \sim q$

In part (b), the students were instructed to use a truth table to test the validity of the argument “If it rains or no one comes, the game will not take place. The game was a success. Therefore, it did not rain”.

The analysis of data shows that out of 398 students who attempted this question, 41.0 per cent scored 0 to 2.5 marks and 59.0 per cent of students scored 3 to 10 marks. Generally, majority of students had average performance on this question. The students’ performance summary is presented in Figure 8.



**Figure 8:** *Students' Performance on Question 7*

The analysis shows that more than 50 per cent of the students demonstrated their competence while responding to the question. In part (a) (i) and (ii), the students managed to construct the truth tables for the given propositions, which were  $(p \wedge q) \rightarrow (p \vee q)$  and  $\sim p \wedge (p \rightarrow q) \rightarrow \sim q$ . Thereafter, they correctly assigned the truth-values to  $p$  and  $q$ .

In part (b), the students were able to identify the appropriate logical connectives and managed to write correctly the symbolic statement of the given argument, that is,  $r$  = it rains,  $p$  = one comes and  $q$  = game takes place. Thereafter, they formulated the symbolic statement that read as  $[(r \vee p) \rightarrow \sim q] \rightarrow \sim r$ . Then, they constructed the truth table with appropriate columns and rows hence they applied the truth table to test the validity of the argument as illustrated in extract 7.1 which is a sample of a response from one of the students who correctly attempted the question.

(a) Construct a truth table for each of the following propositions:

(i)  $(p \wedge q) \rightarrow (p \vee q)$

(ii)  $(\sim p \wedge (p \rightarrow q)) \rightarrow \sim q$

④ ① <sup>solution</sup> Required to construct truth table for  $(p \wedge q) \rightarrow (p \vee q)$

P	q	$(p \wedge q)$	$(p \vee q)$	$(p \wedge q) \rightarrow (p \vee q)$
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	T

$\therefore$  The truth table for proposition  $(p \wedge q) \rightarrow (p \vee q)$

② <sup>solution</sup>

Required to construct the truth table for  $(\sim p \wedge (p \rightarrow q)) \rightarrow \sim q$

P	q	$\sim p$	$\sim q$	$(p \rightarrow q)$	$(\sim p \wedge (p \rightarrow q))$	$(\sim p \wedge (p \rightarrow q)) \rightarrow \sim q$
T	T	F	F	T	F	T
T	F	F	T	F	F	T
F	T	T	F	T	T	F
F	F	T	T	T	T	T

$\therefore$  The truth table for the proposition  $(\sim p \wedge (p \rightarrow q)) \rightarrow \sim q$

(b) Use a truth table to test validity of the following argument: "If it rains or no one comes, the game will not take place. The game was a success. Therefore, it did not rain".

<sup>solution</sup>

let  $p$  = It rain

then  $q$  = ~~The game will~~ one comes.

and  $r$  = The game will.

$\left[ ((P \vee \neg Q) \rightarrow \neg r) \wedge r \right] \rightarrow \neg P$

to go through the Truth Table

P	Q	r	$\neg P$	$\neg Q$	$\neg r$	$(P \vee \neg Q)$	$(P \vee \neg Q) \rightarrow \neg r$	a $\wedge r$	b $\rightarrow \neg P$
T	T	T	F	F	F	T	F	F	T
T	T	F	F	F	T	T	T	F	T
T	F	T	F	T	F	T	F	F	T
T	F	F	F	T	T	T	T	F	T
F	T	T	T	F	F	F	T	T	T
F	T	F	T	F	T	F	T	F	T
F	F	T	T	T	F	T	F	F	T
F	F	F	T	T	T	T	T	F	T

∴ Since the last column all are T (True), Therefore the Argument is Validity.

**Extract 7.1:** A sample of the student's responses to question 7

In Extract 7.1, the student managed to construct the correct truth tables for the given propositions in part (a). Also, in part (b), the student was able to identify the appropriate logical connectives and managed to write the correct symbolic statement which used to test the validity of the given argument.

Despite the strengths demonstrated by most of the students, the analysis shows that some students were unable to respond to the question correctly and scored low marks due to various reasons as explained below:

In part (a) (i), the students failed to recognize that the required truth table had 5 columns and 5 rows. Instead, they constructed the truth table with 5 columns and 6 rows. Moreover, other students were unable to identify the correct entries to be entered in the truth table. For example, one student entered wrong entries in the first column, as seen in extract 7.2. Furthermore, some students failed to understand the meaning of the sign ' $\rightarrow$  (implies)' because they wrote  $T \rightarrow T = F$  and  $F \rightarrow F = F$  in their responses.

Also, in part (a) (ii), the analysis revealed that some students encountered challenges in determining the correct number of columns and rows required in their responses. They constructed truth tables with 6 columns and 6 rows instead of a truth table with 7 columns and 5 rows. Likewise, other students failed to enter the correct truth values in the truth table.



In part (b), some students misinterpreted the statement “If it rains or no one comes” interpreting “If it rains” =  $p$  and “If it does not rain” =  $q$  while “if it does not rain” is the negation of  $p$ . This implies that the student had inadequate knowledge on the concept tested. Furthermore, there were students who correctly let  $p$  = “if it rain”,  $q$  = “the game will take place” and  $r$  = “the game was a success” but incorrectly formed the proposition as  $(p \vee \sim p) \wedge \rightarrow r \wedge \sim p$ .

When testing the validity of the arguments by using truth tables, some students constructed truth tables with the wrong number of columns and rows instead of 9 columns and 9 rows. Extract 7.2 provides a sample response selected from one of the students who incorrectly responded to the question.

(a) Construct a truth table for each of the following propositions:

(i)  $(p \wedge q) \rightarrow (p \vee q)$

(ii)  $(\sim p \wedge (p \rightarrow q)) \rightarrow \sim q$

1)

$p$	$q$	$p \wedge q$	$p \vee q$	$(p \wedge q) \rightarrow (p \vee q)$
T	F	F	F	F
F	T	F	F	F
F	F	F	F	F
F	F	F	T	F
T	T	T	T	T

1)

$p$	$q$	$\sim p \wedge$	$p \rightarrow q$	$\rightarrow \sim q$	$(\sim p \wedge (p \rightarrow q)) \rightarrow \sim q$
T	F	T	F	F	F
F	T	F	F	T	F
T	F	T	F	F	F
F	F	F	T	F	F
F	F	F	F	T	F
T	T	T	T	T	T

(b) Use a truth table to test validity of the following argument: "If it rains or no one comes, the game will not take place. The game was a success. Therefore, it did not rain".

If it rain let be M  
If it not rain let be x

$MAX < MX \vee M$

M	x	MAX	$<MX$	$\vee M$	$MAX < MX \vee M$
T	F	F	F	F	F
F	T	F	F	F	F
F	F	T	F	F	F
F	F	F	T	F	F
T	F	F	F	T	F
T	T	T	T	T	T

**Extract 7.2:** A sample of the student's responses to question 7

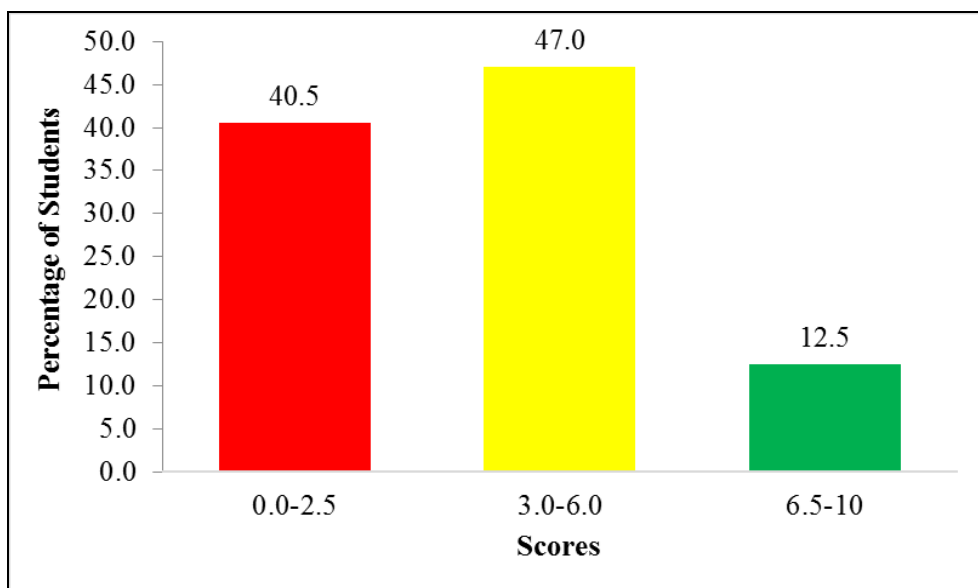
In Extract 7.2, the student failed to realize the required columns and rows of the truth table in part (a), while in part (b), the student misinterpreted the given statement.

## 2.8 Question 8: Variations

This question comprised two parts, (a) and (b). In part (a), the students were given that quantity  $x$  is jointly proportional to the quantity  $y$  and the reciprocal of quantity  $z$ . If  $x=3$  where  $y=2$  and  $z=1$ , then they were required in (i) to write down the equation connecting  $x$ ,  $y$  and  $z$  whereas in (ii) to determine the value of  $z$ , where  $x=6$  and  $y=20$ . In part (b), the question stated; Twelve men working for 10 hours per day take 4 weeks to plant maize in a certain farm. For how long should 20 men work per day in order to plant maize in the same farm for 14 weeks ?

The analysis of data shows that 398 (100%) students attempted this question. Out of them, 12.5% scored 6.5 to 10 marks while 47% of the students scored 3 to 6 marks and 40.5% of the students scored 0 to 2.5

marks. Generally, most students' performance on this question was average. The students' performance summary is presented in Figure 9.



**Figure 9:** Students' Performance on Question 8

The students who answered this question correctly had adequate knowledge of variations. In part (a) (i), the analysis of the data shows that the students were able to demonstrate the relation between direct and inverse variation.

They correctly interpreted  $x \propto y$  and  $x \propto \frac{1}{z}$ . Then, they combined it to

obtain  $x \propto \frac{y}{z}$ . Thereafter, they removed the proportionality symbol and got

$x = \frac{ky}{z}$ , where  $k$  is the proportionality constant. Following that, they

entered the values of  $x = 3$ ,  $y = 2$  and  $z = 1$  into the equation  $x = \frac{ky}{z}$  and

obtained the value of  $k = \frac{3}{2}$ . Finally, they were able to write the equation

connecting  $x$ ,  $y$  and  $z$  as  $x = \frac{3y}{2z}$ . In part (a) (ii), the students correctly

substituted the values of  $x = 6$  and  $y = 20$  to the equation  $x = \frac{3y}{2z}$ . That is,

they wrote  $6 = \frac{3 \times 20}{2 \times z}$  then simplified and obtained the value of  $z = 5$ .

In part (b), according to the analysis, the students correctly interpreted the word problem and managed to formulate the variation model  $M \propto \frac{1}{t}$ . Where  $M$  stood for men and  $t$  for time. Thereafter, they introduced the proportionality constant  $k$  and obtained the equation  $k = M \times t$ . Then, they substituted the values of  $M = 12$  and  $t = 10$ , which resulted in  $k_1 = 12 \times 10 \times 4$  and  $k_2 = 20 \times y \times 2$ . Since  $k_1 = k_2$ , they then simplified to obtain the value of  $y = 12$ . Finally, they concluded that they had to work for 12 hours a day. Extract 8.1 is a sample of a response from one of the students who responded correctly to this question.

8. (a) Quantity  $x$  is jointly proportional to quantity  $y$  and the reciprocal of quantity  $z$ . If  $x = 3$

when  $y = 2$  and  $z = 1$ ;

- (i) write down the equation connecting  $x$ ,  $y$  and  $z$ .
- (ii) determine the value of  $z$ , when  $x = 6$  and  $y = 20$ .

Method

$$x \propto y \propto \frac{1}{z}$$

$$x \propto \frac{y}{z}$$

$$x = k \frac{y}{z}$$

$$\frac{xz}{y} = k \quad \Rightarrow \quad k = \frac{xz}{y}$$

$$k = \frac{3 \times 1}{2}$$

$$k = \frac{3}{2}$$

$$(i) \quad x = \frac{3y}{2z}$$

$$\therefore \text{The equation is } x = \frac{3y}{2z}$$

$$\begin{aligned}
 x &= \frac{3 \times y}{z} \quad \text{where } z = ? \quad , x = 6, y = 20 \\
 6 &= \frac{3 \times 20}{z} \rightarrow 6 = \frac{30}{z} \\
 6z &= 30 \\
 \frac{6z}{6} &= \frac{30}{6} \\
 z &= 5
 \end{aligned}$$

∴ The value of  $z = 5$

- (b) Twelve men working for 10 hours per day take 4 weeks to plant maize in a certain farm. For how long should 20 men work per day in order to plant maize in the same farm for 14 days?

Method

let  $M$  - men

$h$  - hour

$w$  - week

$$m \propto h \propto \frac{1}{w}$$

$$m \propto \frac{h}{w}$$

$$m = \frac{khw}{hw}$$

$$m = \frac{k}{hw}$$

$$k = mhw$$

$$k = 12 \times 10 \times 4$$

$$k = 120 \times 4$$

$$k = 480$$

$$480 = 20 \times 2 \times h$$

$$\frac{480}{40} = \frac{40h}{40}$$

$$h = 12 \text{ hours}$$

∴ It takes 12 hours

**Extract 8.1:** A sample of the student's responses to question 8

In Extract 8.1, the student demonstrated correctly the relation between direct and inverse variation in part (a). Also, in part (b), the student correctly interpreted the word problem and managed to calculate the working time required.

On the other hand, the analysis of data shows that 40.5 per cent of students who attempted the question scored 0 to 2.5 marks as they failed to answer this question correctly due to various challenges.

In part (a) (i), some students were unable to transform and interpret the given variation statement into mathematical models. For example, one among the students responded by adding up the given variables to obtain the expression  $x + y + z$  and later substituted  $x = 3$ ,  $y = 2$  and  $z = 1$  and got  $3x + 2y + z$  instead of  $x = \frac{3y}{2z}$ . Apart from misinterpreting the problem, the analysis shows that the student also lacked computational skills. Furthermore, other students failed to correctly interpret the given mathematical problem such as  $x \propto \frac{1}{yz}$ . Then, they introduced the constant  $k$  to get  $x = \frac{k}{yz}$ , which led to  $k = xyz$ . Thereafter, they substituted  $x = 3$ ,  $y = 2$  and  $z = 1$  to get  $k = 3 \times 2 \times 1 = 6$ . Then, they ended up with the incorrect equation  $x = \frac{6}{yz}$ .

In part (a) (ii), some of the students substituted  $x = 6$ ,  $y = 20$  and  $k = 6$  into incorrectly formulated equations as  $zk = 6 + 20$  to obtain  $6z = 14$ , and later simplified it to get  $z = \frac{7}{3}$  instead of 5. Likewise, another student applied the obtained equation  $k = xyz$  and substituted  $k = 6$ ,  $x = 6$  and  $y = 20$  to obtain  $6 = 6 \times 20 \times z$  which was then simplified to  $6 = 120z$ . Finally they obtained  $z = \frac{1}{20}$  instead of  $z = 5$ .

In part (b), a few students incorrectly perceived the relationship as directly proportional rather than inversely proportional, resulting in the incorrect

	Men	Hours	Week	
approach. One student considered	12	10	4	followed by cross
	20	?	2	

multiplication with wrong computations to obtain  $20 \times 4 \times x = 12 \times 10 \times 4$ .

Thereafter, he/she simplified to obtain  $80x = 240$  and finally obtained 3 hours instead of 12 hours.

While other students applied the same approach as they considered the variation statement as direct variation and assumed the number of workers to be constant as 20, which made them obtain an incorrect value of  $x$ . For

Men   Hours   Week

instance, one wrote

20	10	4
20	14	$x$

and after simplifying by cross

20   14    $x$

multiplication, they obtained  $20 \times 14 \times 4 = 20 \times 10 \times x$ . Further simplification

resulted in  $x = \frac{28}{5} = 5.6$  weeks. Extract 8.2 provides a sample of a response

from one of the students who failed to answer the question correctly.

8. (a) Quantity  $x$  is jointly proportional to quantity  $y$  and the reciprocal of quantity  $z$ . If  $x = 3$  when  $y = 2$  and  $z = 1$ ;
- (i) write down the equation connecting  $x$ ,  $y$  and  $z$ .
  - (ii) determine the value of  $z$ , when  $x = 6$  and  $y = 20$ .

(i) Soln

$$x = 3 \quad y = 2 \quad z = 1$$

$$3x + 2y + z$$

$$x, y, z = 3x + 2y + z$$

$$\therefore 3x + 2y + z$$

Hence equation is  $3x + 2y + z$

(ii) Soln

$$z = ? \quad x = 6 \quad y = 20$$

$$x = 6$$

$$2x = 6 + 20$$

$$\frac{6x}{6} = \frac{14}{6} \frac{7}{3}$$

$$\therefore z = \frac{14}{6} \frac{7}{3}$$

Hence the value of  $z$  is  $\frac{7}{3}$

- (b) Twelve men working for 10 hours per day take 4 weeks to plant maize in a certain farm. For how long should 20 men work per day in order to plant maize in the same farm for 14 days?

Soln

Men	hours	Weeks
12	10	4
20	?	2

$$20 \times 4 \times x = 12 \times 10 \times 4$$

$$80x = 120 \times 4$$

$$\frac{80x}{80} = \frac{240}{80}$$

$$x = 3$$

∴ he should takes 3 hours.

12	10	4
20	3	2

Hence they takes 3 hours to plant maize

**Extract 8.2:** A sample of the student's responses to question 8

In Extract 8.2, the student responded incorrectly by adding up the given variables in part (a), while in part (b), the student incorrectly perceived the relationship be direct proportional.

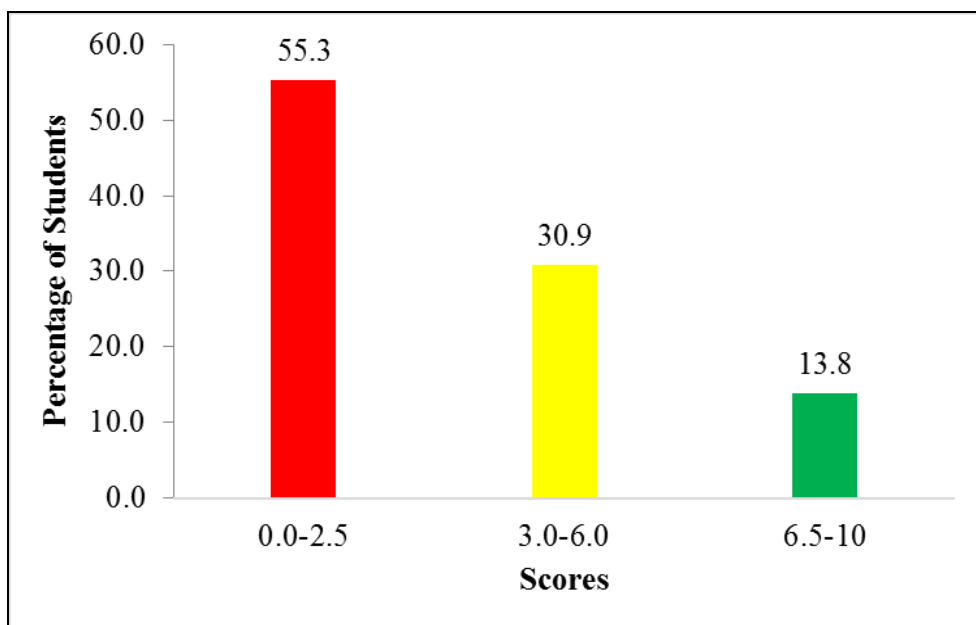
## 2.9 Question 9: Coordinate Geometry

The students were asked to draw the graphs of  $y = x^2 - 7x + 18$  and  $y = x + 2$  for  $-1 \leq x \leq 6$  on the same  $xy$ -plane, hence using the drawn graphs to find the common solution of the given equations.

The analysis of data shows that out of 398 students who attempted the question, 44.7 per cent scored 3 to 10 marks. Among them, 13.8 per cent managed to score 6.5 to 10 marks. The general performance of students in



this question is average. The summary of students' performance is presented in Figure 10.



**Figure 10:** *Students' Performance on Question 9*

The analysis revealed that the students who responded to the question correctly had sufficient knowledge of coordinate geometry as they managed to prepare correctly the table of values for both  $y = x^2 - 7x + 18$  and  $y = x + 2$  as follows:

Table of values for  $y = x^2 - 7x + 18$

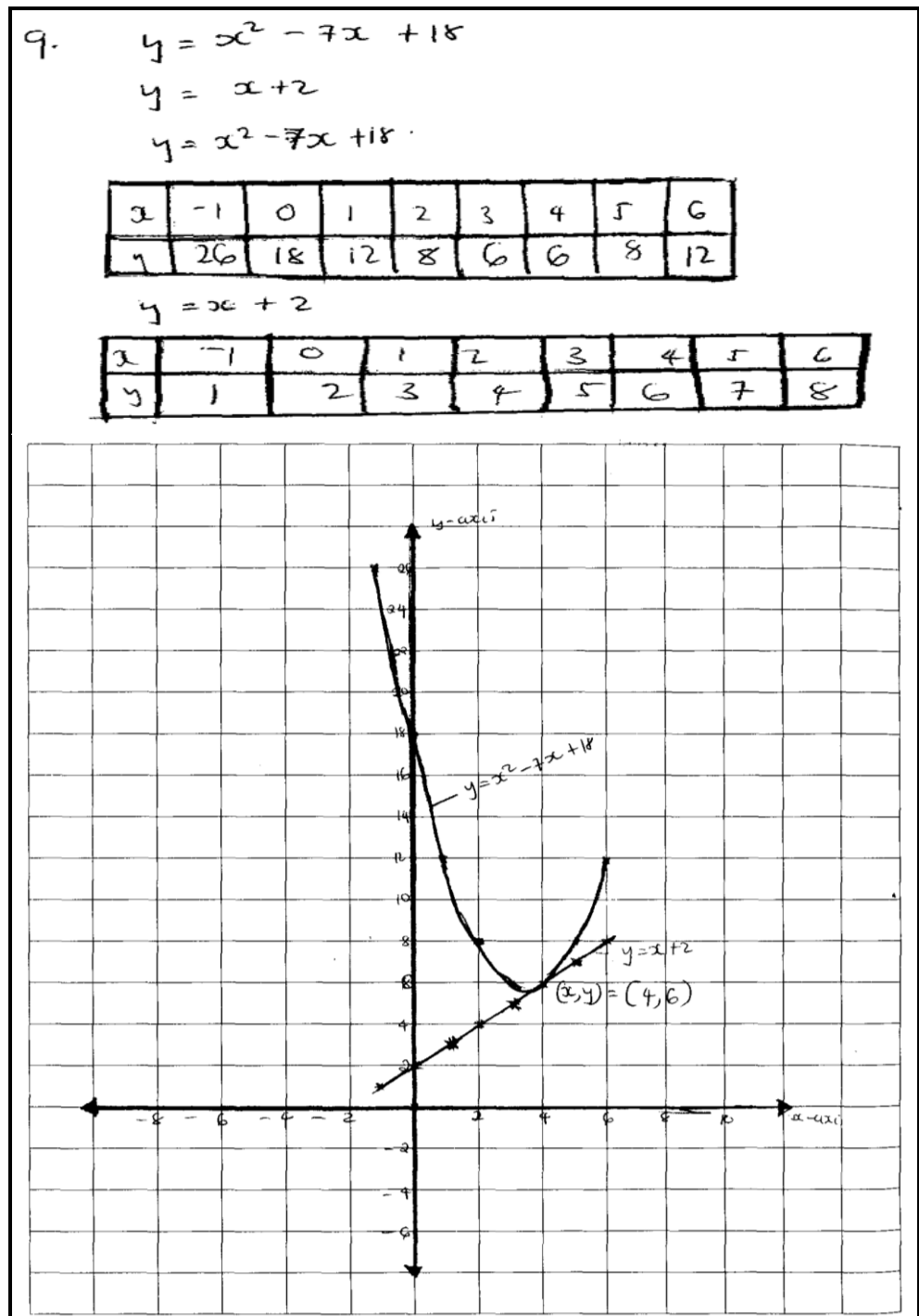
$x$	-1	0	1	2	3	4	5	6
$y$	26	18	12	8	6	6	8	12

Table of values for  $y = x + 2$

$x$	-1	0	1	2	3	4	5	6
$y$	1	2	3	4	5	6	7	8

Thereafter, they applied the ordered pairs obtained from each table of values to plot the graphs. Finally, they used the graphs to determine the common solutions of the given equations as  $(4, 6)$ . Extract 9.1 provides a sample of

a solution from one of the students who performed well on this question and scored all marks.



Extract 9.1: A sample of the student's responses to question 9

In Extract 9.1, the student managed to prepare correctly the table of values for both equations and finally drew the graph of the given functions and correctly identified the common solution.

Despite the strengths demonstrated by some students, there were those who were unable to respond correctly to the question due to some difficulties. Such as, some students failed to construct and applied a correct table of values. Instead, they substituted the wrong values of  $x$ . For example, one student constructed the following table of values.

$x$	-3	-2	-1	0	1	2	3
$y$	8	8	12	18	12	8	8

Thereafter, he/she plotted a graph by using a set of ordered pairs obtained from the table and finally determined the wrong point of intersection. Furthermore, the analysis shows that there were students who applied the concepts of intercept incorrectly to construct the table of values of the given equations, later they used the obtained points to plot the graph. For example, one student did as follows:

For  $y = x^2 - 7x + 18$

and

For  $y = x + 2$

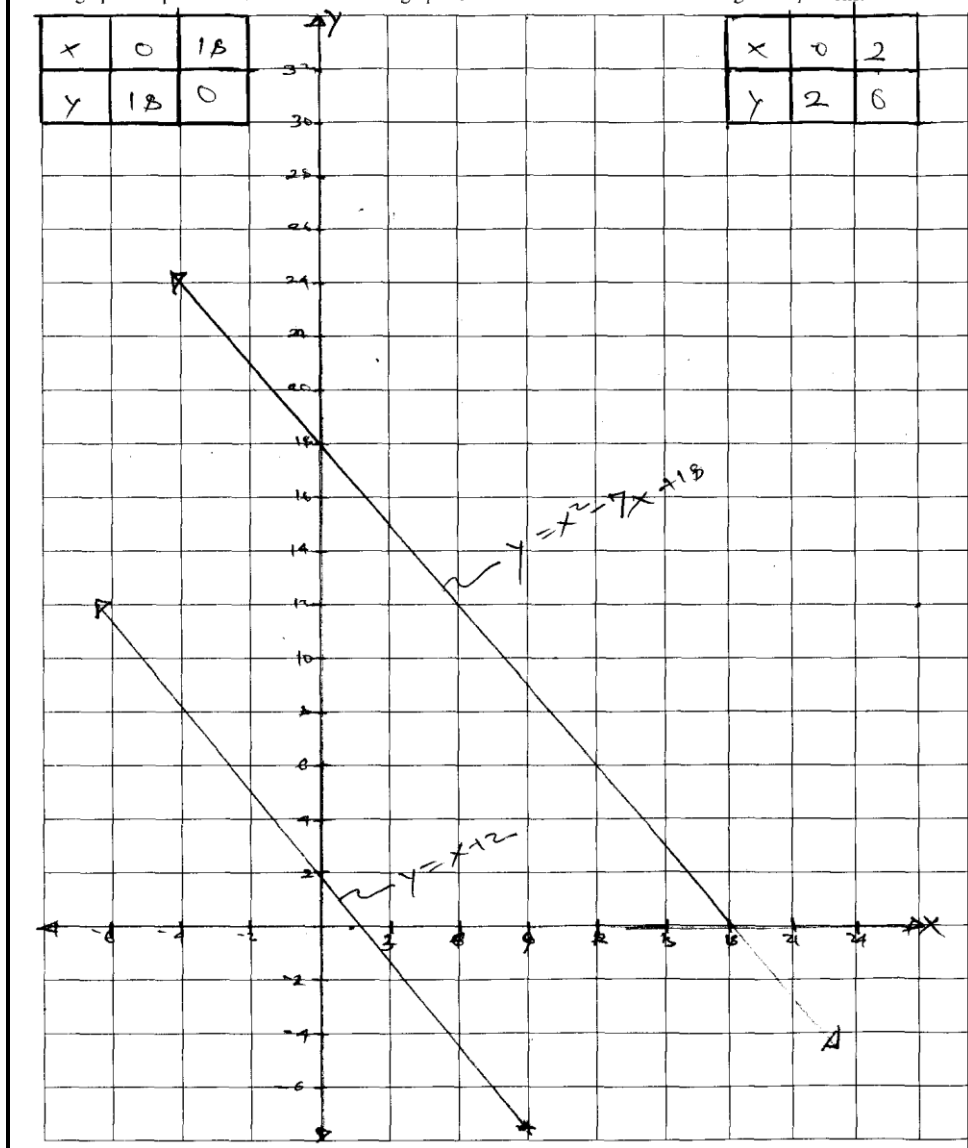
$x$	0	18
$y$	18	0

$x$	0	2
$y$	2	0

Then, he/she applied the obtained points from the table of values to draw the graphs and finally used the incorrectly drawn graphs to determine the point of intersection.

On the other hand, there were other students who misinterpreted the given equations as linear, thus drawing the graphs which are parallel. Then, they failed to obtain the point of intersection. Extract 9.2 provides a sample of a response selected from one of the students who responded to the question incorrectly.

9. Draw the graphs of  $y = x^2 - 7x + 18$  and  $y = x + 2$  for  $-1 \leq x \leq 6$  on the same  $xy$ -plane in the following graphical space. Hence use the obtained graph to find the common solution of the given equations.



**Extract 9.2:** A sample of the student's responses to question 9

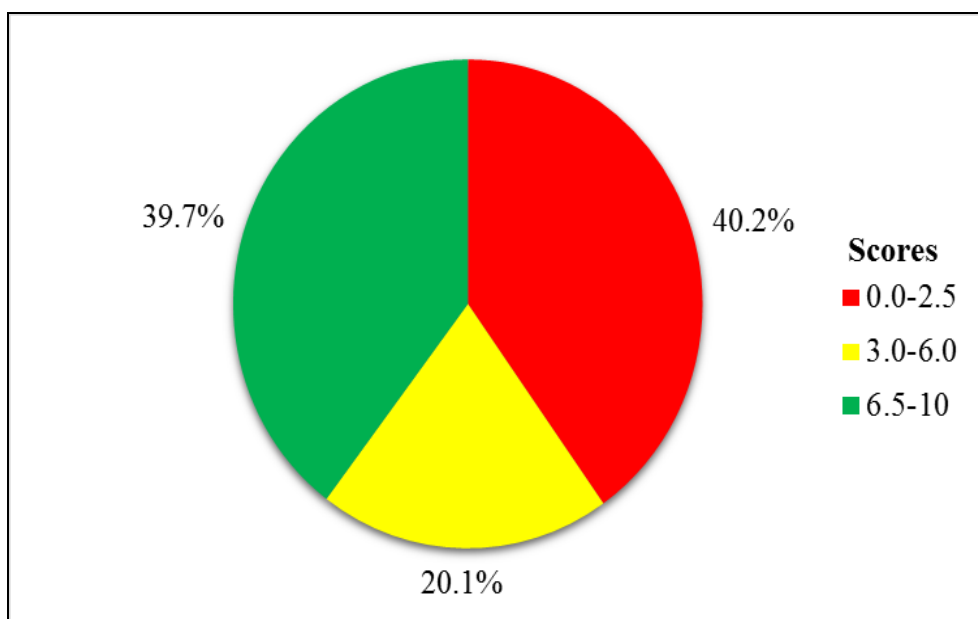
In Extract 9.2, the student was unable to construct and apply the table of values to draw the graph, hence he/she failed to get a common solution.

## 2.10 Question 10: Sets

This question stated as follows: In a class of 63 students, 22 students study Biology, 26 study Chemistry, 25 study Physics, 18 study both Physics and Chemistry, 4 study both Biology and Chemistry, 3 study both Physics and Biology and 1 student studies all the three subjects. (a) They were required to represent this information using a Venn diagram and (b) to use the results of part (a) to determine the number of students who study:

- (i) Biology only.
- (ii) Physics or Chemistry.
- (iii) None of the three subjects.
- (iv) Physics but not Chemistry.

The analysis of data shows that out of all students who attempted this question, 59.8 per cent scored 3 to 10 marks. The analysis also shows that 40.2 per cent of the students scored 0 to 2.5 marks and 20.1 per cent scored 3 to 6 marks. It was also noted that 6 (1.5%) students scored zero while 24 (6%) students scored 10 marks.



**Figure 11:** *Students' Performance in Question 10*

From Figure 11 which presents the summary of performance, it is observed that the students' performance in this question was generally average. In this question, a large number of students proved their ability in solving questions involving three sets. In part (a), competent students managed to use the given data to draw a Venn diagram that involved three sets, namely Biology

(B), Chemistry (C) and Physics (P) and correctly indicated the number of students in the respective sets.

In part (b), the students used the results of part (a) to determine the number of students who study;

(i) Biology only = 16

(ii) Students who study Physics or Chemistry =  $5 + 3 + 1 + 17 + 5 + 2 = 33$

(iii) None of the three subjects. They considered students who study science subjects =  $5 + 16 + 5 + 2 + 17 + 3 + 1 = 49$ . Then, subtracted 49 from the total number of students in a class which is 63, that is  $63 - 49 = 14$ . Finally, they concluded that the number of students for none of the three subjects = 14.

(iv) Physics but not Chemistry =  $2 + 5 = 7$

Extract 10.1 provides a sample of a response selected from one of the students who answered the question correctly.

10. In a class of 63 students, 22 study Biology, 26 study Chemistry, 25 study Physics, 18 study both Physics and Chemistry, 4 study both Biology and Chemistry, 3 study both Physics and Biology and 1 student studies all three subjects.

(a) Represent this information using a Venn diagram.

Solution

Venn diagram.

(b) Use the result of part (a) to determine the number of students who study:

(i) Biology only.	i) <u>16 students study biology only</u>
(ii) Physics or Chemistry.	ii) <u>33 students study physics or chemistry</u>
(iii) None of the three subjects.	iii) <u>14 students study none of the three subjects</u>
(iv) Physics but not Chemistry.	iv) <u>7 students</u>

**Extract 10.1:** A sample of the student's responses to question 10

In Extract 10.1, the student managed to use the given data to draw a Venn diagram that involved three sets in part (a). Also, in part (b), the student correctly used the results of part (a) to determine the required values.

In spite of the good responses from most of the students, there were other students who were unable to respond to the question correctly and had scored low marks. Those students faced the following difficulties;

In part (a), basically the students were required to perform operations to determine the exact number to be entered in every region, but they transformed the data without making any basic operations, and as a result they entered incorrect data in the regions of  $(P \cap C)$ ,  $(B \cap C)$  and  $(P \cap B)$ . Moreover, other students lacked drawing skills for the Venn diagram of sets, thus they were not able to draw the Venn diagram correctly.

In part (b), since the students failed to respond correctly to part (a), all the difficulties they faced led to incorrect responses in this part. Some responded in (b) (i) as Biology only =  $22 + 3 + 1 + 4 = 30$  then took  $63 - 30 = 33$  instead of 16.

Also, in part (b) (ii), to determine Physics or Chemistry they took  $70 - 63 = 13$  instead of  $5 + 3 + 1 + 17 + 5 + 2 = 33$ . In part (b) (iii), for none of the three subjects, they added  $22 + 3 + 1 + 4 + 18 + 26 + 25 = 99$  despite the fact that they were supposed to respond by considering students who study science subjects and subtracting from the total number of students.

In part (b) (iv), for Physics but not Chemistry; they added  $18 + 25 = 43$  instead of  $2 + 5 = 7$ . Further analysis shows that other students responded in part (b) as follows:

(i) Biology only =  $22 - 4 - 3 - 1 = 14$

(ii) Physics or Chemistry =  $25 + 26 = 51$

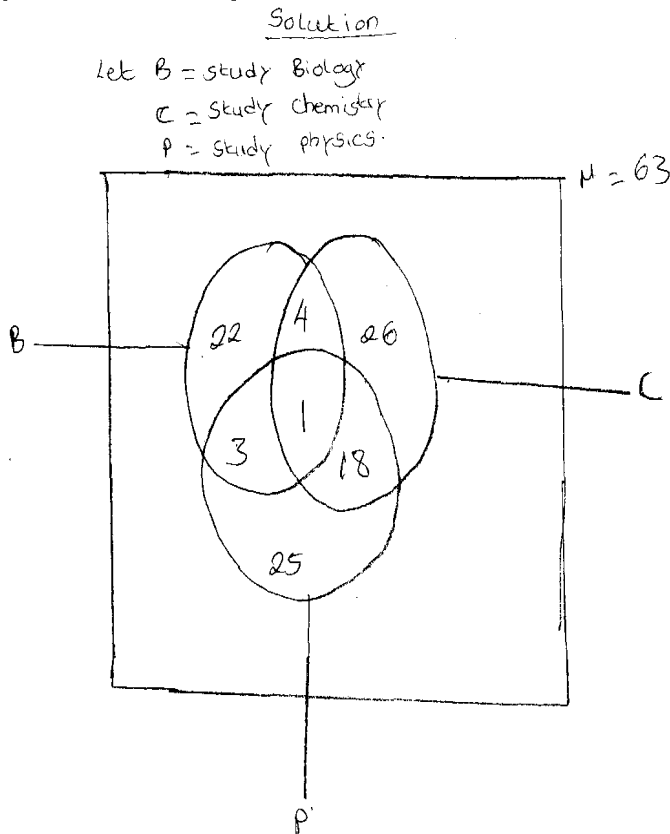
(iii) None of the three subjects = 1

(iv) Physics but not Chemistry =  $25 + 3 + 1 = 29$

Extract 10.2 provides a sample of a response from one of the students who were not able to answer the question correctly.

10. In a class of 63 students, 22 study Biology, 26 study Chemistry, 25 study Physics, 18 study both Physics and Chemistry, 4 study both Biology and Chemistry, 3 study both Physics and Biology and 1 student studies all three subjects.

(a) Represent this information using a Venn diagram.



(b) Use the result of part (a) to determine the number of students who study:

- (i) Biology only.
- (ii) Physics or Chemistry.
- (iii) None of the three subjects.
- (iv) Physics but not Chemistry.

Solution

∴ Biology only  
 $n(B) = 22$   
 $n(U) = 63$   
 $n(B \cup C) = 41$



$$63 = 22 + 4 + 1$$

$$63 = 27$$

$$= 27$$

∴ The student who study biology only is 27

(ii) Physics or chemistry

solution

$$25 + 18 + 26 + 3 + 4 + 1$$

$$51 + 3 + 4 + 1$$

$$54 + 4 + 1$$

$$59$$

∴ The students who study physics or chemistry are 59

(iii) None of the three subjects

solution

$$22 + 4 + 26 + 3 + 1 + 18 + 25 = 63$$

$$26 + 26 + 3 + 1 + 18 + 25 = 63$$

$$55 + 1 + 18 + 25 = 63$$

$$74 + 25 = 63$$

$$99 \Rightarrow 63$$

$$36$$

∴ The students none of the three subjects is 36

(iv) Physics but not chemistry.

solution.

**Extract 10.2:** A sample of the student's responses to question 10

In Extract 10.2, the student transferred the data to Venn diagram wrongly without considering what the question required them to do in part (a). While the wrong response in part (a) affected the response of part (b).

### 3.0 ANALYSIS OF THE STUDENTS' PERFORMANCE ON EACH TOPIC

The Additional Mathematics paper consisted of 10 questions from nine (9) topics, which are *Numbers*, *Algebra*, *Geometrical constructions*, *Locus*, *Coordinate geometry*, *Symmetry*, *Logic*, *Variations* and *Sets*. The analysis of students' performance per topic in 2022 indicated that two (2) topics were well performed. The topics on which students had good performance were *Symmetry* (87.5%) and *Algebra* (70.6%). The students' good performance on these topics was attributed to sufficient knowledge and abilities in the subject areas, having the right competences to employ concepts in answering questions, as well as correctly and accurately interpreting the questions.

On the other hand, there were six (6) topics in which students had average performance including *Coordinate Geometry* (61.8%), *Sets* (59.8%), *Variations* (59.5%), *Logic* (59.0%), *Numbers* (55.2%) and *Geometrical Constructions* (55.0%). The students' average performance on those topics was attributed to their moderate abilities in computation and knowledge of the subject matter.

Further analysis shows that students had poor performance on one (1) topic; *Locus*. In this topic only 21.6 per cent performed well. The poor performance was largely attributed to learners insufficient knowledge and abilities to describe a locus of a point as well as incorrectly interpreting the question. The analysis of the students' performance for each topic is shown statistically in appendix I.

The analysis of students' performance per topic in 2021 shows that five (5) topics were well performed, three (3) topics were averagely performed and one (1) was poorly performed. The data show that the performance of students increased in three (3) topics of *Symmetry* (0.1%), *Locus* (19.1%) and *Coordinate Geometry* (31.6%) for 2022 as compared to 2021. However, the topic of *Numbers* appeared to have the highest decrease of students' performance. The comparison of performance per topic for 2021 and 2022 is represented in appendix II.

## **4.0 CONCLUSION AND RECOMMENDATIONS**

### **4.1 Conclusion**

The analysis showed that the general performance on Additional Mathematics Assessment in 2022 was good (76.88%). The performance has improved by 1.47 per cent compared to that of 2021. Analysis of performance in different topics has shown that there were six (6) topics having average performance and one (1) having poor performance in 2022. The reasons that contributed to poor performance on the topics include misinterpretation of word problems, poor computation skills, failure to understand the questions asked, lack of knowledge and skills of the tested concepts and failure to recall the appropriate formulae.

In general, the report has shown the key areas where the students demonstrated their strengths and weaknesses. It is anticipated that this analysis will be useful to education stakeholders in light of their position, and that the recommendations presented in this report will aid in raising students' performance on the upcoming Additional Mathematics assessments.

### **4.2 Recommendations**

In order to raise the standard of performance in this subject, the following recommendations are made:

- (a) Since students showed lack of competence in some areas of the assessment, it is recommended that students be advised to read more books and do enough exercises in order to improve their competence in Additional Mathematics.
- (b) Since some students performed poorly in some areas, it might be a signal that they were not adequately taught. It is therefore recommended that, the teachers should be committed to their teaching profession, teach according to the syllabus and cover all topics on time to give learners ample time to practise.
- (c) Teachers should constantly evaluate their students in order to spot areas of weakness and offer them appropriate help for their learning. Some weakness spotted were indicative of lack of guidance to the students.
- (d) The quality assurance officers should make thorough follow up in schools, focusing mainly on the effective and timely implementation

of the syllabus. That is recommended because poor performance might be a sign of lack of supervision.

**APPENDIX I: Students' Performance on Each Topic in 2022**

S/N	Topic	Question Number	Percentage of Students who Scored an Average of 30% or Above	Remarks
1.	Symmetry	6	87.5	Good
2.	Algebra	2	70.6	Good
3.	Coordinate Geometry	5 & 9	61.8	Average
4.	Sets	10	59.8	Average
5.	Variations	8	59.5	Average
6.	Logic	7	59.0	Average
7.	Numbers	1	55.2	Average
8.	Geometrical Constructions	3	55.0	Average
9.	Locus	4	26.1	Weak

## APPENDIX II: Comparison of Students' Performance in 2021 and 2022

S/N	Topic	2021			2022		
		Question Number	Percentage of Students who Scored an Average of 30% or Above	Remarks	Question Number	Percentage of Students who Scored an Average of 30% or Above	Remarks
1.	Symmetry	6	87.4	Good	6	87.5	Good
2.	Algebra	2	81.6	Good	2	70.6	Good
3.	Coordinate Geometry	5	30.2	Average	5 & 9	61.8	Average
4.	Sets	9	63.2	Average	10	59.8	Average
5.	Variations	8 & 10	71.1	Good	8	59.5	Average
6.	Logic	7	63.0	Average	7	59.0	Average
7.	Numbers	1	93.4	Good	1	55.2	Average
8.	Geometrical Constructions	3	80.0	Good	3	55.0	Average
9.	Locus	4	07	Poor	4	26.1	Weak

